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★Probabilistic methods in the theory of arithmetic functions.

Macmillan Lectures in Mathematics, 2.

Macmillan Co. of India, Ltd., New Delhi, 1978. vii+118 pp. Rs. 29.50.

This is an account of some results in probabilistic number theory obtained during the last decade. Most of the results given in this book are due to the author.

The book begins with results on probability that are used in the monograph. Chapter 1 includes the Erdos-Wintner theorem, which gives necessary and sufficient conditions for a real-valued additive arithmetic (a.a.) function to have a limiting distribution. The proof is based on Novoselov’s techniques. In Chapter 2, the spectra and types of limiting distributions are considered. Chapter 3 deals with smoothness properties of limiting distributions. It is known that it is either discrete, continuous singular or absolutely continuous. Necessary and sufficient conditions for a limiting distribution to be discrete are known. However, there are no criteria to distinguish between the absolutely continuous case and the singular one. The author gives some sufficient conditions under which the limiting distribution is singular. In particular, it is shown that every bounded real-valued a.a. function has a singular distribution. No a.a. function can have a uniform distribution.

Chapter 4 is devoted to a conjecture of Erdos. In 1947 P. Erdos conjectured that if the natural density of positive integers \( m \), such that a real-valued a.a. function \( f(m) \) belongs to some bounded interval, exists and is positive, then \( f(m) \) has a limiting distribution. In Chapter 4, some partial solutions of this conjecture (due to P. M. Paul) are given.

Chapter 5 contains the proof of G. Halasz’s results on the sums of multiplicative arithmetic functions. In Chapter 6, necessary and sufficient conditions for a real-valued multiplicative arithmetic function to have a limiting distribution are given. The proof is new. In Chapter 7, necessary and sufficient conditions for a real-valued a.a. function to have limiting distribution mod 1 are obtained. Chapter 8 extends some of the results of previous chapters to functions of pairs of positive integers. In Chapter 9, the notion of \( \beta \)-density is introduced. The \( \beta \)-density of a set \( A \) of positive integers is defined as \( \lim_{t \to \infty} t^{-1} \# \{ m \in A, t \leq m < t + t^\beta \} \), when it exists. Sufficient conditions for an a.a. function to have a limiting distribution in the sense of \( \beta \)-density are given. In Chapter 10, a short review of some re-
sults not treated in this monograph is given. Some open problems are discussed.

The book is a very welcome addition to the literature in probabilistic number theory.  

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