Overview: SSB of Weinberg-Salam theory

• Group: \( SU(2) \otimes U(1) \). Couplings: \( g_2, g_1 \)

• Gauge transformation of multiplet of matter fields:

\[
\psi 
\mapsto
\psi \mapsto e^{ig_2 \omega \cdot T} e^{ig_1 \omega_0 Y/2} \psi
\]

\( Y \) is weak hypercharge of multiplet.

• Ingredients for gauge-invariant Lagrangian: Field-strength tensor and covariant derivative (as from Yang-Mills).

• Write gauge-invariant Lagrangian, with Higgs part

• With VEV for scalar field (“Higgs” field):
  – Unbroken U(1) symmetry; identify massless gauge field as photon
  – Broken symmetries: 3 massive gauge bosons.

• Treat as parameters: \( e, v, \theta_W \)
Key formulae

- Field strength tensors:
  \[ B_{\mu \nu} = \partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu}, \]
  \[ W_{\mu \nu}^a = \partial_{\mu}W_{\nu}^a - \partial_{\nu}W_{\mu}^a + g_2 \epsilon_{abc}W_{\mu}^bW_{\nu}^c \]

- Covariant derivative on Higgs doublet of scalar fields
  \[ D_{\mu} \phi = \partial_{\mu} \phi - \frac{ig_2}{2} W_{\mu}^a \sigma^a - \frac{ig_1 Y}{2} B_{\mu} \phi \]

- Lagrangian density:
  \[ \mathcal{L} = -\frac{1}{4} W_{\mu \nu}^a W^{a, \mu \nu} - \frac{1}{4} B_{\mu \nu} B^{\mu \nu} - D_{\mu} \phi^\dagger D^{\mu} \phi - \lambda \left( \phi^\dagger \phi - \frac{\mu^2}{2\lambda} \right)^2 \]
  + fermion and Yukawa terms