VECTOR TIME SERIES AND INNOVATION RESPONSE ANALYSIS

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1. The Wold decomposition theorem, and the vector auto-regressive (VAR) *and vector moving average* (VMA) *representations*

Let X_t be a k-variate covariance stationary time series process: $X_t \in \mathbb{R}^k$, $E(X_t) = \mu$, $E[(X_t - \mu)(X_{t-m} - \mu)^T] = \Xi(m)$, where the expectation vector μ and the covariance matrices $\Xi(m)$ do not depend on the time index t. The multivariate Wold decomposition theorem states that the process X_t has the Wold representation

$$X_t = \sum_{s=0}^{\infty} \Gamma_s U_{t-s} + W_t, \tag{1}$$

where the $k \times k$ matrices Γ_s are such that

$$\Gamma_0 = I_k, \quad \sum_{s=1}^{\infty} \Gamma_s \Gamma_s^T \ converges, \tag{2}$$

the process U_t is k-variate white noise:

$$U_t \in \mathbb{R}^k, \quad E(U_t) = 0, \quad E(U_t U_t^T) = \Sigma, \quad E(U_t U_{t-m}^T) = O \ for \ m \neq 0, \tag{3}$$

and $W_t \in \mathbb{R}^k$ is a linear deterministic process: there exists a k-vector c_0 and $k \times k$ matrices C_s such that without error,

$$W_t = c_0 + \sum_{s=1}^{\infty} C_s W_{t-s}, \quad and \ E[U_t W_{t-m}^T] = O \quad for \ m = 0, \pm 1, \pm 2, ... \tag{4}$$

Assuming that the deterministic process W_t is constant, it follows from **(1)** that W_t must be equal to the expectation of X_t: $W_t = E(X_t) = \mu$. Then the Wold decomposition theorem implies that the covariance stationary process X_t is a Vector Moving Average (VMA) process, possibly of infinite order:

$$X_t = \mu + \sum_{m=0}^{\infty} \Gamma_m U_{t-m} = \mu + \Gamma(L)U_t, \ where \ \Gamma_0 = I_k, \ \Gamma(L) = I_k + \sum_{m=1}^{\infty} \Gamma_m L^m. \tag{5}$$

Now suppose that there exists a matrix valued lag polynomial $C(L)$ such that $C(L)\Gamma(L) = I_k$. Note that $C(0) = I_k$ because $\Gamma(0) = I_k$, so that we can write

$$\Gamma(L)^{-1} = C(L) = I_k - \sum_{m=1}^{\infty} C_m L^m. \tag{6}$$

Then

$$X_t = \eta + \sum_{s=1}^{\infty} C_s X_{t-s} + U_t, \tag{7}$$

where $\eta = C(1)\mu = \Gamma(0)^{-1}\mu$. This is the Vector Auto-Regressive (VAR) representation.

Next, assume in addition to the covariance stationarity condition that X_t is a Gaussian process: for arbitrary $m \geq 1$ and arbitrary indices $t_1 < t_2 <.....< t_m$, the vector $(X_{t_1}^T,....,X_{t_m}^T)^T$ is jointly normally distributed. Then U_t is a Gaussian process, and since the U_t's are serially uncorrelated, they are independent [*Exercise*: Why?] and normally distributed: U_t is i.i.d. $N_k[0,\Sigma]$. Consequently, X_t is then strictly stationary: the distribution of $(X_{t_1}^T,....,X_{t_m}^T)^T$ only depends on the differences of the indices t_j, and not on their levels, and moreover,

$$E(X_t|X_{t-1},X_{t-2},...) = \eta + \sum_{s=1}^{\infty} C_s X_{t-s}, \tag{8}$$

because $E(U_t) = 0$ and U_t is independent of X_{t-m} for $m > 0$, so that

$$E(U_t|X_{t-1},X_{t-2},....) = E(U_t) = 0. \tag{9}$$

2. VAR(p) models

All linear vector time series models are approximations of model **(7)**. In particular, the assumption of the VAR(p) model is that in model **(7)**, $C_s = O$ for $s > p$:

$$X_t = \eta + \sum_{s=1}^{p} C_s X_{t-s} + U_t. \tag{10}$$

A necessary condition for the strict stationarity of the $VAR(p)$ model is that the the error process U_t is strictly stationary, and the lag polynomial

$$C(L) = I_k - C_1 L - \dots - C_p L^p \tag{11}$$

can be inverted: $C(L)^{-1} = \Gamma(L)$, where $\Gamma(L)$ is the same as before, because then $X_t = \mu + \Gamma(L)U_t$, where the right-hand side is a moving average of a stationary process and therefore stationary itself.

As is well-known, the inverse of $C(L)$ is:

$$C(L)^{-1} = [c^{i,j}(L)] = \frac{1}{\det C(L)}[\mathrm{cof}_{j,i}\{C(L)\}]. \tag{12}$$

Thus, denoting

$$C^*(L) = [\mathrm{cof}_{j,i}\{C(L)\}] \tag{13}$$

we have

$$C^*(L)C(L) = \det(C(L))I \tag{14}$$

hence:

$$\det(C(L))X_t = C^*(1)\eta + C^*(L)U_t. \tag{15}$$

Note that $C^*(L)$ consists of finite-order lag polynomials, and that $\det(C(L))$ is a finite-order lag polynomial. Now it follows from the condition for stationarity of univariate AR(p) processes that:

PROPOSITION 1: *A necessary condition for the* (*strict*) *stationarity of the VAR*(p) *process* **(10)** *is that all the roots of* $\det(C(L))$ *are located outside the complex unit circle*.

3. Granger causality

Consider the strictly stationary bi-variate vector time series process $Z_t = (x_t, y_t)^T$. As is well-known (or it ought to be), the best one-step ahead forecast of Z_t given the whole past of the process Z_t is the conditional expectation

$$E[Z_t|Z_{t-1},Z_{t-2},........] = \begin{pmatrix} E[x_t|Z_{t-1},Z_{t-2},.....] \\ E[y_t|Z_{t-1},Z_{t-2},.....] \end{pmatrix}. \qquad \textbf{(16)}$$

Now if the past of the y_t process does not contribute to the best forecast of x_t, one says that y_t does *not* Granger cause x_t:

DEFINITION 1: *y_t does not Granger-cause x_t if*

$$E\,[x_t - E(x_t|x_{t-1},y_{t-1},x_{t-2},y_{t-2},.......)\,]^2 = E\,[x_t - E(x_t|x_{t-1},x_{t-2},.....)\,]^2. \qquad \textbf{(17)}$$

If y_t Granger-causes x_t then one can predict x_t better using the whole past of the x_t and y_t processes than using only the past of x_t:

$$E\,[x_t - E(x_t|x_{t-1},y_{t-1},x_{t-2},y_{t-2},.......)\,]^2 < E\,[x_t - E(x_t|x_{t-1},x_{t-2},.....)\,]^2. \qquad \textbf{(18)}$$

Suppose that Z_t is a Gaussian $VAR(p)$ process:

$$Z_t = \eta + C_1 Z_{t-1} + ... + C_p Z_{t-p} + U_t, \quad U_t \text{ is i.i.d. } N_2(0,\Sigma). \qquad \textbf{(19)}$$

Then

$$E(Z_t|Z_{t-1},Z_{t-2},....) = \eta + \sum_{s=1}^{p} C_s Z_{t-s} \qquad \textbf{(20)}$$

hence

$$E(x_t|Z_{t-1},Z_{t-2},...) = \eta_1 + \sum_{s=1}^{p} (c_{1,1,s}x_{t-s} + c_{1,2,s}y_{t-s}), \qquad \textbf{(21)}$$

where η_1 is the first component of η and

$$C_s = \begin{pmatrix} c_{1,1,s} & c_{1,2,s} \\ c_{2,1,s} & c_{2,2,s} \end{pmatrix}. \qquad \textbf{(22)}$$

Now y_t does *not* Granger cause x_t if $c_{1,2,s} = 0$ for $s = 1,..,p$, so that then the matrices C_s are lower-triangular, as then the lagged y_t's disappear from the right-hand side of **(21)**.

Finally, the above argument easily extends to the case where y_t and/or x_t are vectors themselves.

4. Sims' nonstructural VAR innovation response analysis

In his seminal paper "Macroeconomics and Reality", Sims (1980) works with a vector X_t of macro-economics variables (dimension $k = 6$). He assumes for X_t a stationary VAR(p) process:

$$C(L)X_t = X_t - C_1X_{t-1} - ... - C_pX_{t-p} = \eta + U_t, \text{ with } U_t \text{ i.i.d. } N(0,\Sigma). \quad \textbf{(23)}$$

Since by stationarity all the roots of det($C(L)$) are located outside the unit circle, the lag polynomial $C(L)$ is invertible: $C(L)^{-1} = \Gamma(L)$. We can now write the VAR(p) process as a VMA(∞) process:

$$X_t = \mu + \sum_{s=0}^{\infty} \Gamma_s U_{t-s}, \quad \Gamma_0 = I, \quad \textbf{(24)}$$

which is just the Wold representation.

Since the innovations U_t are *i.i.d.* $N(0,\Sigma)$ we have for $m \geq 0$

$$E(X_{t+m}|U_t) = \Gamma_m U_t + \mu. \quad \textbf{(25)}$$

The expected "impact" of the first component $u_{1,t}$ of U_t on X_{t+m} is:

$$E(X_{t+m}|u_{1,t}) = E\,[E(X_{t+m}|U_t)|u_{1,t}\,] = \Gamma_m E(U_t|u_{1,t}) + \mu. \quad \textbf{(26)}$$

The *innovation response* of $u_{1,t}$ on X_{t+m} is now defined as

$$E(X_{t+m}|u_{1,t}) - E(X_{t+m}) = \Gamma_m E(U_t|u_{1,t}) = \Gamma_m \begin{pmatrix} u_{1,t} \\ E(u_{2,t}|u_{1,t}) \\ \vdots \\ E(u_{k,t}|u_{1,t}) \end{pmatrix}. \quad \textbf{(27)}$$

We see that the innovation $u_{1,t}$ has two effects on X_{t+m}: a direct effect and an indirect effect via $E(u_{j,t}|u_{1,t})$ for $j > 1$.

Sims (1980, 1982) proposes to calculate $E(U_t|u_{1,t})$ as follows. Observe that Var(U_t) = Σ. Since Σ is positive definite we can write $\Sigma = \Delta\Delta^T$, where Δ is a lower triangular matrix:

$$\Delta = \begin{pmatrix} \delta_{1,1} & 0 & 0 & ..... & 0 \\ \delta_{2,1} & \delta_{2,2} & 0 & ..... & 0 \\ \delta_{3,1} & \delta_{3,2} & \delta_{3,3} & ..... & 0 \\ : & : & : & & : \\ \delta_{k,1} & \delta_{k,2} & \delta_{k,3} & ..... & \delta_{k,k} \end{pmatrix} \tag{28}$$

with positive diagonal elements. Let $e_t = \Delta^{-1}U_t$. Then e_t is i.i.d. $N(0,I)$. Note that $u_{1,t} = \delta_{1,1}e_{1,t}$ with $e_{1,t}$ the first element of e_t. We can now write:

$$E(U_t|u_{1,t}) = \Delta E(e_t|e_{1,t}) = \Delta \begin{pmatrix} 1 \\ 0 \\ : \\ 0 \end{pmatrix} e_{1,t} = \delta_1 e_{1,t} \tag{29}$$

with δ_1 the first column of Δ. Thus the response of $u_{1,t}$ on X_{t+m} is $\Gamma_m\delta_1 e_{1,t}$. Sims replaces in his analysis $e_{1,t}$ by its standard error (=1). He calls it a unit shock. Thus the *innovation response* of a unit shock in the first component of X_t on X_{t+m} is given by $\Gamma_m\delta_1$, for $m = 0,1,2....$

We have seen that U_t can be written as $U_t = \Delta e_t$, with Δ a lower triangular matrix and e_t i.i.d. $N(0,I)$. Thus:

$$U_t = \begin{pmatrix} u_{1,t} \\ u_{2,t} \\ : \\ u_{i,t} \\ : \\ u_{k,t} \end{pmatrix} = \begin{pmatrix} \delta_{1,1}e_{1,t} \\ \delta_{2,1}e_{1,t} + \delta_{2,2}e_{2,t} \\ : \\ \sum_{j=1}^{i} \delta_{i,j}e_{j,t} \\ : \\ \sum_{j=1}^{k} \delta_{k,j}e_{j,t} \end{pmatrix}, \tag{30}$$

hence

$$u_{i,t} = \sum_{j=1}^{i} \delta_{i,j}\, e_{j,t} \qquad (i = 1,..,k). \tag{31}$$

Rather than identifying the innovation response of a shock in variable 2 on the future values of X_t

by $E(X_{t+m}|u_{2,t})$, Sims considers the *net* effect of $u_{2,t}$ on X_{t+m}:

$$E(X_{t+m}|u_{1,t},u_{2,t}) - E(X_{t+m}|u_{1,t}) = E(X_{t+m}|e_{1,t},e_{2,t}) - E(X_{t+m}|e_{1,t}) \quad \textbf{(32)}$$

The equality follows from the fact

$$\begin{pmatrix} u_{1,t} \\ u_{2,t} \end{pmatrix} = \begin{pmatrix} \delta_{1,1} & 0 \\ \delta_{2,1} & \delta_{2,2} \end{pmatrix} \begin{pmatrix} e_{1,t} \\ e_{2,t} \end{pmatrix} \quad \textbf{(33)}$$

is a one-to-one mapping:

$$\begin{pmatrix} e_{1,t} \\ e_{2,t} \end{pmatrix} = \frac{1}{\delta_{1,1}\delta_{2,2}} \begin{pmatrix} \delta_{2,2} & 0 \\ -\delta_{2,1} & \delta_{2,2} \end{pmatrix} \begin{pmatrix} u_{1,t} \\ u_{2,t} \end{pmatrix}. \quad \textbf{(34)}$$

Since

$$E(X_{t+m}|e_{1,t},e_{2,t}) = \Gamma_m(\delta_1 e_{1,t} + \delta_2 e_{2,t}), \quad E(X_{t+m}|e_{1,t}) = \Gamma_m \delta_1 e_{1,t} \quad \textbf{(35)}$$

with δ_1 and δ_2 the first two columns of Δ, the *net* effect of $u_{2,t}$ on X_{t+m} is:

$$E(X_{t+m}|u_{1,t},u_{2,t}) - E(X_{t+m}|u_{1,t}) = \Gamma_m \delta_2 e_{2,t} \quad \textbf{(36)}$$

Again, Sims replaces $e_{2,t}$ by its standard error 1, so that $\Gamma_m \delta_2$ is the innovation response of a unit shock in variable 2 on X_{t+m}.

In general we have for $i > 1$:

$$\begin{aligned} &E(X_{t+m}|u_{1,t},\dots,u_{i,t}) - E(X_{t+m}|u_{1,t},\dots,u_{i-1,t}) \\ &= E(X_{t+m}|e_{1,t},\dots,e_{i,t}) - E(X_{t+m}|e_{1,t},\dots,e_{i-1,t}) = \Gamma_m \delta_i e_{it} \end{aligned} \quad \textbf{(37)}$$

with δ_i the i-th column of Δ. Again, Sims replaces $e_{i,t}$ by its standard error 1. Thus the innovation response of a unit shock in the i-th component of X_t on X_{t+m} is $\Gamma_m \delta_i$.

Summarizing:

DEFINITION 2: *Denoting* $R_m = (r_{i,j}(m)) = \Gamma_m \Delta$, *the innovation response of a unit shock in variable j on variable i is given by* $r_{i,j}(m)$, $m = 0,1,2,......$, *where* $r_{i,j}(m)$ *is the element of* R_m *in the i-th row and j-th column.*

The graphs in Sims' (1980) article are the plots of the functions $r_{i,j}(m)$ for $m = 0,1,2,...$

Remark 1. Since the innovations e_t are the errors of the VAR model for $\Delta^{-1}X_t$, it is in general not possible to measure the components of e_t in the unit of measurement of the corresponding components of X_t. For example, let X_t be bivariate, with component 1 measured in US dollars and component 2 measured in euros. Then the same applies to the corresponding VAR errors $u_{1,t}$ and $u_{2,t}$ in **(33)**. Hence it follows from **(34)** that the unit of measurement of $e_{1,t}$ is the same as for $u_{1,t}$ (US $), but the unit of measurement of $e_{2,t}$ is a combination of US dollars and euros. On the other hand, the units of measurement of the innovation responses are the same as for the variables involved.

Note that the matrix Δ is not unique. It depends on the *order* of the components of X_t. Therefore, in conducting innovation response analysis one has to determine the order of the shocks to the system in advance.

The matrixes $C_1,...,C_p$ and the vector η of intercepts can be estimated consistently by OLS. Let $\hat{C}_1, ..., \hat{C}_p, \hat{\eta}$ be these OLS estimators and let

$$\hat{U}_t = X_t - \hat{\eta} - \sum_{i=1}^{p} \hat{C}_i X_{t-i} \quad (t = p+1,..,n). \tag{38}$$

be the vectors of OLS residuals. Then

$$\hat{\Sigma} = \frac{1}{n-p} \sum_{t=p+1}^{n} \hat{U}_t \hat{U}_t^T \tag{39}$$

is a consistent estimator of Σ (the variance matrix of U_t). Under the normality assumption, these estimators are just the maximum likelihood estimators.

Given the order of the variables in X_t we can calculate the lower triangular matrix $\hat{\Delta}$ such that $\hat{\Sigma} = \hat{\Delta}\hat{\Delta}^T$. Thus the main problem is how to estimate the matrices Γ_m for $m = 0,1,2,...$. This can be done as follows.

First, note that $\Gamma_0 = I$. Next, observe that the matrices Γ_m can be obtained from backwards substitution of the recursive relation $\Gamma_m = C_1\Gamma_{m-1} + ... + C_p\Gamma_{m-p} + E_m$, where $\Gamma_j = O$ for $j < 0$, $\Gamma_0 = I$, $E_j = O$ for $j \neq 0$, and $E_0 = I$. Replacing C_j by its OLS estimate $\hat{C}_j$, we then get consistent estimates $\hat{\Gamma}_m$ of Γ_m. Denoting $\hat{R}_m = (\hat{r}_{i,j}(m)) = \hat{\Gamma}_m\hat{\Delta}$, the estimated innovation response of a unit shock in variable j on variable i is $\hat{r}_{i,j}(m)$.

For each m, $\hat{r}_{i,j}(m)$ is a nonlinear but continuously differentiable function of the elements of the matrices $\hat{C}_i$, i = 1,..,p, and $\hat{\Sigma}$, and the vector $\hat{\beta}$ of these stacked elements has asymptotically a multivariate normal distribution:

$$\sqrt{n}(\hat{\beta} - \beta) \to N(0,\Omega) \ \ in \ \ distr., \tag{40}$$

where β is the vector of corresponding elements of the matrices C_i, i = 1,..,p, and Σ. Therefore, it follows by the so-called delta method that for each i, j, and m,

$$\sqrt{n}(\hat{r}_{i,j}(m) - r_{i,j}(m)) \to N(0,\xi_{i,j,m}^T\Omega\xi_{i,j,m}) \ \ in \ \ distr., \tag{41}$$

where

$$\xi_{i,j,m} = \partial r_{i,j}(m)/\partial\beta^T. \tag{42}$$

Hence

$$\frac{\sqrt{n}(\hat{r}_{i,j}(m) - r_{i,j}(m))}{\sqrt{\hat{\xi}_{i,j,m}^T\hat{\Omega}\hat{\xi}_{i,j,m}}} \to N(0,1) \ \ in \ \ distr., \tag{43}$$

where

$$\hat{\xi}_{i,j,m} = \partial\hat{r}_{i,j}(m)/\partial\hat{\beta}^T, \tag{44}$$

and $\hat{\Omega}$ is a consistent estimator of Ω. [*Exercise*: Assuming normality, how would you determine $\hat{\Omega}$?] On the basis of this result it is possible to endow each of the estimated innovation responses $\hat{r}_{i,j}(m)$ with asymptotic confidence intervals. See Baillie (1987).

5. *Bernanke-Sims' structural VAR innovation response analysis*

A disadvantage of Sims' (1980, 1982) nonstructural VAR approach is that the only way to incorporate economic theory into the VAR model is through the order of the variables in X_t. Therefore, Bernanke (1986) and Sims (1986) propose the following structural VAR(p) model:

$$BX_t = a_0 + \sum_{s=1}^{p} A_s X_{t-s} + e_t, \quad e_t \sim i.i.d.\ N_k(0,I). \tag{45}$$

The matrix B is the matrix of structural coefficients. This matrix represents the contemporaneous interaction between the variables in X_t, similar to the classical structural equations model.

Assuming that the matrix B is nonsingular, this structural model is related to the non-structural Var(p) model **(10)**:

$$X_t = B^{-1}a_0 + \sum_{s=1}^{p} B^{-1}A_s X_{t-s} + B^{-1}e_t = \eta + \sum_{s=1}^{p} C_s X_{t-s} + U_t. \quad \textbf{(46)}$$

Therefore, the matrix B links the nonstructural errors U_t to the innovations e_t:

$$BU_t = e_t, \text{ where } e_t \sim N_k(0, I). \quad \textbf{(47)}$$

Note that there are many matrices B for which this is true, for example let $B = \Sigma^{-1/2}$.

Given that B is specified such that B is invertible, **(47)** reads

$$U_t = B^{-1}e_t, \quad \textbf{(48)}$$

hence B^{-1} now takes over the role of the lower triangular matrix Δ in non-structural VAR innovation response analysis. Thus,

DEFINITION 3: *Denoting* $R_m^{(s)} = (r_{i,j}^{(s)}(m)) = \Gamma_m B^{-1}$, *the (structural) innovation response of a unit shock in variable j on variable i is given by* $r_{i,j}^{(s)}(m)$, $m = 0,1,2,......$, *where* $r_{i,j}^{(s)}(m)$ *is the element of* $R_m^{(s)}$ *in the i-th row and j-th column.*

Remark 2. Note that Remark 1 applies to the structural case as well.

Since

$$\Sigma = B^{-1}(B^T)^{-1} = (B^T B)^{-1}, \text{ hence } B^T B = \Sigma^{-1}, \quad \textbf{(49)}$$

and Σ contains only $k + (k^2 - k)/2$ different elements, we can identify no more than $k + (k^2 - k)/2$ elements of B. Therefore, we have to set at least $(k^2 - k)/2$ elements of B equal to given constants (usually zeros). This is where economic theory comes in the picture.

However, even is the matrix B contain $k + (k^2 - k)/2$ or less non-zero entries, identification is not guaranteed. For example, consider the case

$$B = \begin{pmatrix} b_1 & 0 & b_5 \\ 0 & b_2 & b_4 \\ b_6 & 0 & b_3 \end{pmatrix}. \qquad \textbf{(50)}$$

Then

$$B^T B = \begin{pmatrix} b_1 & 0 & b_6 \\ 0 & b_2 & 0 \\ b_5 & b_4 & b_3 \end{pmatrix} \begin{pmatrix} b_1 & 0 & b_5 \\ 0 & b_2 & b_4 \\ b_6 & 0 & b_3 \end{pmatrix} = \begin{pmatrix} b_1^2 + b_6^2 & 0 & b_1 b_5 + b_3 b_6 \\ 0 & b_2^2 & b_2 b_4 \\ b_1 b_5 + b_3 b_6 & b_2 b_4 & b_3^2 + b_5^2 \end{pmatrix} = \Sigma^{-1}, \qquad \textbf{(51)}$$

which effectively consists of five different nonlinear equations in six unknowns.

Given appropriate restrictions on B, we can estimate B by solving the nonlinear system of equations

$$\hat{B}^T \hat{B} = \hat{\Sigma}^{-1} \qquad \textbf{(52)}$$

in the just-identified case, or by maximum likelihood in the over-identified case, where there are less unknown elements of B than $k + (k^2 - k)/2$. Replacing the lower triangular matrices Δ and $\hat{\Delta}$ in Sims' approach by B^{-1} and $\hat{B}^{-1}$, respectively, now yields the structural innovation responses.

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