Errata for “Introduction to the Mathematical Theory of Control”

by A. Bressan and B. Piccoli, AIMS series on Applied Mathematics, 2007

Chapter 2

p.17. In (2.12), the last integral should be
\[ \int_{t_0}^t \beta(s) \exp \left( \int_s^t \alpha(\sigma) \, d\sigma \right) \, ds. \]

p.21. In the formula on line 17, the last term should be \( e^{C(t-t_0)} - 1 \).

p.24, line 14. “the function \( x(t) = M(t, s)\xi \) provides the solution to the Cauchy problem”.

Chapter 3

p.48, line 9. Should be:
\[ |f(t, x, u) - f(t, x', u)| \leq L|x - x'|. \]

p.49, line 6. “in a finite number of steps”

p.49, formula (3.35) should be
\[ \langle f(t_i, \tilde{x}(t_i), u_i), p_i \rangle > \langle f(t_i, \tilde{x}(t_i), \tilde{u}(t_i)), p_i \rangle - \frac{\varepsilon L}{3}. \]

p.55. The formula on line 14 should be:
\[ t_1 = \inf \{ t' < \tau; \phi(x(t)) \leq t + \varepsilon(\tau - t) \text{ for all } t \in [t', \tau] \}, \]
removing the factor 2.

p.65. The equation on line 4 from bottom should be
\[ \dot{x} \in F^\sharp(t, x) = \text{co}F(t, x). \]
Chapter 4

p.75, line 7: “... to design a feedback control $u = u(x)$...”

p.82. The equation on line 8 should be

$$\dot{x} = (A + B\bar{F})x + v_1u, \quad u \in \mathbb{R}.$$  

Chapter 5

p.92. Equation (5.11) should be

$$\dot{x}(t) = A(t) \cdot x(t) + h(t, u(t)),$$

p.94. A comma is missing in equation (5.19). It should be:

$$F^+(t, x) = \{(y_0, y) \in \mathbb{R}^{1+n}; \ y_0 \geq L(t, x, \omega), \ y = f(t, x, \omega) \text{ for some } \omega \in U\}$$

Chapter 6

p.106. Line 5 from bottom should be:

$$p_1(t) = \cos(T - t), \quad p_2(t) = \sin(T - t).$$

p.106. Equation (6.22), should be:

$$u^*(t) = \text{sign}(p_2(t)) = \text{sign}(\sin(T - t))$$

p.111, the last formula should be: $\dot{v}(t) = D_x f(t, x^*(t), u^*(t)) v(t)$.

p.115, in formula (6.47), all three limits should have the form $\lim_{\varepsilon \to 0^+} \frac{1}{\varepsilon} \int \cdots$

p.119. The statement above (6.72), should be: ” This yields a minimization problem ”.

p.128, problem 6.3, lines 8, 10, 11. Replace with $(x_1, x_2)(0) = (0, -1)$. Then $\tilde{y} = (b, 1)$, and $\hat{y} = (x_1, 1)$. (Notice that in this second case the coordinate $x_1$ is free).
Chapter 7

p.133. To be consistent with (7.3), the integral in (7.1) should be \( \int_t^T \).

p.138. Theorem 7.3.1, part (ii): “then \( V(t, x(t, u^*)) \) is constant for \( t \in [t_0, T] \).”

p.140. line 6. In Step 3. of the proof, the equation should be:

\[
\inf_{\omega \in U} \left\{ \partial_
u V(t, x(t, u^*)) + \nabla_y V(t, x(t, u^*)) \cdot f(t, x(t, u^*), \omega) \right\} \leq \frac{1}{\nu}.
\]

p.140, equation (7.22) should be

\[
V(s, y) = \inf_{u \in U} \left\{ \int_s^T L(t, x(t), u(t)) \, dt + \psi(T, x(T)); \ x(s) = y \right\}.
\]

p.147, line 12, should be “\( y_1 > \phi(y_2) \).”

Chapter 8

p.177. Equation (8.19) should be

\[
F(x_\varepsilon, u_\varepsilon(x_\varepsilon), \nabla \psi(x_\varepsilon)) \leq \varepsilon \Delta \psi(x_\varepsilon).
\]

p.180, lines 6–8 from bottom:

\[
\Phi_\varepsilon(x, y) \leq 0 \quad \text{if } |x - y|^2 \geq 4M\varepsilon.
\]

Hence (8.28) implies

\[
|x_\varepsilon - y_\varepsilon| \leq 2\sqrt{M\varepsilon}.
\]

p.182. Equation (8.42) should be:

\[
\phi_t(t_\varepsilon, x_\varepsilon) + H(t_\varepsilon, x_\varepsilon, \nabla \phi(t_\varepsilon, x_\varepsilon)) \leq -\frac{\varepsilon}{(T - t_\varepsilon)^2}.
\]

p.184, line 4: “takes a minimum at the point \( (t_\varepsilon, x_\varepsilon) \).”

p.188, lines 11-12: “hence, by (8.54) we deduce

\[
J(s, y', u) \leq J(s, y, u) + \int_s^T C e^{C(t-s)} |y' - y| \, dt + C e^{C(T-s)} |y - y'|.
\]
p.188. Equation (8.64) should be:

\[ V(s, y') \leq V(s, y) + \varepsilon + (C + 1)e^{CT}|y - y'|. \]

p.188. In the first two equations in Step 3, the right hand side should contain \( + \ C(s' - s) \), rather than \( -C(s' - s) \).

p.192. The second equation in (8.83) should be: \( |h(x, u) - h(y, u)| \leq L|x - y| \).

**Appendices**

p.281. In the equation (A.30) one should have: \( |u - u'| \leq \delta \varepsilon \).

p.281. Lines 16-17 should be: “By (A.30), the point \( y' = f(x', u) \in F(x') \) satisfies \( |y' - y| < \varepsilon \).”

p.282. Line 3 should be: \( \xi = (\xi_1, \cdots, \xi_n) \).

p.282. Line 8 from bottom should be: \( \varphi(t_k) = (v, y_k) \).

p.282. Line 6 from bottom should be: \( \bar{y} \in F(\tau) \).

p.288, line 5: “choose a sequence of points \( x_\nu \notin K \) with \( x_\nu \to w \).”