Comparison of Probit and Logit Analysis

The following figure compares the standard normal density $f(x)$ with the density $g(x)$ of the **rescaled** Logit distribution

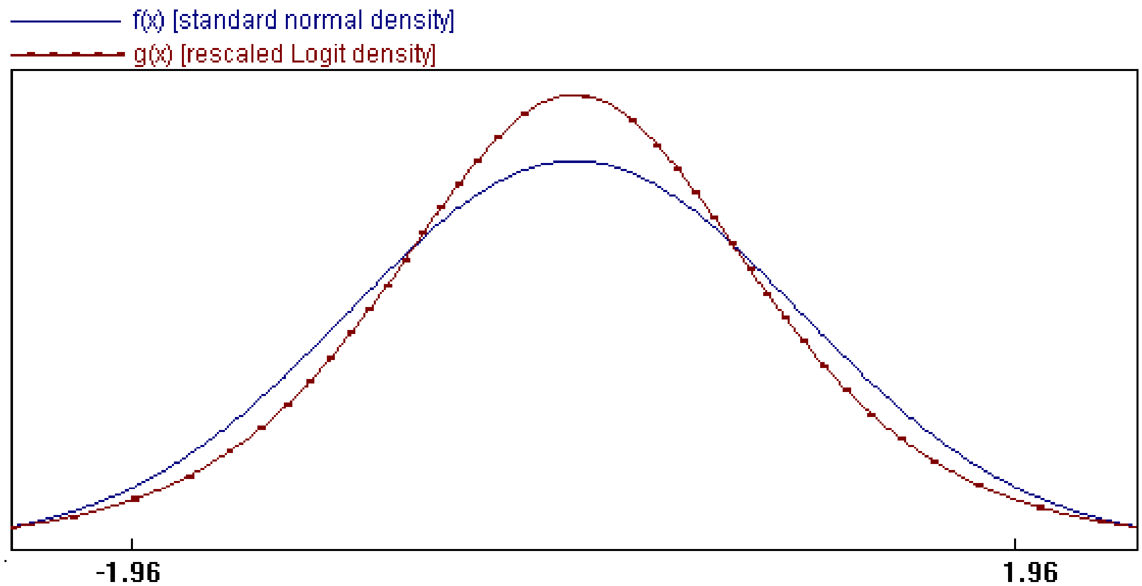
$$G(x) = \frac{1}{1 + \exp(-x/\sigma)},$$

i.e.,

$$g(x) = \frac{1}{\sigma} G(x) (1 - G(x)),$$

where σ is chosen such that $G(1.96) = 0.975$ as for the standard normal distribution. This is the case for

$$\sigma = 0.5349985$$



We see that $f(x)$ is somewhat flatter than $g(x)$. Nevertheless we may expect that Probit and Logit analyses for the same data yield similar result, taking into account the rescaling. To check this, I have generated data according to a standard Probit model

$$\Pr\left[Y_j = 1 | X_j\right] = F\left(\alpha + \beta X_j\right), \; F(x) = \int_{-\infty}^{x} \frac{\exp\left(-u^2/2\right)}{\sqrt{2\pi}} du,$$

for $j = 1, ..., n = 500$, with $\alpha = \beta = 1$, and the X_j's drawn from the standard normal distribution. If we estimate this model as a standard Logit model, we may expect that the Logit estimates $\widehat{\alpha}_L$, $\widehat{\beta}_L$ are related to the Probit estimates $\widehat{\alpha}_P$, $\widehat{\beta}_P$ as follows

$$\widehat{\alpha}_P \approx \sigma \widehat{\alpha}_L, \; \widehat{\beta}_P \approx \sigma \widehat{\beta}_L$$

The Probit estimation results are

$$\widehat{\alpha}_P = 1.117529, \; \widehat{\beta}_P = 1.147434$$

and the Logit estimation results are

$$\widehat{\alpha}_L = 1.928988, \; \widehat{\beta}_L = 2.019920$$

Thus,

$$\widehat{\alpha}_P / \widehat{\alpha}_L = 0.579334, \; \widehat{\beta}_P / \widehat{\beta}_L = 0.568059$$

which are reasonably close to $\sigma = 0.5349985$. Therefore, the Probit parameter estimates are between 50% and 60% smaller in absolute value than the corresponding Logit parameter estimates.