Bayesian change point model with Gamma hyperpriors: full conditionals

Our goal is to draw samples from the 5-dimensional posterior distribution $f(k, \theta, \lambda, b_1, b_2|Y)$. The posterior distribution is

$$f(k, \theta, \lambda, b_1, b_2|Y) \propto \prod_{i=1}^{k} \frac{\theta^{Y_i}e^{-\theta}}{Y_i!} \prod_{i=k+1}^{n} \frac{\lambda^{Y_i}e^{-\lambda}}{Y_i!} \times \frac{1}{\Gamma(0.5)b_1^{0.5}} \theta^{-0.5}e^{-\theta/b_1} \times \frac{1}{\Gamma(0.5)b_2^{0.5}} \lambda^{-0.5}e^{-\lambda/b_2}$$

(1)

From 1 we can obtain full conditional distributions for each parameter by ignoring all terms that are constant with respect to the parameter.

For $\theta$:

$$f(\theta|k, \lambda, b_1, b_2, Y) \propto \prod_{i=1}^{k} \frac{\theta^{Y_i}e^{-\theta}}{Y_i!} \times \frac{1}{\Gamma(0.5)b_1^{0.5}} \theta^{-0.5}e^{-\theta/b_1}$$

(2)

For $\lambda$:

$$f(\lambda|k, \theta, b_1, b_2, Y) \propto \prod_{i=k+1}^{n} \frac{\lambda^{Y_i}e^{-\lambda}}{Y_i!} \times \frac{1}{\Gamma(0.5)b_2^{0.5}} \lambda^{-0.5}e^{-\lambda/b_2}$$

(3)

For $k$:

$$f(k|\theta, \lambda, b_1, b_2, Y) \propto \prod_{i=1}^{k} \frac{\theta^{Y_i}e^{-\theta}}{Y_i!} \prod_{i=k+1}^{n} \frac{\lambda^{Y_i}e^{-\lambda}}{Y_i!}$$

(4)

For $b_1$:

$$f(b_1|k, \theta, \lambda, b_2, Y) \propto \frac{1}{b_1^{0.5}} e^{-\theta/b_1} \times b_1^{c_1-1}e^{-b_1/d_1}$$

(5)

For $b_2$:

$$f(b_2|k, \theta, \lambda, b_1, Y) \propto \frac{1}{b_2^{0.5}} e^{-\lambda/b_2} \times b_2^{c_2-1}e^{-b_2/d_2}$$

(6)

$f(b_1|k, \theta, \lambda, b_2, Y)$ and $f(b_2|k, \theta, \lambda, b_1|Y)$ are not well known densities. We can use a Metropolis-Hastings accept-reject step to sample from their full conditionals.