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On the characteristic function of the distribution of the values of additive arithmetic functions.


Let \( f \) be a real-valued additive \((f(mn) = f(m) + f(n) \text{ for } (m, n) = 1)\) arithmetic function satisfying \( f(2^k) = kf(2) \) for \( k = 1, 2, \ldots \). Let \( g \) be a real-valued multiplicative \((g(mn) = g(m)g(n) \text{ for } (m, n) = 1)\) arithmetic function with \( g(2^k) = g^k(2) \) for \( k = 1, 2, \ldots \). The author proves the following two theorems. Theorem 1: If \( F \) is the distribution of \( f \), then there exists a discrete infinitely divisible distribution \( G \) such that \( F \ast G \) is infinitely divisible with discrete Levy function (in the Levy representation of the logarithm of the characteristic function), and no normal factor. Hence such \( f \) cannot have a gamma distribution, nor can it have any stable distributions. In particular, \( f \) cannot have an exponential distribution. From this follows Theorem 2: \( g \) cannot have a uniform distribution.