Exercises for Chapter 1
Introduction to Probability Theory

1. A family has 2 children. What is the conditional probability that both are boys given that at least one of them is a boy?

2. An urn contains 2 black balls and 3 white balls. Two balls are selected from the urn at random without replacement. Find the probability that both balls are black.

3. Three men went to a party and hang their coats in a closet. When they left, each of them randomly picked a coat. What is the probability that they all get their own coat?

4. Two dice are rolled. What is the probability that at least one is six? If the two faces are different, what is the probability that at least one is six?

5. A coin is to be tossed until a head appears twice. What is the sample space for this experiment? If the coin is fair, what is the probability that it will be tossed exactly four times?
6. Suppose each of 3 persons tosses a coin. If the outcome of one of the tosses differ from the other outcomes, then the game ends. If not, then the persons start over and retoss their coins. Assuming fair coins, what is the probability that the game will end with the first round of tosses?

7. A communication system transmits 3 signals 0, 1, 2 with probability $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{4}$ respectively. At the receiving end, there are errors. When 0 is sent, the probability of receiving 0 is $1 - \epsilon$ and receiving 1 $\epsilon$. When 1 is sent, the probability of receiving 1 is $1 - \epsilon$ and receiving 2 $\epsilon$. When 2 is sent, the probability of receiving 2 is $1 - \epsilon$ and receiving 0 $\epsilon$.

(a) What is the probability of output 0,1,2?
(b) If the output is 1, what is the probability of input being 0? 1? 2?

8. There are 2 urns. The first contains 2 white and 1 black balls. The second contains 1 white and 5 black balls. A ball is randomly drawn from the first urn and put in the second. Then a ball is randomly drawn from the second urn and it is white. What is the probability the ball drawn from the first urn is white?
9. An urn contains $b$ black balls and $r$ red balls. One ball is picked at random and put back. Another $c$ balls of the same color are put in the urn. Then a ball is picked at random from the urn again. The ball is red. What is the probability that the ball first picked is black given the second pick is red?

10. Bill and George go target shooting together. Both shoot at a target at the same time. Suppose Bill hits the target with prob. 0.7 whereas George independently hits the target with prob. 0.4.

(a) Given that exactly one shot hits the target. What is the prob. that it comes from George?
(b) Given that the target is hit, what is the prob. that George hits it?

11. A gambler has in his pocket a fair coin and a two-headed coin. He selects one of the coin at random, and then he flips it. It shows head. He flips the same coin a second time and again it shows heads. Now

(a) What is the prob. that it is the fair coin?
(b) Suppose that he flips the same coin a third time and it shows tail. What is the prob. that it is the fair coin?