Nov. 7 Statistic for the day:
Percent of American adults who claim they voted in the 1994 election: 56%
Percent who actually voted: 39%

Assignment (same as last class): Read Chapter 20
Do exercises 1, 2, 4, and 6 on pages 382-383.

Rule of sample proportions (p. 359)
IF: 1. There is a population proportion of interest
   2. We have a random sample from the population
   3. The sample is large enough so that we will see at least five
     of both possible outcomes
THEN: if numerous samples of the same size are taken and the sample
proportion is computed every time, the resulting histogram will:
   1. be roughly bell-shaped
   2. have mean equal to the true population proportion
   3. have standard deviation equal to:
      \[
      \sqrt{\frac{\text{population proportion}}{\text{sample size}}} \times \frac{1}{1 - \text{population proportion}}
      \]

Rule of sample means (p. 363)
IF: 1. The population of measurements of interest is bell-shaped, OR
   2. A large sample (at least 30) is taken.
THEN: If numerous samples of the same size are taken and the sample
mean is computed every time, the resulting histogram will:
   1. be roughly bell-shaped
   2. have mean equal to the true population mean
   3. have standard deviation estimated by:
      \[
      \frac{\text{sample standard deviation}}{\sqrt{\text{sample size}}}
      \]

Example: SAT math scores
Suppose nationally we know that the SAT math test has a
mean of 100 points and a standard deviation of 100 points.
Draw by hand a picture of what you expect the distribution
of sample means based on samples of size 100 to look like.

Sample means have a normal distribution
Mean: 500
Standard deviation: \[
\frac{100}{\sqrt{100}} = \frac{100}{10} = 10
\]
So draw a bell shaped curve, centered at 500, with 95%
of the bell between 500 – 20 = 480 and 500 + 20 = 520

Back to proportions:
Suppose the true proportion is known
Recall that in craps, \( P(\text{win}) = \frac{244}{495} = 0.493 \) for each game.
Q: If you play 100 games of craps, what proportion will you win?
A: Dunno.

Q: Okay, I guess that was obvious. But can we at least give a range of possible proportions that will be
valid most (say, 95%) of the time?
A: Sure. Use the Rule of Sample Proportions.
Back to proportions (cont’d):
Suppose the true proportion is known
Recall that in craps, \( P(\text{win}) = \frac{244}{495} = 0.493 \) for each game.

The RoSP says that the sampling distribution of the proportion of wins in 100 games will be:

- roughly normal,
- with mean 0.493,
- and standard deviation \( \sqrt{\frac{0.493 \cdot 0.507}{100}} = 0.05 \)

Therefore, the 68-95-99.7 rule says that 95% of the time, the proportion of wins in 100 games will be between 0.493−2×0.05 and 0.493+2×0.05.

\[ \text{(0.393 and 0.593)} \]

Definition of "95% confidence interval for the true population proportion":
An interval of values computed from the sample that is almost certain (95% certain in this case) to cover the true but unknown population proportion.

The plan:
1. Take a sample
2. Compute the sample proportion
3. Compute the estimate of the standard deviation of the sample proportion:
   \[ \sqrt{\frac{\text{sample proportion} \cdot (1 - \text{proportion})}{\text{sample size}}} \]
4. 95% confidence interval for the true population proportion:
   \[ \text{sample proportion} \pm 2 \times \text{SD} \]

Other confidence coefficients
The confidence intervals we’ve been constructing (using ±2×stdev) are called 95% confidence intervals.

The confidence coefficient is 95%, or .95.

It means that we cover the middle 95% of the normal curve associated with sample proportions, and this requires 2 standard deviations.

We can change the confidence coefficient by using the normal table (p. 157) to determine the appropriate number of standard deviations.

Other confidence coefficients:
An example
Suppose we want a 90% confidence interval instead of 95%.

How many standard deviations span the middle 90% of the normal curve?

90% confidence interval (see p. 157)
Since 90% is in the middle, there is 5% in either end.

So find z for .05 and z for .95.

We get \( z = \pm 1.64 \)

90% confidence interval: sample proportion ± 1.64×stdev
Back to the original example of sample size 200, 14% left-handed in sample.

We found the sample stdev to be .025. Hence, to construct a 90% CI we have

\[ .140 - 1.64 \times .025 = .140 - .041 = .099 \]

\[ .140 + 1.64 \times .025 = .140 + .041 = .181 \]

90% confidence interval: .099 to .181

(95% confidence interval was: .090 to .190)

So the 90% CI is shorter (a more precise estimate) but less accurate. It has a 10% chance of missing the true population proportion.

Imagine that someone in your family has just handed a baby to you. Imagine yourself holding the baby.

With which arm are you holding the baby?

Study 1: monkeys

Salk observed 42 rhesus monkeys in Bronx Zoo holding babies. 40 held the baby on the left.

Suppose this is a sample of Rhesus monkeys. Find a 90% confidence interval for the proportion of monkey mothers who hold baby on left.

1. sample proportion: \( \frac{40}{42} = .95 \)
2. sample size: 42
3. standard deviation of sample proportion: .034
4. number of standard deviations for 90%: 1.64
5. 90% confidence interval: \( .95 \pm 1.64 \times .034 \)
\[ .895 \text{ to } 1 \]

Study 2: mothers both right and left handed

Of 255 right handed mothers, 83% held baby on left. They said it was more natural since it frees the right hand for doing things.

Of 32 left handed mothers, 78% held baby on left. They said it was better to hold baby in dominant arm.

Right handed: 98% confidence interval
1. sample proportion: .83
2. sample size: 255
3. standard deviation of the sample proportion: .024
4. number of standard deviations for 98%: 2.33
5. 98% confidence interval: \( .83 \pm 2.33 \times .024 \)
\[ .774 \text{ to } .886 \]

Left handed mothers 90% confidence interval:
1. sample proportion: .78
2. sample size: 32
3. standard deviation: .073
4. number of standard deviations for 90%: 1.64
5. 90% confidence interval: \( .78 \pm 1.64 \times .073 \)
\[ .66 \text{ to } .90 \]
Study 3: shoppers

Researchers loitered around a supermarket parking lot and recorded in which arm the shoppers carried their grocery bags.

Of 438 shoppers, 50% carried bags on left.

95% confidence interval:
1. sample proportion: .50
2. sample size: 483
3. standard deviation of sample proportion: .024
4. number of standard deviations for 95%: 2
5. 95% confidence interval: .50 ± 2(.024)
   .452 to .548

Study 4: paintings and sculpture

Of 466 paintings and sculpture of the Madonna and child, 80% held baby on left.

98% confidence interval:
1. sample proportion: .80
2. sample size: 466
3. standard deviation of sample proportion: .019
4. number of standard deviations for 98%: 2.33
5. 98% confidence interval: .80 ± 2.33(.019)
   .756 to .844