Combinatory Analysis 2018: Partitions, q-Series, and Applications

Penn State University
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Talk Titles and Abstracts

Invited Talks

Krishnaswami Alladi (University of Florida)

On the local distribution of the number of small prime factors - a variation of the classical theme

Abstract: The global distribution of $\nu_y(n)$, the number of (distinct) prime factors of n which are $< y$, plays a crucial role in the proof of the celebrated Erdös-Kac theorem on the distribution of $\nu(n)$, the number of distinct prime factors of n. Although much is known about the "local distribution" of $\nu(n)$, namely the asymptotics of the function $N_k(x) = \sum_{n \le x, \nu(n)=k} 1$, little attention has been paid to the local distribution of $\nu_y(n)$. In discussing the asymptotic behavior of $N_k(x,y) = \sum_{n \le x, \nu_y(n)=k} 1$, we noticed a very interesting variation of the classical theme that seems to have escaped attention. To explain this phenomenon, we will obtain uniform asymptotic estimates for $N_k(x,y)$ by a variety of analytic techniques such as those of Selberg, and of Buchstab-De Bruijn (involving difference-differential equations). This is joint work with my recent PhD student Todd Molnar. Tenenbaum has approached this problem from a different approach and shown that some of our results could be sharpened.

George Andrews (Penn State University)

Dyson's Most Beautiful Identity, Sequences in Partitions and Mock Theta Functions

Abstract: We begin with background on "Dyson's Most Beautiful Identity," a Rogers-Ramanujan type identity related to the modulus 27. From there we move on to the development of research on sequences in partitions starting with Sylvester. A new theorem on sequences in partitions is presented which in turn leads to L.J. Rogers' Modulus 14 identities. All this naturally leads up to the results of Dyson and Bailey for Rogers-Ramanujan type identities related to the modulus 27. Finally a new proof of these results leads to "9th order" mock theta functions.

Siegfried Baluyot (University of Illinois at Urbana-Champaign)

Critical zeros of the Riemann zeta-function

Abstract: This talk will be a brief survey of current knowledge about the proportion of the zeros of the Riemann zeta-function that are on the critical line. We will present different methods of detecting the zeros, and then discuss recent advances towards pushing this proportion higher. This is joint work with Nicolas Robles, Arindam Roy, and Alexandru Zaharescu.

Alexander Berkovich (University of Florida)

25 Years Later. Some Highlights

Abstract: Ever since 1993, my scientific career has deeply been influenced by George Andrews. George became a father figure, friend and an inspiration to me. In this talk, I briefly discuss some theorems I discovered with his constant feedback, interest and encouragement. Here is a partial list: Finite Analogue of Goellnitz (Big) Partition Theorem, Tri-Pentagonal Number Theorems, Partition Inequalities.

Bruce Berndt (University of Illinois at Urbana-Champaign)

The Final Problem: An Unproved Identity from the Lost Notebook Connected with the Dirichlet Divisor Problem

Abstract: A certain identity in the Lost Notebook associated with the Dirichlet Divisor Problem may be interpreted in three ways. Proofs have been given for two interpretations, but a proof has never been given for the third (and most natural) interpretation, i.e., the formulation which is actually given by Ramanujan.

Heng Huat Chan (National University of Singapore, Singapore)

Wronskians of theta functions and Ramanujan's series for $1/\pi$

Abstract: In this talk, we show the connection between Wronskians of theta functions and Ramanujan's series for $1/\pi$.

Sylvie Corteel (CNRS et Université Paris Diderot, France)

Lecture Hall Tableaux

Abstract: The lecture hall partitions were introduced by Bousquet-Mlou and Eriksson in 1997 by showing that they are the inversion vectors of elements of the parabolic quotient $\tilde{C}_n/C_n$. Since 1997, a lot of beautiful combinatorial techniques were developed to study these objects and their generalizations. These use partition analysis, bijective combinatorics, basic hypergeometric series, geometric combinatorics, real rooted polynomials... Some of those results can be found in the survey paper by C. D. Savage "The Mathematics of lecture hall partitions". Here we take a different approach and show that these objects are also (dual) moments of the Little q-Jacobi polynomials. Recently, multivariate moments were introduced by Corteel and Williams in the context of asymmetric exclusion processes. The benefit of this new approach is that the multivariate moments of the multivariate Little q-Jacobi polynomials give rise of tableau analogues of Lecture Hall partitions. We define two tableau analogues of lecture hall partitions and we show that their generating function is a beautiful product. This uses a mix of orthogonal polynomials techniques, non intersecting lattice paths and q-Selberg integral. This is joint work with Jang Soo Kim (SKKU).

Kimmo Eriksson (Mälardalen University, Sweden)

Bulgarian solitaire and international assessments: A talk with two themes

Abstract: I will talk about my recent and ongoing research in two different areas. The first theme is generalizations of the iterative process on integer partitions known as Bulgarian solitaire. The second theme is what we can learn about effective mathematics education from analysis of big data generated by the large-scale international assessments TIMSS and PISA.

Frank Garvan (University of Florida)

Higher order mock theta conjectures

Abstract: The Mock Theta Conjectures were identities stated by Ramanujan for his so called fifth order mock theta functions. Andrews and Garvan showed how two of these fifth order functions are related to rank differences mod 5. Hickerson was first to prove these identities and was also able to relate the three Ramanujan seventh order mock theta functions to rank differences mod 7. Based on work of Zwegers, Zagier observed that the two fifth order functions and the three seventh order functions are holomorphic parts of real analytic vector modular forms on $SL_2(Z)$. Zagier gave an indication how these functions could be generalized. We give details of these generalizations and show how Zagier's 11th order functions are related to rank differences mod 11.

Shishuo Fu (Chongqing University, China)

Multipartitions revisited: cranks, congruences and inequalities

Abstract: In the spirit of Andrews' 2008 survey, we study multipartitions for their own sake. We shall present results on multipartitions that are highly analogous to those of ordinary partitions. These include three generalized notions of cranks (k-cranks), two of which explain congruences enjoyed by various k-colored partition functions, k-crank moments, and some inequalities. Our methods of proof range from q-series manipulation to combinatorial analysis. We conclude with two unimodality conjectures. These are joint work with Shane Chern and Dazhao Tang.

Mike Hirschhorn (University of New South Wales, Australia)

An elementary proof of the congruences modulo powers of 5 for the cubic partition function

Abstract: Let $p^*(n)$ be the number of partitions of n with even parts in two colors (the so-called "cubic" partitions). It is known that if δ_α denotes the reciprocal of 8 modulo 5^α then for $\alpha \geq 2$ and $n \geq 0$,

$$p^*(5^\alpha n + \delta_\alpha) \equiv 0 \pmod{5^{\lfloor \frac{\alpha}{2} \rfloor}}.$$

We give an elementary proof of this.

Mourad Ismail (University of Central Florida)

The Ramanujan Function, Old and new questions

Abstract: We discuss earlier and more recent results about the Ramanujan function, its zeros and various properties.

William Keith (Michgan Technological University)

Unimodality of sums and differences of q-binomials

Abstract: Establishing the unimodality of differences of q-binomials, and products thereof, is in general a challenging question. Using a result of Kirillov and Reshetikhin on the distribution of the major index of standard Young tableaux, we establish unimodality for a large class of such formulas. A particular case of our results gives a positive answer to a question of Sagan on the unimodality of the major index statistic for 321-avoiding permutations.

Christian Krattenthaler (University of Vienna, Austria)

p-Divisibility of combinatorial (and other) sequences

Abstract: If one looks at the number of involutions in the symmetric group S_n, then one can observe a (non-obvious) high divisibility by powers of 2. This phenomenon generalizes to numbers $I(n,p)$ of "p-volutions" in S_n, that is, to the number of elements w of S_n with $w^p = id$. More precisely,

$$p^{[n/p]-[n/p^2]} \text{ divides } I(n,p)$$

as shown by Dress and Yoshida, and independently by Grady and Newman in the early 1990s, and this divisibility is sharp for infinitely many n. A fancy way to describe this result is that the number of representations of the cyclic group C_p in S_n, that is, $|Hom(C_p, S_n)|$, has the above described p-divisibility. Further work of Tomoyuki Yoshida, together with coworkers, attempted to extend this result from C_p to all finite Abelian groups G.

I shall present a complete solution to the problem. It involves three ingredients: (1) the well-known relation between $|Hom(G, S_n)|$ and the numbers of subgroups in G of a given index; (2) Dwork's criterion for an exponential of a formal power series to have integer coefficients; (3) some well-known facts about Kostka-Foulkes polynomials. All this will be thoroughly explained.

This is joint work with Thomas Müller.

Brandt Kronholm (University of Texas at Grand Valley)

Restricted partitions: Recent results and methods

Abstract: In this presentation we will discuss several recent results on partitions restricted by both the number and sizes of parts. Our emphasis will be on the methods used to obtain these results. We will begin by offering two q-series proofs of the following fact: The number of partitions of n into at most four parts, denoted $p(n,4)$, is divisible by 3 exactly 50% of the time. However, the second proof will make use of a novel approach revealing far more information about the result than the first proof.

These novel methods have been used to obtain many other compelling results. We will discuss a complete characterization of the maximal coefficients, $p(n,3,N)$, of Gaussian polynomials $\left[{M \atop 3}\right]$ and a surprising extension of a theorem on first differences of restricted partitions. These same methods were used to build evidence for a recent theorem on infinite families of congruences for $p(n,m,N)$. They were also used to provide a new decomposition of partitions, which in turn allows us to define statistics called supercranks that combinatorially witness every instance of divisibility of $p(n,3)$ by any prime of the form $6j-1$.

Kagan Kursungoz (Sabanci University, Turkey)

Andrews-Gordon type identities for Kanade-Russell conjectures

Abstract: Kanade and Russell conjectured six integer partition identities in 2015 using computer software they developed. The conjectures eluded proof (or highly unlikely, falsification) so far. We will consider one side of their identities only, and construct multiple series that resemble the series side of Andrews-Gordon identities as generating functions of the said partitions. The construction works in slightly greater generality than the identities. Namely, one obtains multiple series as generating functions of similar type of partitions whether or not Kanade and Russell's software outputs an identity.

Jeremy Lovejoy (CNRS et Université Paris Diderot, France)

Colored Jones polynomials, q-series, and modular forms

Abstract: In this talk I will discuss joint work with Kazuhiro Hikami, in which we use Bailey pairs and the Rosso-Jones formula to compute the cyclotomic expansion of the colored Jones polynomial of a certain family of torus knots. As an application we find quantum modular forms dual to the generalized Kontsevich-Zagier series. As another application we obtain formulas for the unified WRT invariants of certain 3-manifolds, some of which are mock theta functions. I will also touch on joint work with Robert Osburn, in which we compute a formula for the colored Jones polynomial of double twist knots.

Ken Ono **(Emory University)**

Polya's Program for the Riemann Hypothesis and Related Problems

Abstract: In 1927 Polya proved that the Riemann Hypothesis is equivalent to the hyperbolicity of Jensen polynomials for Riemanns Ξ-function. This hyperbolicity has only been proved for degrees $d = 1, 2, 3$. We prove the hyperbolicity of 100% of the Jensen polynomials of every degree. We obtain a general theorem which models such polynomials by Hermite polynomials. This theorem also allows us to prove a conjecture of Chen, Jia, and Wang on the partition function. This is joint work with Michael Griffin, Larry Rolen, and Don Zagier.

Peter Paule **(Research Institute for Symbolic Computation (RISC), Johannes Kepler University Linz, Austria)**

A unified algorithmic framework for Ramanujan's congruences modulo powers of 5,7, and 11

Abstract: In 1919 Ramanujan conjectured three infinite families of congruences for the partition function modulo powers of 5, 7, and 11. In 1938 Watson proved the 5-case and (a corrected version of) the 7-case. In 1967 Atkin proved the remaining 11-family using a method significantly different from Watson's. In joint work with Silviu Radu (RISC), we set up a new algorithmic framework which brings all these cases under one umbrella. In his paper Atkin remarked that, in comparison with the 5 and 7-case, his proof for 11 is "indeed 'langweilig', as Watson suggested." In our framework we find the 11-case particularly interesting.

James Sellers **(Penn State University)**

Extending Parity Results for Generalized Frobenius Partition Functions

Abstract: In his 1984 AMS Memoir, George Andrews defined two families of generalized Frobenius partition functions which he denoted $\phi_k(n)$ and $c\phi_k(n)$ where $k \geq 1$. Both of these functions "naturally" generalize the unrestricted partition function $p(n)$ since $p(n) = \phi_1(n) = c\phi_1(n)$ for all n. In his Memoir, Andrews proved (among many other things) that, for all $n \geq 0$, $c\phi_2(5n+3) \equiv 0 \pmod 5$. Soon after, many authors proved congruence properties for various generalized Frobenius partition functions, typically for small values of k. In this talk, I will discuss a variety of these past congruence results. I will then transition to very recent parity results satisfied by $\phi_k(n)$ and $c\phi_k(n)$ for (different) infinite sets of values for k. This is joint work with George Andrews.

Andrew Sills **(Georgia Southern University)**

MacMahon's partial fractions

Abstract: Cayley used ordinary partial fractions decompositions of $1/[(1-x)(1-x^2)...(1-x^m)]$ to obtain direct formulas for the number of partitions of n into at most m parts for several small values of m. No pattern for general m can be discerned from these, and in particular the rational coefficients that appear in the partial fraction decomposition become quite cumbersome for even moderate sized m.

Later, MacMahon gave a decomposition of $1/[(1-x)(1-x^2)...(1-x^m)]$ into what he called "partial fractions of a new and special kind" in which the coefficients are "easily calculable number[s]" and the sum is indexed by the partitions of m.

While MacMahon derived his "new and special" partial fractions using "combinatory analysis," the aim of this talk is to give a fully combinatorial explanation of MacMahon's decomposition. In particular, we will observe a natural interplay between partitions of n into at most m parts and weak compositions of n with m parts.

Richard Stanley (MIT and University of Miami)

A tale of two triangles

Abstract: We will discuss two triangles analogous to Pascal's triangle (or the arithmetic triangle). The first is a multiplicative analogue of Pascal's triangle, and the second is a slight modification of Stern's diatomic array. A typical result is the following: let

$$\prod_{i=0}^{n-1}(1+x^{2^i}+x^{2^{i+1}}) = \sum a_j x^j.$$

Then

$$\sum a_j^3 = 3 \cdot 7^{n-1}.$$

Dennis Stanton (University of Minnesota)

Marking and shifting parts in partition theorems

Abstract: Refinements, analytic and combinatorial, are given for the following results: Rogers-Ramanujan identities, Gollnitz-Gordon identities, Andrews-Gordon identities, Eulers odd=distinct theorem, and others. A single generic part is weighted or marked. Corresponding results are given for replacing a part by a larger part, shifting it. This corresponds to taking a certain subset of the original set of partitions given by differences.

Ole Warnaar (University of Queensland, Australia)

On modular Nekrasov-Okounkov formulas

Abstract: The Nekrasov-Okounkov formula is a far-reaching generalisation of Euler's classical product formula for the partition generating function. In this talk I will discuss several modular analogues of the Nekrasov-Okounkov formula based on Littlewood's decomposition of partitions into cores and quotients, and variations thereof.

Ae Ja Yee (Penn State University)

Truncated Hecke-Rogers type series

Abstract: The recent work of George Andrews and Mircea Merca on the truncated version of Euler's pentagonal number theorem has opened up a new study on truncated theta series. Since then several papers on the topic have followed. In collaboration with Chun Wang, I was able to generalize it to Hecke-Rogers type double series, which are associated with some interesting partition functions. In this talk, I will present some of our results.

Doron Zeilberger **(Rutgers University)**

Symbolic computation and partitions: From "Pencil with Power-Steering" to "Self-Driving Pencil"

Abstract: George Andrews famously asserted that, to him, a computer algebra system is "just a pencil with power-steering", but it is already starting to be a "self-driving pencil".

Contributed Talks

Moussa Ahmia (University of Mohamed Seddik Ben yahya Jijel)

q-Log-concave and unimodal sequences of q-multinomial coefficients and symmetric function

Abstract: We prove the strong log-concavity and the unimodality of the sequences associated with new class of symmetric function proposed in our paper entitled "Connection between bi^{s}nomial coefficients with their analogs and symmetric functions" published in Turkish Journal of Mathematics. This work generalizes some results of Sagan in the paper entitled "Log concave sequences of symmetric functions and analogs of the Jacobi- Trudi determinants".

As applications of our results, we establish the strong q-log-concavity of q-multinomial coefficients $\begin{bmatrix} n \\ k \end{bmatrix}_q^{(s)}$ and their sequence lying along a ray $\left\{ \begin{bmatrix} n_0+j\alpha \\ k_0-j\beta \end{bmatrix}_q^{(s)} \right\}_{j\geq 0}$ with $\alpha\beta > 0$, also we prove the unimodality of the bi^{s}nomial coefficients and their sequence lying along a ray $\left\{ \binom{n_0+j\alpha}{k_0-j\beta}_s \right\}_{j\geq 0}$ with $\alpha\beta > 0$.

Zafer Selcuk Aygin (Nanyang Technological University, Singapore)

Combinations of ranks and cranks of partitions moduli 6, 9 and 12 and their comparison with the partition function

Abstract: Let $L \in \{6, 9, 12\}$. We determine the generating functions of certain combinations of three ranks and three cranks modulo L in terms of eta quotients. Then using the periodicity of signs of these eta quotients, we compare their values with the values of $\frac{p(n)}{L/3}$. This talk is based on joint work with Song Heng Chan.

Cristina Ballantine (College of the Holy Cross)

A combinatorial proof of Andrews' theorem related to Euler's identity

Abstract: Let $a(n)$ be the number of partitions of n such that the set of even parts has exactly one element, $b(n)$ be the difference between the number of parts in all odd partitions of n and the number of parts in all distinct partitions of n, and $c(n)$ be the number of partitions of n in which exactly one part is repeated. Beck conjectured that $a(n) = b(n)$ and Andrews, using generating functions, proved that $a(n) = b(n) = c(n)$. We give combinatorial proofs of the identities $a(n) = b(n)$ and $c(n) = b(n)$. Our proof relies on bijections between a set of partitions and a multiset in which equal partitions are distinguished by decorations with binary words.

Gaurav Bhatnagar (University of Vienna, Austria)

The determinant of an elliptic Sylvesteresque matrix

Abstract: We evaluate the determinant of a matrix whose entries are elliptic hypergeometric terms and whose form is reminiscent of Sylvester matrices. A hypergeometric determinant evaluation of a matrix of this type has appeared in the context of approximation theory, in the work of Feng, Krattenthaler and Xu. Our determinant evaluation is an elliptic extension of their evaluation, which has two additional parameters (in addition to the base q and nome p found in elliptic hypergeometric terms). We also extend the evaluation to a formula transforming an elliptic determinant into a multiple of another elliptic determinant. This transformation has two further parameters. The proofs of the determinant evaluation and the transformation formula require an elliptic determinant lemma due to Warnaar, and the application of two C_n elliptic formulas

that extend Frenkel and Turaev's ${}_{10}V_9$ summation formula and ${}_{12}V_{11}$ transformation formula, results due to Warnaar, Rosengren, Rains, and Coskun and Gustafson. This is joint work with Christian Krattenthaler.

Hannah Burson (University of Illinois at Urbana Champaign)

Combinatorial interpretations of some identities from Ramanujan's lost notebook

Abstract: In his lost notebook, Ramanujan stated five identities related to the false theta series

$$f(q) = \sum_{n=0}^{\infty} (-1)^n q^{n(n+1)/2}.$$

The analytic proofs of these identities were first found by Andrews in 1981 and vary in difficulty. In this talk, we introduce new combinatorial interpretations of these identities. We illustrate how to work with these interpretations by giving a new bijective proof of one of these five identities.

Zhu Cao (Kennesaw State University)

Elementary proofs of some of Ramanujan's identities

Abstract: In this talk, we give new proofs to a list of Ramanujan's identities using primitive roots of unity and exact covering systems. Some new identities will be presented as applications. This is joint work with Yong Hu.

Song Heng Chan (Nanyang Technological University, Singapore)

Ranks of partitions

Abstract: We will give a quick introduction on the ranks of partitions and present some of the latest developments in the topic. We will also share our new results arising from recent joint works with several groups of authors.

Shane Chern (Penn State University)

An infinite family of congruences for 1-shell totally symmetric plane partitions

Abstract: Let $s(n)$ denote the number of 1-shell totally symmetric plane partitions of n. In this paper, we prove an infinite family of congruences modulo powers of 5 for $s(n)$. Namely,

$$s\left(2 \cdot 5^{2\alpha-1} n + 5^{2\alpha-1}\right) \equiv 0 \pmod{5^{\alpha}}.$$

Edmund Y.-M. Chiang (The Hong Kong University of Science and Technology)

A Picard theorem for the Askey-Wilson operator

Abstract: We derive a full-fledged Nevanlinna Theory for the Askey-Wilson operator which includes a Picard-type theorem as a consequence. In particular, this allows us to gain insight that zero/pole-sequences of many infinite q-products are indeed deficient in the sense of Nevanlinna Theory when interpreted in this difference sitting. This is a join work with Shaoji Feng.

Mithun Das (Harish-Chandra research institute, Inida)

Sign change of various kind of Z-function

Abstract: The Z-function is a real valued function along the line $\frac{1}{2}+it$ for real t, introduced by G. H. Hardy in 1914 to study the Zeta function along the line $\frac{1}{2}+it$. In my talk I will show that the Lebesgue measure of both positive and negative values of Z-function associated with certain kind of L-function have positive density in the interval $[T, T+H]$. This is a joint work with S. Pujahari.

Madeline Dawsey (Emory University)

Effective error bounds for Andrews' smallest parts function

Abstract: We prove Chen's conjectured inequalities for the Andrews spt-function. The proof of these inequalities is complicated by the problem that the recently obtained Rademacher-type exact formula by Ahlgren and Andersen is conditionally convergent. Instead, we consider a different formula from Ahlgren and Andersen which expresses $\operatorname{spt}(n)$ as a finite sum of algebraic numbers, in the spirit of earlier work of Bruinier and Ono for $p(n)$. We obtain the first known effective error bounds for $\operatorname{spt}(n)$,

$$\operatorname{spt}(n) = \frac{\sqrt{3}}{\pi\sqrt{24n-1}} e^{\pi\sqrt{24n-1}/6} + E_s(n),$$

where for an explicitly defined constant C and a certain logarithmic expression $q(n)$, we have

$$|E_s(n)| < C \cdot 2^{q(n)} (24n-1)^2 e^{\pi\sqrt{24n-1}/12}.$$

Colin Defant (Princeton University)

Connected components of ranges of divisor functions

Abstract: For each complex number c, the divisor function σ_c is the arithmetic function given by $\sigma_c(n) = \sum_{d|n} d^c$. We will touch upon a very small sliver of the rich and exhilarating history behind these classical functions, leading into a discussion of recent questions concerning their ranges. More specifically, we consider the basic topological properties of the closure of the range of σ_c. We have shown that this set has nonempty interior and has finitely many connected components when $\Re(c) \leq 0$ and $c \neq 0$ (this was already proven by Carlo Sanna in the case in which c is real). Furthermore, the number $\mathcal{N}(c)$ of connected components tends to ∞ as $\Re(c) \to -\infty$. We will mention some recent results due to Nina Zubrilina, who found asymptotic estimates for $\mathcal{N}(-r)$ when $r \geq 1$ is real. Zubrilina also discovered a surprising new property of the number 4. Her work leads to some interesting open problems and conjectures.

Atul Dixit (Indian Institute of Technology Gandhinagar, India)

Partition implications of a three parameter q-series identity

Abstract: A generalization of a beautiful q-series identity found in the unorganized portion of Ramanujan's second and third notebooks is obtained. As a consequence, we derive a new three-parameter identity which is a rich source of partition-theoretic information. It is used to obtain a generalization of a recent result of Andrews, Garvan and Liang, which itself generalizes the famous result of Fokkink, Fokkink and Wang. This three-parameter identity also leads to several new weighted partition identities as well as a natural proof of a recent result of Garvan. We also obtain a new result consisting of an infinite series involving a special case of Fine's function $F(a, b; t)$, namely, $F(0, q^n; cq^n)$. For $c = 1$, this gives Andrews' famous identity for $\operatorname{spt}(n)$ whereas for $c = -1$ and 0, it gives new relations that the divisor function $d(n)$ has with other

partition-theoretic functions such as the largest parts function lpt(n). This is joint work with Bibekananda Maji.

Jehanne Dousse (University of Zurich, Swiss)

Andrews' generalizations of Schur's theorem

Abstract: Schur's celebrated partition identity states that for all n, the number of partitions of n into distinct parts congruent to 1 or 2 modulo 3 equals the number of partitions of n with difference at least 3 between two consecutive parts, such that no two consecutive multiples of 3 appear. George Andrews gave several proofs of Schur's theorem using recurrences and q-difference equations. Two of these proofs led him to prove two beautiful generalizations of Schur's theorem. In this talk, we will discuss these generalizations and a recent theorem which unifies them (together with their overpartition versions).

Alexander Dunn (University of Illinois at Urbana-Champaign)

Maass forms and the mock theta function $f(q)$

Abstract: Let $f(q) := 1 + \sum_n \alpha(n)q^n$ be the well-known third order mock theta of Ramanujan. In 1964, George Andrews proved an asymptotic formula of the form

$$\alpha(n) = \sum_{c \leq \sqrt{n}} \psi(n) + O_\epsilon(n^\epsilon),$$

where $\psi(n)$ is a complicated expression involving generalized Kloosterman sums and the I-Bessel function. Confirming a conjecture of Andrews, Bringmann and Ono proved in 2009 that this series converges to $\alpha(n)$ when extended to ∞. Here we obtain a power savings bound for the error in Andrews' formula using the spectral theory of Maass forms of half-integral weight. We also obtain bounds for the size of the error term incurred by truncating Rademacher's analytic formula for the ordinary partition function $p(n)$; this improves a recent result of Ahlgren and Andersen. This is a joint work with Scott Ahlgren.

Larry Ericksen

Continued fractions and polynomials related to hyperbinary representations

Abstract: A hyperbinary expansion of an integer n is an expansion of n as a sum of powers of 2, each power being used at most twice. Stern's diatomic sequence has long been connected with the concept of hyperbinary representations. We define 2-parameter generalized Stern polynomials in two variables, which explicitly characterize the individual hyperbinary expansions. We then identify several classes of finite and infinite continued fractions involving a single term in their partial numerators, and we obtain a class of companion polynomials with which we determine the denominators of these continued fractions. Finally we present sparser subsequences of the Stern polynomials, based on certain Lucas functions, which lead to further infinite classes of continued fractions. This is joint work with Karl Dilcher.

Ankush Goswami (University of Florida)

A q-analogue of Euler's $\zeta(6) = \pi^6/945$

Abstract: Recently, Zhi-Wei Sun obtained a q-analogue of Euler's $\zeta(2) = \pi^2/6$ and $\zeta(4) = \pi^4/90$. The q-analogue for the latter identity was also simultaneously and independently obtained by me. Motivated by this, I also obtained the q-analogue of $\zeta(6) = \pi^6/945$. In fact, we will see that the q-analogue for $\zeta(6)$ is more difficult as compared to $\zeta(2)$ and $\zeta(4)$. This difficulty arises due to an extra term that shows up in the

identity, however in the limit $q \to 1^-$, this term goes to zero. We will also state the q-analogue for Euler's $\zeta(2k)$ for $k \in \mathbb{Z}^+$. In the general case, however we have to deal with two cases separately, namely k even and k odd.

Min-Joo Jang (University of Cologne, Germany)

Quantum modular forms and singular combinatorial series

Abstract: Since Dyson defined the rank of a partition, a number of studies have been done on this statistic. For example, a celebrated result of Bringmann and Ono showed that the rank generating function is essentially a mock modular form. Andrews introduced k-marked Durfee symbols and more generally defined the ranks for them. In particular, when $k = 1$ one recovers Dyson's rank. In this talk, we establish the quantum modular properties of this combinatorial series, the rank generating function for k-marked Durfee symbols. This is joint work with Amanda Folsom, Susie Kimport, and Holly Swisher.

Chris Jennings-Shaffer (University of Cologne, Germany)

A depth 2 mock modular form coming from a partition function and conjecture of Andrews, Dixit, Schultz, and Yee

Abstract: Among the numerous integer partition smallest parts functions, one introduced by Andrews, Dixit, Schultz, and Yee is for an overpartition analogue of the third order mock theta function $\omega(q)$. It turns out this smallest parts function is essentially a mock modular form, but the modular properties of the underlying partition function are unclear and were left as conjecture by Andrews, Dixit, Schultz, and Yee. In joint work with Bringmann and Mahlburg, the underlying partition generating function was shown to be essentially a depth 2 mock modular form. This means the generating function can be "completed" to a function that transforms like a modular form, and the image of the "completion" under the Maass Lowering operator can be expressed in terms of harmonic Maass forms. The proof of this fact requires both q-series techniques and the mock modular forms machinery of Zwegers.

Shashank Kanade (University of Denver)

Some new q-series conjectures

Abstract: Representation theory of affine Lie algebras and more generally, vertex operator algebras, leads to interesting q-series identities. Especially, the principally specialized characters of standard modules for affine Lie algebras are related to important identities such as the Rogers-Ramanujan, Andrews-Gordon, Andrews-Bressoud, Capparelli and Gollnitz-Gordon identities. In this talk, I will present some new conjectures related to affine Lie algebras $D_4^{(3)}$ and $A_9^{(2)}$ which were obtained jointly with Matthew C. Russell.

Sun Kim (University of Cologne, Germany)

A generalization of the modular equations of higher degrees

Abstract: Ramanujan's modular equations of prime degrees 3, 5, 11, 7 and 23 are associated with elegant colored partition theorems. In 2005, Warnaar established a general identity which implies the modular equations of degrees 3 and 7. In this talk, we discuss a generalization of the remaining modular equations of degrees 5, 11 and 23. This generalization also implies new colored partition theorems and many identities conjectured by Sandon and Zanello.

Louis Kolitsch (The University of Tennessee at Martin)

Some theorems about partitions with restricted parts

Abstract: In this talk several results about partitions with restricted parts will be presented with analytical and combinatorial proofs. The restrictions involve gap conditions on the parts, the repetition of parts, and the sizes of the parts.

Tri Lai (University of Nebraska-Lincoln)

Cyclically symmetric Lozenge tilings of a hexagon with four holes

Abstract: The work of Mills, Robbins, and Rumsey on cyclically symmetric plane partitions yields a simple product formula for the number of lozenge tilings of a regular hexagon, which are invariant under rotation by 120 degrees. In this paper, we generalize this result to a hexagon in which four triangles have been removed. In particular, one triangle has been moved from the center of the hexagon and three more satellite triangles of are missing along the lines connecting the center of the hexagon and the vertices of the central removed triangle. We show that if the satellite triangles have the side even, then the cyclically symmetric tilings of the region is enumerated by a simple product formula. The case with odd-side satellite triangles does not yield a simple formula. (This is joint work with Ranjan Rohatgi, arXiv:1705.01122)

Lance Littlejohn (Baylor University)

Computing $\zeta(2n)$ via spectral theory and Green's functions

Abstract: In this talk, we derive the classic formula

$$\zeta(2n) = \frac{2^{2n-1}\pi^{2n}B_n}{(2n)!} \quad (n \in \mathbb{N}),$$

where $\zeta(\cdot)$ is the Riemann zeta function and B_n is the n^{th} Bernoulli number, via a differential equations/spectral theory approach. Specifically, we calculate the Green's function for the linear operator A^n, where

$$A : \mathcal{D}(A) \subset L^2[0,1] \to L^2[0,1]$$

is the self-adjoint Fourier sine operator defined by

$$\begin{cases} Af = -f'' \\ f \in \mathcal{D}(A) = \{f : [0,1] \to \mathbb{C} \mid f, f' \in AC[0,1]; f'' \in L^2[0,1]; f(0) = f(1) = 0\}. \end{cases}$$

From this Green's formula, the evaluation of $\zeta(2n)$ follows.

This work is joint with Baylor colleagues Fritz Gesztesy, Klaus Kirsten and Hagop Tossounian.

Xinrong Ma (Soochow University, China)

Transformations of basic hypergeometric series from the viewpoint of substitution of parameter operators

Abstract: This talk is about an operator method arising from the usual substitution of parameters to transformation formulas of basic hypergeometric series. As applications, some concrete substitution of parameter operators implied by some known and fundamental transformation formulas are defined, thereby applications to the ${}_{r+1}\phi_r$ series for $r = 1, 2$ are exploited. Among these applications, three results are of interest: one is an observation that Heine's transformations are both finite and closed under composition

operation; the second one is the equivalency of Watson's transformation of ${}_2\phi_1$ series and Bailey's well–known theta function identity; the third one is a generalization of Morita's connection formula for two independent solutions to the q–confluent hypergeometric equation. This is joint work with Jin Wang.

Bibekanand Maji **(Indian Institute of Technology Gandhinagar, India)**

Two-parameter generalization of Ramanujan's formula for $\zeta(2m+1)$ and its implications

Abstract: In this talk, we are going to obtain transformation formulas for a generalized Lambert series, which has been studied by Kanemitsu, Tanigawa, and Yoshimoto. While extending the work of Kanemitsu et al. we found a beautiful two-parameter generalization of the Ramanujan's famous formula for $\zeta(2m+1), m > 0$ and the transformation formula for the logarithm of Dedekind eta-function $\eta(z)$. An identity relating $\zeta(2N+1), \zeta(4N+1), \cdots, \zeta(2Nm+1)$ is obtained for N odd and $m \in \mathbb{N}$. Certain transcendence results of Zudilin- and Rivoal-type are obtained for odd zeta values and generalized Lambert series. A criterion for transcendence of $\zeta(2m+1)$ and a Zudilin-type result on the irrationality of Euler's constant γ is also given. This is a joint work with Atul Dixit, Rajat Gupta, and Rahul Kumar.

Amita Malik **(Rutgers University)**

Parity of certain restricted partition function

Abstract: It is expected that the partition function is even approximately half the time. In 1959, Kolberg showed that it takes even (odd) values infinitely often. Since then, there has been great interest in finding non-trivial lower bounds for the number of terms up to N where the partition function is even (odd) and the best known bounds are still far from the expected bound. In this talk, we discuss the parity of certain restricted partition function.

Mircea Merca **(University of Craiova, Academy of Romanian Scientists, Romania)**

Non-trivial linear partition inequalities and the Prouhet-Tarry-Escott problem

Abstract: An asymptotic classification for the linear homogeneous partition inequalities of the form $\sum_{i=1}^{r} p(n+x_i) \leqslant \sum_{i=1}^{s} p(n+y_i)$ has recently been introduced. In this paper, we investigate partition inequalities of this form when $r = s$. From an asymptotic point of view, such partition inequalities are considered to be non-trivial because they have the same number of terms on both sides. In this context, we provide a very general method for proving the non-trivial partition inequalities. This is a numerical method that does not involve q-series and connects the non-trivial linear homogeneous partition inequalities with the Prouhet-Tarry-Escott problem: if $\{x_1, x_2, \ldots, x_r\} \stackrel{k}{=} \{y_1, y_2, \ldots, y_r\}$ $(k \geqslant 0)$, then for n large enough the expression $\sum_{i=1}^{r} \big(p(n+x_i) - p(n+y_i)\big)$ has the same sign as $\sum_{i=1}^{r} (x_i^{k+1} - y_i^{k+1})$.

Hayan Nam **(University of California, Irvine)**

Tiling proof of Euler's pentagonal number theorem and generalizations

Abstract: In two papers, Little and Sellers introduced an exciting new combinatorial method for proving partition identities which is not directly bijective. Instead, they consider various sets of weighted tilings of a $1 \times \infty$ board with squares and dominoes, and for each type of tiling they construct a generating function in two different ways, which generates a q-series identity. Using this method, they recover quite a few classical q-series identities, but Euler's Pentagonal Number Theorem among them. In this talk, we introduce a key parameter when constructing the generating functions of various sets of tilings which allows us to recover

Euler's Pentagonal Number Theorem along with an infinite family of generalizations. This talk is based on joint work with Dennis Eichhorn and Jaebum Sohn.

Neville Robbins (San Francisco State University)

Integer triangles with 120 or 60 degree angles

Abstract: Using elementary means, we solve the equations $x^2 + xy + y^2 = z^2, x^2 - xy + y^2 = z^2$, where x, y, z are integers and $(x, y) = 1$.

Matthew Russell (Rutgers, The State University of New Jersey)

Bijective proofs of some new MacMahon-style partition identities

Abstract: Roughly a century ago, MacMahon proved a partition identity about partitions with no consecutive integers appearing as parts (or, equivalently, no part appearing exactly once). In recent years, there has been much interest in providing bijective proofs of this identity, along with some of its various generalizations. Some new families of MacMahon-style identities were conjectured after computer searches that were joint work of the speaker with Shashank Kanade and Debajyoti Nandi. We will use bijective ideas drawn from the work of Sylvester and others to provide proofs of these identities.

Manjil Pratim Saikia (University of Vienna, Austria)

Refined enumeration of Alternating Sign Matrices

Abstract: Alternating Sign Matrices (ASMs) are square matrices with entries in the set $\{0, 1, -1\}$ such that all row and column sums equal 1 and the non-zero entries alternate in sign in each row and column. ASMs are counted by a simple product formula, which also counts other combinatorial objects. We can also count ASMs with respect to their symmetry classes as well as boundary conditions (in each boundary row and column, there appears exactly one non-zero entry, a 1). In this talk, we will give new enumeration formulas for some of the symmetry classes of ASMs where one or more boundary conditions are fixed.

Michael Schlosser (University of Vienna, Austria)

Bilateral identities of the Rogers–Ramanujan type

Abstract: The Rogers–Ramanujan identities are deep identities which have found interpretations in combinatorics, probability theory, statistical mechanics, representations of Lie algebras, vertex algebras, and conformal field theory. In this talk, we present a number of bilateral identities of the Rogers–Ramanujan type which we have obtained by analytic means. Our closed form bilateral summations (involving sums which are indexed over the full range of integers with nonvanishing terms, just as in Ramanujan's ${}_1\psi_1$ summation formula) appear to be the first of their kind and include bilateral companions to the Rogers–Ramanujan and Göllnitz–Gordon identities. We also give multilateral companions to the Andrews-Gordon identities.

Maxie Schmidt (Georgia Institute of Technology)

New Connections Between Partitions and Multiplicative Functions

Abstract: Subtle and explicit connections between partitions and special classical functions in multiplicative number theory are decidely hard to come by in the body of literature surrounding the theory of partitions. A more general representative theory which connects special and restricted partition functions to multiplicative functions such as Euler's totient function $\phi(n)$, the Möebius function $\mu(n)$, von Mangoldt's prime characterizing function $\Lambda(n)$, the number of distinct prime factors of n denoted by $\omega(n)$, and the generalized sum-of-divisors functions $\sigma_\alpha(n)$ has been an unapproached topic up until recently. The work of Merca and Schmidt on so-termed "Lambert series factorization theorems" and partition-related matrix products over 2017–2018 has provided a general framework for connecting the theory of partitions with special multiplicative functions from more classical number theory (cf. arXiv papers 1706.00393, 1706.02359, 1712.00611, and 1712.00608). Additional work of Merca and Schmidt published in the Ramanujan Journal and to appear in the American Mathematical Monthly in 2018 respectively provide new identities and a generalization of Stanley's theorem connecting $\mu(n)$ and $\phi(n)$ to the partition functions $p(n)$ and $S_{n,k}^{(r)}$ which denotes the number of k's in all partitions of n into parts of size at least r. We will briefly summarize these recent characteristic factorization theorem approaches, the methodology to their proofs, and a plethora of new identities connecting $p(n)$ and $q(n)$ to special functions as well as new series representations for the Riemann zeta function $\zeta(s)$. We will also present new variants of these factorization theorems which expand a broader class of divisor sums which arise in applications as a part of our talk.

Robert Schneider (Emory University)

Toward an algebra of partitions

Abstract: Much like the positive integers $\mathbb{Z}^+$, the set $\mathcal{P}$ of integer partitions ripples with interesting patterns and relations. Now, as Alladi-Erdős point out, the prime decompositions of integers are in bijective correspondence with the set of partitions into prime parts, if we associate 1 to the empty partition. Might some number-theoretic theorems arise as images in $\mathbb{Z}^+$ (i.e. in prime partitions) of greater algebraic and set-theoretic structures in $\mathcal{P}$? In the 1970s, Andrews developed a beautiful theory of partition ideals using ideas from lattice theory, teasing the possibility of a universal algebra of partitions. Looking in a similar direction for arithmetic structures in the partitions, we show that many objects from elementary and analytic number theory are special cases of general partition-theoretic and q-series theorems: a multiplicative arithmetic of partitions that specializes to classical cases; a class of "partition zeta functions" containing $\zeta(s)$ and other Dirichlet series as well as exotic non-classical cases; partition formulas for arithmetic densities of subsets of $\mathbb{Z}^+$ such as kth-power-free integers; and other phenomena at the intersection of the additive and multiplicative branches of number theory.

Nicolas Smoot (Research Institute for Symbolic Computation, Austria)

A Family of Congruences for Rogers–Ramanujan Subpartitions

Abstract: Recent work at the Research Institute for Symbolic Computation has led to the development of powerful new techniques in the study of partition congruences. As an example, the Rogers–Ramanujan subpartition of a given partition λ is the largest subpartition of λ in which the parts are nonrepeating, nonconsecutive, and larger than the remaining parts of λ. Let $R_l(m)$ be the number of partitions of m containing a Rogers–Ramanujan subpartition of length l, and $A_1(m) = \sum_{l\geq 0} l \cdot R_l(m)$. Choi, Kim, and Lovejoy have conjectured the existence of a remarkable family of congruences for $A_1(m)$, in the spirit of Ramanujan's famous congruences for partition numbers with respect to powers of 5. We present recent progress in proving these congruences. This work provides an example of the importance of computer algebra to the continuation of research in partition theory.

Kenneth Stolarsky (University of Illinois at Urbana-Champaign)

Large element subsets and planar curves

Abstract: Let $[n]$ denote the set of the first n integers. We use generating functions to enumerate subsets of $[n]$ whose smallest element is large compared to card(S), where card(S) satisfies congruence conditions.

This leads to the study of certain non-convex planar curves on which the poles of the generating functions are located. The Turan-Erdos theorem on distribution of polynomial zeros is employed here. This work is joint with George Andrews.

Armin Straub (University of South Alabama)

Gauss congruences

Abstract: The Gauss congruences are a natural generalization of the more familiar Fermat and Euler congruences. Interesting families of combinatorial and number theoretic sequences are known to satisfy these congruences. Though a general classification remains wide open, Minton characterized constant recursive sequences satisfying Gauss congruences. We consider the natural extension of this question to Laurent coefficients of multivariate rational functions. This talk is based on joint work with Frits Beukers and Marc Houben.

Pee Choon Toh (Nanyang Technological University, Singapore)

On the crank function of cubic partition pairs

Abstract: Define $b(n)$ the counting function for the number of cubic partition pairs of n by its generating function as follows.

$$\sum_{n=0}^{\infty} b(n)q^n = \frac{1}{(q;q)_\infty^2 (q^2;q^2)_\infty^2}.$$

It is known that $b(n)$ satisfies the following Ramanujan type congruence:

$$b(5n+4) \equiv 0 \pmod 5.$$

We will introduce a crank function for cubic partition pairs which explains the above congruence. We will also study the function $c(n)$ which counts the number of cubic partition pairs of n, weighted by the parity of the crank. Joint work with Byungchan Kim.

Ali K. Uncu (RISC, Johannes Kepler University, Austria)

Counting on 4-decorated Ferrers diagrams

Abstract: Integer partitions with even parts below odd parts (EO-partitions) have recently been studied extensively by G. E. Andrews. In this talk, we will present some new weighted partition and composition identities. In particular, we will discuss a (not so conventional) connection between a subset of the EO-partitions and the Rogers–Ramanujan partitions.

Khoi Vo (California State University, Long Beach)

Density of subset of solitary numbers

Abstract: The question of the density of the set of solitary numbers has been left open for a long time. It has been conjectured that its value is zero. Recently, Loomis has brought up new sub-classes of solitary numbers. In this presentation, we will proceed in proving the density of these sub-classes of solitary numbers are indeed all zero. This result helped us one step further on the way of proving the conjecture.

Ian Wagner (Emory University)

Hyperbolicity of the partition Jensen polynomials

Abstract: Given an arithmetic function $a : \mathbb{N} \to \mathbb{R}$, one can associate a naturally defined, doubly infinite family of Jensen polynomials. Recent work of Griffin, Ono, Rolen, and Zagier shows that for certain families of functions $a : \mathbb{N} \to \mathbb{R}$, the associated Jensen polynomials are eventually hyperbolic (i.e., eventually all of their roots are real). This work proves Chen, Jia, and Wang's conjecture that the partition Jensen polynomials are eventually hyperbolic as a special case. Here, we make this result explicit. Let $N(d)$ be the minimal number such that for all $n \geq N(d)$, the partition Jensen polynomial of degree d and shift n is hyperbolic. We prove that $N(3) = 94$, $N(4) = 206$, and $N(5) = 381$, and in general, that $N(d) \leq (3d)^{24d}(50d)^{3d^2}$.

Chen Wang (University of Vienna, Austria)

A proof of Borwein's conjecture

Abstract: Borwein conjectured in 1990 that the m-th coefficient of the polynomials $(q;q)_{3n}/(q^3;q^3)_n$ is always non-negative when 3 divides m, and non-positive when 3 does not divide m. In this talk, I will present a proof of Borwein's conjecture, utilizing the saddle point method and some expansions, first given by Andrews in 1994, to give explicit upper and lower bounds on the coefficients.

Chun Wang (East China Normal University, China)

Truncated Hecke-Rogers type series - Part II

Abstract: In 2012, Andrews and Merca investigated the truncated version of the series of Euler. Their work has inspired several mathematicians to work on the topic. Recently, Yee and I generalized the study on truncated theta series to several Hecke-Rogers type series, which are associated with certain partition functions. In her talk, Yee presents some of those results. In this talk, I will continue to discuss further results from our joint work.

Liuquan Wang (Wuhan University, China)

Modular forms and k-colored generalized Frobenius partitions

Abstract: Let k and n be positive integers. Let $c\phi_k(n)$ denote the number of k-colored generalized Frobenius partitions of n and $\mathrm{C}\Phi_k(q)$ be the generating function of $c\phi_k(n)$. In this talk, we discuss a new way to study $\mathrm{C}\Phi_k(q)$ by using the theory of modular forms and discover some surprising properties of $\mathrm{C}\Phi_k(q)$. In particular, we find alternative representations of $\mathrm{C}\Phi_k(q)$ for all k less than 18 and a general congruence for $c\phi_k(n)$ modulo arbitrary prime powers. We also reveal some relations between $c\phi_k(n)$ and the ordinary partition function $p(n)$.
