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On the distribution of arithmetic functions.


Let $P_N$ be an additive set function on subsets of a finite set $U_N$. That is, if $A$ and $B$ are disjoint subsets of $U_N$, then $P_N(A \cup B) = P_N(A) + P_N(B)$. Assume that the elements of $U_N$ are integers and that $P_N$ satisfies the following two properties: (i) for the elements $A(a)$ of $U_N$ that are divisible by $a$, $P_N(A(a)) = g(a) + o(1)$ as $N \to +\infty$, where $g(a)$ is a multiplicative function and (ii) for any complex valued additive arithmetical function $f(n)$, the Turan-Kubilius inequality extends to the measure $P_N$ as the argument $n$ goes through $U_N$. Under these conditions, the reviewer proved [Z. Wahrscheinlichkeitstheorie und Verw. Gebiete 18 (1971), 261–270; MR 45#5101] that if $f(n)$ above is real valued and if $B(x) = \{ n: n \in U_N, f(n) < x \}$, then as $N \to +\infty$, $\lim P_N(B(x)) = F(x)$ exists for all continuity points of $F(x)$. The author reobtains the following special case of this result: $U_N$ is the set of all integers between $N$ and $N + N^t$, where $0 < t < 1$ is fixed, and $P_N$ is the relative frequency. Here assumption (i) is evident and the author makes assumptions to guarantee (ii).