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Mixtures of global and local Edgeworth expansions and their applications. (English. English summary)


Let \( \{Y_j\} \) be an i.i.d. sequence of random variables with a lattice distribution. For an array of constants \( \{a_{j,N}\} \), the authors obtain Edgeworth expansions for

\[
P\left\{ \sum_{j=1}^{N} a_{j,N}(Y_j - E(Y_j)) \in H, \sum_{j=1}^{N} Y_j = n \right\}
\]

as a combination of global and local expansions. From this, expansions for conditional probabilities

\[
P\left\{ \sum_{j=1}^{N} a_{j,N}(Y_j - E(Y_j)) \in H \mid \sum_{j=1}^{N} Y_j = n \right\}
\]

are derived using local expansions for \( P\{\sum_{j=1}^{N} Y_j = n\} \). For the case where \( Y_1 \) is absolutely continuous, Edgeworth expansions are obtained for \( (\sum_{j=1}^{N} a_{j,N}Y_j)/(\sum_{j=1}^{N} Y_j) \).

These results are applied to derive Edgeworth expansions for bootstrap distributions, for Bayesian bootstrap distributions, and for the distribution of statistics based on samples from finite populations. The Bayesian bootstrap is shown to be second-order correct for smooth positive “priors”, whenever the third cumulant of the “prior” is equal to the third power of its standard deviation. (It follows that among the standard gamma “priors”, the only one that leads to second order correctness is the one with mean 4.) Similar results are established for the weighted bootstrap when the weights are constructed from random variables with a lattice distribution.

{Reviewer’s remark: In Equation (10) of Theorem 1, it appears that \( d_N(t) \) should replace \( D_N(t) \).} 

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