Let $f$ be a real-valued additive arithmetic function. In 1946, P. Erdős [Ann. of Math. (2) 47 (1946), 1–20; MR 7, 416] proved that if $f$ is nondecreasing or satisfies $f(n+1) - f(n) \to 0$ as $n \to \infty$, then for some constant $A$, $f(n) = A \log n$ for all $n$. Among others, Wirsing and Katai have generalized these problems. The present book mainly deals with the exposition of a method for characterizing real-valued additive arithmetic functions in terms of their differences $f(an+b) - f(An+B)$. To give a typical result let $a > 0$, $A > 0$ and $b, B$ satisfy $\Delta = aB - Ab \neq 0$. Then for some positive constants $c$ and $d$ one has

$$\sum^* \frac{1}{q} |f(q) - F(x) \log q|^2 \leq$$

$$d \sup_{x<y} \frac{1}{y} \sum_{x<n<y} |f(an+b) - f(An+B)|^2,$$

where $\sum^*$ denotes sum over all prime powers $q \leq x$ and $(q,a;A|\Delta|) = 1$, and the function $F$ can be defined in terms of $f(q)$ with prime powers $q \leq x$.

In particular, additive functions for which $f(an+b) - f(An+B)$ tends to a finite limit are characterized in Chapter 13 and an extension of a result of Wirsing is derived in Chapter 14.

Most of the results presented in this book are new and are due to the author. He combines algebraic methods with methods of elementary functional analysis to derive these results. These are then applied in Chapters 16, 17, 18 and 19, in particular, to the problem of representing a given integer as a product of rationals of prescribed type.

A list of unsolved problems is given in the last chapter. Some of these problems relate to finding necessary and sufficient conditions for the mean values of $g(p(n))$ to exist, where $p$ is a polynomial with integer coefficients and $g$ is an arithmetic function. For conditions under which $f(p(n))$ has a distribution, when $f$ is additive, see a paper by I. Katai [Acta Math. Hungar. 20 (1969), 69–87; MR 38 5728] and the reviewer's work [Sankhya Ser. A 34 (1972), 323–334; MR 50#2109; MR errata; EA 50].
The first ten chapters of this volume are devoted to the exposition of a method for characterizing additive functions in terms of their differences \( f(an+b) - f(An+B) \). Generalized Turan-Kubilius inequalities are developed in Chapter 1. Chapter 5 contains results from functional analysis. Some results on the large sieve are collected in Chapter 6. Chapter 7 studies the behaviour of additive functions on short arithmetic progressions which have large differences. These results are used in Chapter 8 to get inequalities similar to (1). Chapter 11 contains some historical remarks. An account of the necessary algebra is given in Chapter 15. Quantitative characterizations of Shannon’s entropy function, using the results on differences of additive functions, are developed for finite probability spaces in Chapter 20. Necessary and sufficient conditions are obtained, in Chapter 21, for the frequencies \(# \{ n \leq x : f(n+1) - f(n) \leq y\beta(x) \} \)/x to possess a limiting distribution, where \( f \) is a real-valued additive function. In Chapter 22, it is shown that a sufficiently dense sequence of integers would multiplicatively generate a power of almost every positive integer.

\( \text{Gutti J. Babu (1-PAS-S)} \)