Bargaining on behalf of price-insensitive consumers

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Abstract

There are economic settings in which decision makers are indifferent to prices. For example, a patient seeking medical treatment is indifferent to the treatment’s cost, if it is covered by his insurance program. We study price-bargaining between suppliers and an intermediary, where the latter represents consumers who are characterized by price insensitivity. We show that when the intermediary bargains simultaneously with all suppliers, then if the suppliers have sufficient bargaining power the resulting prices are too high; specifically, prices exceed the value of the good/service being delivered. Sequential negotiations prevent this pathology. The intermediary’s payoff is independent of the negotiations order, but the suppliers’ payoffs are not; under some conditions, a supplier’s payoff is increasing in the supplier’s position in the sequence.

Keywords: Bargaining; Overpricing; Price insensitivity.

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1 Introduction

In most economic settings, agents care about prices and costs. Consumers care about the prices of the goods they buy, firms about their production costs, and so on. There are, however, settings in which decision makers do not consider prices when taking their decisions. For example:

- A patient seeking medical treatment that is covered by medical insurance—he ignores the treatment’s price when taking his decision since it is paid by the insurer;
- A military commander choosing which (or how much) ammunition to use in battle—he is unlikely to consider its costs.

A common feature of these examples is the vertically structured economy in which an intermediary (e.g., an insurance company, Department of Defense) bargains prices with suppliers (e.g., hospitals, military contractors), under the assumption that the negotiated prices will have minimal to no effect on the end users’ decisions. In what follows we study such negotiations. For concreteness, and to fix ideas, we consider an insurer who bargains with hospitals for per-treatment prices, and whose goal is to maximize the consumers’ expected surplus. However, our results apply to any environment which is characterized by the aforementioned vertical structure and price insensitivity.

We start by considering simultaneous negotiations, where the insurer Nash bargains with each hospital separately, taking the prices with all other hospitals as given—an approach that originated in Horn and Wolinsky (1988). We follow the term coined in Collard-Wexler (2014) and call it *Nash-in-Nash* (NiN). In the NiN model, the insurer bargains with each hospital, holding the contracts with all other hospitals fixed. The Nash product is formed by comparing the hospital-network’s surplus with and without the bargained-with hospital. Consider the case of two hospitals, $A$ and $B$. Consider bargaining with $A$, given the negotiated price with $B$. 

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The network’s surplus without $A$ depends not only on $B$’s price but also on $A$’s patients that would substitute to $B$ if $A$ left the network, and their value for $B$. Since patients are price-insensitive, they will go to hospital $B$ even if their value for $B$ is lower than $B$’s price. Thus, these patients create a negative surplus when $A$ is not in the network. As a result, $A$’s marginal value per-patient when added to the network (given $B$’s price) is higher than its actual per-patient value, since its addition to the network prevents the aforementioned negative surplus creation. We show that in any network with two or more hospitals, as long as some patients substitute to an in-network hospital when their most preferred hospital is out of the network, prices exceed patient valuations, given that the hospitals’ bargaining power is sufficiently large. We call this phenomenon *Nash overpricing*. In Theorem 1 we show that if hospitals’ bargaining power is large enough, Nash overpricing will occur for every hospital in the network, which we refer to as *complete Nash overpricing*.

NiN is a common framework for studying bargaining problems, in particular in applied theoretical work.\(^1\) Our main contribution lies in highlighting that when consumers are price-insensitive, NiN may lead to unexpected and undesirable outcomes; specifically, to Nash overpricing. We also suggest a fairly simple solution to the problem, as follows.

Overpricing can be resolved if, instead of simultaneously, the insurer interacted with the hospitals sequentially. A straightforward backward induction argument shows that this approach rules out Nash overpricing: With $J$ hospitals ordered in a sequence, given that prices with hospitals $\{1, \cdots, J-1\}$ have been determined, the insurer faces a standard 2-player bargaining problem with hospital $J$, in which his payoff is positive. In the negotiation with hospital $J-1$ this is taken into account, and so negotiations with hospital $J-1$ are also a standard 2-player bargaining problem in which the insurer’s payoff is positive, and so on. Under sequential nego-

\(^1\)See Collard-Wexler et al. (2019) and the references therein.
tions the insurer’s payoff is independent of the order of negotiations. The reason is that the price formula that are obtained from the maximization of Nash products internalize the effects of earlier negotiations on later ones, and this internalization turns out to be perfect: switching from any negotiations order to any other results in price-adjustments that exactly off-set the changes induced by price-insensitivity. By contrast, hospitals do care about the order of negotiations. For example, in a two-hospital setting the first hospital’s price is lower than the second hospital’s price, because once there is disagreement with the first hospital, and it “drops out” of the game, the second hospital becomes a monopolist. By contrast, there is no such favorable effect for the first hospital if/when the second hospital drops out.

The rest of the paper is organized as follows. Section 2 lays down the environment, Section 3 considers simultaneous negotiations within this environment (NiN), Section 4 considers sequential negotiations, and Section 5 concludes. The overpricing problem that we identify in the NiN model is derived under the assumption that the pool of hospitals is exogenous. In Appendix A we show that being able to exclude some hospital from the pool (ex ante exclusion), as well as excluding some hospitals after contracts with them have already been signed (ex post exclusion), do not provide a satisfactory solution to overpricing. Thus, the overpricing problem is robust. Appendix B collects proofs that are omitted from the main text.

2 The environment

An insurer bargains with \( J \geq 2 \) hospitals on behalf of a mass of heterogeneous patients. Under the full network, which comprises all \( J \) hospitals, the quantity of patients treated by hospital \( j \) is \( q_j > 0 \). The expected value for a patient who goes to

\[^2\text{Other works in the literature describe settings in which, contrary to ours, the order of negotiations does matter; e.g., Manea (2018) and Xiao (2018).}\]
hospital \( j \) (given the full network) is \( v_j > 0 \).\(^3\) Regardless of what hospitals are in the network, patients always have the (outside) option of not seeking medical treatment, which is associated with the value zero.

If a hospital, say \( j \), is not part of the network, then it is not available for patients to seek treatment. This event only affects those patients who prefer to be treated at \( j \), who then go to their next preferred hospital. The hospital choice of patients that chose hospital \( l \neq j \) when \( j \) is in the network does not change. The mass of additional consumers for hospital \( k \) when hospital \( j \) is dropped from the network and those patients’ expected value are denoted by \( q_{k,-j} \) and \( v_{k,-j} \), respectively.

Hospitals can treat patients with a marginal cost \( c_j \geq 0 \). Therefore, under the full network hospital \( j \)’s profit is:

\[
\pi_j = (p_j - c_j)q_j.
\]

If hospital \( j \) is out of the network, it receives zero profit. However, if another hospital, say \( k \), is out of the network, \( j \) receives more patients and its profits become:

\[
\pi_{j,-k} = (p_j - c_j)(q_j + q_{j,-k}).
\]

The hospitals are symmetric if \( \{q_j, v_j, q_{k,-j}, v_{k,-j}, c_j\} \) are independent of \( k \) and \( j \).

We make the following assumptions:

1. (I) For all \( j \): \( \sum_{k \neq j} q_{k,-j} > 0 \);
2. (II) For all \( j \) and \( k \): \( q_{j,-k} > 0 \Rightarrow v_j > v_{j,-k} \);
3. (III) For all \( j \): \( c_j < \min\{v_{j,-k} : q_{j,-k} > 0\} \);
4. (IV) For all \( j \) and \( k \) with \( q_{k,-j} > 0 \): \( v_j - c_j > v_{k,-j} - c_k \).

(1) says that for every hospital \( j \), at least some patients have a second-choice-hospital that they prefer over the outside option. (II) requires that patients whose

\(^3\)The expectation is over patients: the patients are heterogeneous, and distinct patients for whom \( j \) is the most preferred hospital may value it differently.
first choice of a hospital is \(j\), on average value hospital \(j\) more than patients for whom \(j\) is the second choice. (III) says that the surplus from providing service to patients for whom the service provider is the second choice is still a positive surplus. This has two important implications. First, it implies—because of (I) and (II)—that \(c_j < v_j\), and so the surplus from the full network is positive. Second, it means that negative surplus for the insurer—the thing around which our paper pretty much revolves—is \textit{only} because of overpricing, not because of providing service by “technologically expensive/inefficient” hospitals. Finally, (IV) is a bound that means that the surplus generated by the first-choice hospital is large enough; specifically, it is greater than the surplus that would have been generated had that first choice been removed from the network and then we looked at what surplus the patients who remain in the network generate in any other hospital.

It is worth noting that in all our examples the cost, for simplicity, is taken to be zero. This means (III) and (IV) follow from (I) and (II) in this case.

The insurer maximizes patient surplus. Therefore, the insurer’s value from the full network, given a price vector \(p = (p_1, \ldots, p_J)\), is:

\[
F(p) = \sum_{j=1}^{J} (v_j - p_j) q_j.
\]

Similarly, the insurer’s surplus from the network without hospital \(j\), given the remaining hospital prices, is:

\[
F_{-j}(p) = \sum_{k \neq j} \left[ (v_k - p_k) q_k + (v_{k,-j} - p_k) q_{k,-j} \right].
\]

Note that by assumptions (I) and (II), equations (1) and (2) imply that the insurer’s objective is non-linear; that is, the following holds for every \(j\):

\[
F(p) \neq F_{-j}(p) + (v_j - p_j) q_j.
\]

This non-linearity is a direct consequence of price-insensitivity.

We study two bargaining protocols between the insurer and the hospitals, to be
specified in the next sections; the (asymmetric) Nash bargaining solution is common to both of them, which justifies the following terminology: we say that *Nash over-pricing* occurs if there is some hospital *j* whose price exceeds its value, i.e., *p*<sub>*j*</sub> > *v*<sub>*j*</sub>. If this is true for every *j*, then we say that there is *complete Nash overpricing*.

### 3 Simultaneous negotiations

We start by considering the case where prices between the insurer and each hospital are set following the Nash bargaining solution, holding all other prices fixed; the hospital’s bargaining power parameter is *β* ∈ (0, 1). We refer to this model as the *NiN model*, and to its prices as *NiN prices*.

The NiN prices \( (p^N_1, \cdots, p^N_J) \) satisfy:

\[
p^N_j = \max_{p_j} \left[ F(p_j, p^N_{-j}) - F_{-j}(p^N_{-j}) \right]^{(1-\beta)} \cdot \left[ q_j(p_j - c_j) \right]^\beta.
\]

Maximization of the Nash product gives:

\[
p^N_j = \beta [v_j - \frac{\sum_{k \neq j} (v_k - p^N_k) q_{k,-j}}{q_j}] + (1 - \beta) c_j.
\]  

(3)

A solution to this system of equations is called an *equilibrium*.

**Theorem 1.** *In the NiN model, an equilibrium exists, and it is unique. There exists a \( \bar{\beta} < 1 \), such that if the hospital’s bargaining power parameter satisfies \( \beta \in (\bar{\beta}, 1) \), then each of the NiN prices exceeds the value of service in the corresponding hospital. That is,*

\[
p^N_j > v_j \quad \forall j = 1, \cdots, J.
\]

*Namely, there is complete Nash overpricing.*

**Proof.** We start by establishing existence and uniqueness. Note that we may assume that prices never exceed some (possibly large) bound *M*: clearly, no hospital can obtain a price that exceeds its value plus all the “adverse selection prevention” that
its addition to the network can bring about. Then the RHS of (3) describes a map from \([0, M]^J\) into itself. Though this is a map of vector-to-vector, it can be viewed as an operator on functions because a vector is a constant function. It is easy to check that this operator—i.e., the RHS of (3)—satisfies Blackwell’s sufficient conditions for contraction (monotonicity and discounting). Therefore, (3) has a unique solution; that is, an equilibrium exists, and is unique.

We now turn to complete Nash overpricing. Consider (3):

\[
p_j^N = \beta v_j - \beta \sum_{k \neq j} (v_{k,-j} - p_k^N) q_{k,-j} + (1 - \beta) c_j.
\]

Suppose that \(q_{k,-j} = 0\) for all distinct \(k\) and \(j\). Then \(p_j^N \to v_j\) as \(\beta \to 1\).

For every distinct \(k\) and \(j\) define \(\epsilon_{kj}\) by:

\[
\epsilon_{kj} \equiv p_k^N - v_{k,-j}.
\]

By assumption, \(v_k > v_{k,-j}\) for all distinct \(j\) and \(k\) for which \(q_{k,-j} > 0\). Therefore, there exists an \(\epsilon > 0\) such that for all sufficiently large \(\beta\)'s it holds that \(\epsilon_{kj} > \epsilon\) for all such \(k\) and \(j\). Suppose that \(\beta\) is sufficiently large such that an \(\epsilon > 0\) as above exists. From here on, this \(\beta\) and \(\epsilon\) are fixed.

Let then \(p_0\) be the vector of prices that solves the above system (uniquely), when \(q_{k,-j} = 0\) for all distinct \(k\) and \(j\). Because \(\beta\) is large enough, \((p_0^1, \cdots, p_0^J) > (v_1, \cdots, v_J)\), or simply \(p_0 > v\).

Now, consider the following \(J\)-step process:

Step 1: Increase \(\{q_{k,-1}\}_{k>1}\) from zero to their true values. These \((J-1)\) coefficients only appear in the formula for \(p_1^N\), and this increase has two effects. The “first order” effect is to increase only \(p_0^1\). Specifically, it increases it by \(\frac{\beta}{q_1} \sum_{k \neq 1} \epsilon_{k1} q_{k,-1} > \frac{\beta}{q_1} \epsilon \sum_{k \neq 1} q_{k,-1} > 0\), where the second inequality is by assumption. Next, the increase in \(p_1^0\) affects the other prices, because \(p_1^N\) appears in the formula for any other \(p_j^N\), \(j \neq 1\). The increase in \(p_1^0\) increases any other price. Call the resulting price vector

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4Existence and uniqueness follow from the abovementioned contraction argument.
Clearly, \( p^1 > p^0 \).

Step 2: Increase \( \{q_{k,-2}\}_{k \neq 2} \) from zero to their true values. Again, this has two effects, both of which increase prices. A new vector with higher prices is then obtained, \( p^2 > p^1 \).

Repeating the above process with Steps 3, \( \cdots \), \( J \) results in the vector \( p^J = p^N > v \), and completes the proof.

We now turn to an example that illustrates Nash overpricing is a relatively simple two-hospital setting. In Appendix A we investigate a couple of modifications of the NiN framework and show that neither of them can provide a satisfactory solution to overpricing. In this sense, the overpricing problem is robust in the NiN framework.

**Example: Two symmetric hospitals**

Consider two hospitals, \( A \) and \( B \). The cost of serving a patient is zero. The market has four types of patients, \( \{ab, a0, ba, b0\} \). Patients of type \( ab \) (resp. \( ba \)) have a value of \( u^h = 10 \) from being served by hospital \( A \) (resp. \( B \)) and \( u^l = 5 \) from being served by the other hospital. Patients of type \( a0 \) (resp. \( b0 \)) have a value of \( u^h = 10 \) from hospital \( A \) (resp. \( B \)) but would leave the network if hospital \( A \) (resp. \( B \)) leaves the network. In other words, the \( ab \) and \( a0 \) types prefer \( a \) over the alternatives but disagree on their second choice hospital (\( B \) or out-of-network). The \( ab \) and \( ba \) types (equivalently \( a0 \) and \( b0 \)) disagree on whether their first option is \( A \) or \( B \).

With both hospitals in the network, the insurer expects a unit mass of patients for each hospital. There are \( \alpha \) patients of type \( ab \) and the same for \( ba \), and \( (1 - \alpha) \) patients of each of types \( a0 \) and \( b0 \).

Thus:

\[
F(p) = 20 - p_A - p_B ; \text{ and } F_{-A}(p) = \alpha \cdot (5 - p_B) + (10 - p_B) .
\]

Under Nash bargaining, if we hold \( p_B \) fixed then \( p_A \) solves:

\[
\max_p (F - F_{-A})^{(1-\beta)} \cdot p^\beta
\]
The price response is given by:

\[
p_A^N = \beta \cdot \left[10 (1 - \alpha) + \alpha \left(5 + p_B^N\right)\right]. \tag{4}
\]

Equation (4) shows that A obtains a fraction \(\beta\) of the surplus it generates. The \((1 - \alpha)\) new patients each account for 10 utils. The \(\alpha\) patients that instead would have went to \(B\) only gain 5 directly from going to their preferred hospital, but also save the payment of \(p_B\).

Of these two consumer segments \((1 - \alpha)\) and \(\alpha\), prices may surpass value only because of the second \((\alpha)\) group. In particular, whenever \(p_B > 5\), the insurer actually generates negative surplus (net of prices) serving these patients without \(A\) in the network. The surplus that \(A\) generates to these patients is then higher than it’s ex-post per-patient value of 10. If the hospitals’ bargaining power is sufficiently high so that hospitals obtain most of the surplus they generate, price will be higher than the ex-post value.

Formally, prices are obtained by solving (4) and the symmetric equation for \(p_B\). This gives:

\[
p_j^N = 5 \beta \frac{2 - \alpha}{1 - \beta \alpha},
\]

for both \(j \in \{A, B\}\). The following is easy to verify:

\[
p_j^N \leq 10 \iff \beta \leq \frac{2}{2 + \alpha}. \tag{5}
\]

Since the value from being served by the top choice is 10, it follows that for this example, \(\bar{\beta}\) from Theorem 1 equals \(\frac{2}{2+\alpha}\).

The example is easily generalized for any positive \(u^h\) and \(u^t\). In particular, letting \(u^t = \lambda u^h\) for \(\lambda \in (0, 1)\) the insurer generates negative surplus (i.e., \(p_j^N > u^h\)) iff:

\[
\beta \geq \frac{1}{1 + \alpha(1 - \lambda)}.
\]

In particular, for \(\alpha, \lambda \in (0, 1)\), there is some \(\bar{\beta} < 1\) such that for any \(\beta > \bar{\beta}\) the price of each service is higher than its value.

\(^{5}\text{For } \lambda = 0.5 \text{ one obtains (5).}\)
4 Sequential negotiations

The difficulty arising from NiN lies in the hypothetical “disagreement event” associated with each negotiation. For example, in the two-hospital case, when $A$ is out of the network (i.e., there is disagreement with $A$) its entry-contribution exceeds its true value because its absence from the network generates an adverse shift in patients going to $B$ which would be welfare reducing at $B$’s price. Avoiding this outcome crucially depends on constructing a bargaining mechanism where the impact of disagreement in a certain problem on other bargaining problems is taken into account. We now take this approach, in our sequential Nash model, which is as follows: the hospitals are ordered in a (fixed, commonly known) sequence, and if negotiation breaks down with some hospital $j$, all subsequent negotiations assume that $j$ is not in the insurer’s network.

It is easy to see that the insurer’s surplus is positive under sequential negotiations. Consider again the two-hospital case with $A$ going first and $B$ going second. Given any outcome of negotiations with $A$, the bargaining problem with $B$ must result in a non-negative addition to the insurer’s overall surplus (or else the insurer will not sign a deal with $B$). Additionally, one can map any possible outcome in the $A$-negotiations, say $o$, to the subsequent bargaining problem with $B$, say $P(o)$. Since, as just observed, the insurer’s surplus in $P(o)$ is non-negative given any possible $o$, the bargaining problem with $A$ boils down to a standard bargaining problem in which both parties make positive profits. This idea generalizes to any length of hospital-sequence.

The sequential order of negotiations does not affect the insurer’s surplus, but it does affect negotiated prices and hospital payoffs. For example, in the two-hospital case where $A$ is first, disagreement with $A$ automatically makes $B$ a monopolist. In contrast, disagreement with $B$ cannot have such a favorable effect on $A$’s bargaining position, since it can only happen after the interaction with $A$ has concluded. Specifically, either (i) a deal with $A$ has already been signed and so $A$’s price is fixed, or
(ii) $A$ has dropped out, and is no longer in the network. We show that if there are only two hospitals, or if there are $J$ symmetric hospitals, then prices are increasing in the hospital’s position in the negotiation-sequence.

The economic environment is the same as the one considered in NiN, the only difference is that the Nash products reflect the order of negotiations. We start by presenting the two-hospital case and then turn to the general case.

### 4.1 Two hospitals

The insurer negotiates first with hospital $A$. Consider the insurer’s bargaining with hospital $B$ given that bargaining with hospital $A$ has concluded. Since $p_A$ has already been determined, this bilateral negotiation is effectively equivalent to the NiN bargaining setup. Without hospital $B$, the insurer’s surplus is determined by his agreement with hospital $A$. The insurer’s surplus with only $A$ is:

$$F = q_A(v_A - p_A).$$

For any price $p_B$ the insurer’s value with both hospitals in the network is:

$$F = q_A(v_A - p_A) + q_B(v_B - p_B).$$

Since the additional surplus accruing to the insurer from adding hospital $B$ to a network with only hospital $A$ is bounded below by zero (else $B$ would not be added), it is given by:

$$F - F_B = \max\{0, q_B(v_B - p_B) - q_A(v_A - p_A)\}.$$ 

For the moment, we assume that the addition of $B$ is worthwhile, hence:

$$F - F_B = q_B(v_B - p_B) - q_A(v_A - p_A).$$

After making some calculations, we will verify, ex post, that this assumption is indeed correct.
The Nash product is \( [F - F_{-B}]^{1-\beta} \cdot q_B(p_B - c_B)]^\beta \). Maximizing it gives the price:

\[
p_B = \beta \frac{q_B v_B - q_{A,-B}(v_{A,-B} - p_A)}{q_B} + (1 - \beta)c_B. \tag{6}
\]

Substituting this \( p_B \) into the expression for \( F \) we obtain:

\[
F = q_A(v_A - p_A) + q_B v_B - q_B \left[ \beta \frac{q_B v_B - q_{A,-B}(v_{A,-B} - p_A)}{q_B} + (1 - \beta)c_B \right] \tag{7}
\]

\[
= q_A(v_A - p_A) + q_B(v_B - c_B)(1 - \beta) + \beta q_{A,-B}(v_{A,-B} - p_A).
\]

Now consider bargaining with \( A \). Without \( A \), the insurer will bargain with \( B \), when the insurer’s outside option is zero. Maximizing the Nash product for this problem gives:

\[
F_{-A} = (1 - \beta)(q_B(v_B - c_B) + q_{B,-A}(v_{B,-A} - c_B)). \tag{6}
\]

Therefore,

\[
F - F_{-A} = q_A(v_A - p_A) + \beta q_{A,-B}(v_{A,-B} - p_A) - (1 - \beta)q_{B,-A}(v_{B,-A} - c_B).
\]

Maximizing the Nash product \( [F - F_{-A}]^{1-\beta} \cdot [q_A(p_A - c_A)]^\beta \) gives:

\[
p_A = \beta \frac{q_A v_A + \beta q_{A,-B}v_{A,-B} - (1 - \beta)q_{B,-A}(v_{B,-A} - c_B)}{q_A + \beta q_{A,-B}} + (1 - \beta)c_A. \tag{8}
\]

Equipped with these formulas, we can turn to the results.

**Proposition 1.** In the sequential Nash model with two hospitals, the insurer’s surplus is independent of the order of negotiations.

**Proof.** Consider the case where hospital \( A \) is first and \( B \) is second. Plugging \( p_A \) from (8) into the expression for the surplus, (7), gives the surplus:

\[
(1 - \beta)q_A(v_A - c_A) + (1 - \beta)q_B(v_B - c_B) + \beta(1 - \beta)q_{A,-B}(v_{A,-B} - c_A) +
\]

\[
\beta(1 - \beta)q_{B,-A}(v_{B,-A} - c_B).
\]

Clearly, the same expression obtains if \( B \) goes first. \(\square\)

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\(6\)The insurer obtains a \((1 - \beta)\)-fraction out of the surplus.
We now turn to verify that the above derivation is valid; namely, that it is worthwhile to have the second hospital join the network at the second stage, and the optimum network is the full one. W.l.o.g, it suffices to show that the full network brings greater surplus than the one consisting only of $B$. Utilizing the expression derived in Proposition 1’s proof, what needs to be verified is:

\[
(1 - \beta)q_A (v_A - c_A) + (1 - \beta)q_B (v_B - c_B) + \beta(1 - \beta)q_{A,B} (v_{A,B} - c_A) + \\
+ \beta(1 - \beta)q_{B,A} (v_{B,A} - c_B) > (1 - \beta)q_B (v_B - c_B) + (1 - \beta)q_{B,A} (v_{B,A} - c_B),
\]

which simplifies to:

\[
q_A (v_A - c_A) + \beta q_{A,B} (v_{A,B} - c_A) > (1 - \beta)q_{B,A} (v_{B,A} - c_B).
\]

By definition, $q_A \geq q_{B,A}$, and by assumption (IV) we have that $v_A - c_A > v_{B,A} - c_B$. Thus, the first term on the LHS is larger than the RHS. The remaining element on the LHS is positive, by assumption (III).

The intuition behind Proposition 1 is that when bargaining with the first hospital in the sequence, $A$, the insurer internalizes the effect that this bargaining will have on the next problem in the sequence, namely negotiations with $B$. This is exemplified by the various terms on the RHS of (8). Most importantly, the denominator $q_A + \beta q_{A,B}$ expresses this internalization. If, for example, $q_{A,B}$ is large, then one implication of signing a deal with $A$ is that if there will be disagreement with $B$ later on in the negotiations sequence, many of $B$’s would-be patients would now go to $A$, and in order to prevent the possibility of overpricing a la NiN, the price needs to be adjusted sufficiently downwards, and this is exactly what happens according to the price formula.

We now turn to the hospitals’ profits.

Denote by $\pi_1^j$ the profit of hospital $j$ if it is the first hospital in the sequence, and denote by $\pi_2^j$ its profit if it is second.
Proposition 2. Consider the sequential Nash model with two hospitals. There exists a $\beta^{**} < 1$ such that if $\beta \in (\beta^{**}, 1)$ then $\pi^1_j < \pi^2_j$ for both $j = A, B$.

Proof. Wlog, consider hospital $A$. Since the quantities are unaffected by price and by the order of negotiations, it is enough to show that for all sufficiently large $\beta$'s we have $p^1_A < p^2_A$, where $p^l_A$ is the price corresponding to $\pi^l_A$. It is enough to verify that that’s the case when $\beta = 1$.

Setting $\beta = 1$ in (8) gives:

$$p^1_A = \frac{q_A v_A + q_{A,-B} v_{A,-B}}{q_A + q_{A,-B}}.$$  \hfill (9)

Setting $\beta = 1$ in the analog of (6) gives:

$$p^2_A = \frac{q_A v_A - q_{B,-A}(v_{B,-A} - p^1_B)}{q_A}.$$  

We argue that $\frac{q_A v_A + q_{A,-B} v_{A,-B}}{q_A + q_{A,-B}} < \frac{q_A v_A - q_{B,-A}(v_{B,-A} - p^1_B)}{q_A}$. Simplifying this expression we get:

$$q_A q_{A,-B}(v_{A,-B} - v_A) < -q_{B,-A}(v_{B,-A} - p^1_B)(q_A + q_{A,-B}).$$

The LHS is negative since $v_{A,-B} < v_A$. Therefore, it is enough to prove that the RHS is positive, or that $p^1_B > v_{B,-A}$. Clearly, it is enough to show that $p^1_A > v_{A,-B}$. This follows immediately from (9), since $v_A > v_{A,-B}$. \hfill \Box

The intuition behind Proposition 2 is that being last in the negotiations sequence confers a monopolistic position on the hospital, and this has no counterpart for the first position in the sequence.

4.2 An arbitrary number of hospitals

In the case of $J \geq 3$ hospitals we add the following assumption: we assume that patients leave the insurer if their top two options leave the network. To see the importance of this assumption, suppose that the hospitals are ordered from 1 to $J$, and consider negotiations with hospital $j - 1$. If there is disagreement in these
negotiations, then the insurer moves on to bargain with hospital $j$. Now, in order to formulate the Nash product for these negotiations, it is important to know what happens in case there is disagreement with $j$; in particular, we need to know what would happen to the patients who had $j - 1$ as their top choice, and would choose $j$ if $j - 1$ is out of the network. Our assumption allows us to ignore these patients; that is, to assume that they leave the network. The following is a generalization of Proposition 1.

**Proposition 3.** In the sequential Nash model with $J$ hospitals, the insurer’s surplus is independent of the order of negotiations.

The intuition behind Proposition 3 is the same as the one behind Proposition 1. The following is a generalization of Proposition 2, under the restriction of symmetry. In its statement, $\pi^l$ is the profit of a hospital if it is in the $l$-th position in the sequence.

**Proposition 4.** Consider the sequential Nash model with $J$ symmetric hospitals. There exists a $\tilde{\beta} < 1$ such that if $\beta \in (\tilde{\beta}, 1)$ then $\pi^l$ is strictly increasing in $l$.

## 5 Conclusion

We have studied bargaining between an intermediary and suppliers in which the total surplus from a suppliers’ network is non-linear in the quantity sold by each supplier. This feature makes sense when the intermediary acts on behalf of price-insensitive users. We showed that under the common NiN approach, suppliers may charge unit prices that surpass the unit value of its service, because of the negative surplus which is created from directing users to their second-best choices. If—contrary to the NiN approach—negotiations happen in a sequence, this overpricing problem cannot arise.

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7In discussions with a former CEO of a Blue Cross and Blue Shield in upstate New York (Klein 2015), he indicated, without this assumption being mentioned, that as an insurer he expected back-lash but not significant loss of business if one hospital left the network, but a drastic reaction and loss of business if two (or more) hospitals leave.
6 Appendix A: Exclusion in the NiN framework

Below we consider two types of hospital-exclusion in the NiN framework.

6.1 Ex ante exclusion

In the NiN model with ex-ante exclusion a group of \( J \) hospitals is selected out of a finite pool of potential hospitals, and only then, in a second stage, the NiN interaction occurs with the selected hospitals. The solution concept for this model is subgame perfect equilibrium: the insurer selects a group of hospitals such that its surplus will be maximized, given the second-stage NiN.

**Proposition 5.** In the NiN model with ex-ante exclusion there exists a \( \beta^* < 1 \) such that if \( \beta \in (\beta^*, 1) \) then in any subgame perfect equilibrium the selected network consists of a single hospital.

**Proof.** Let \( \mathcal{X} \) denote the hospital pool. By Theorem 1, for each subset of hospitals \( X \subset \mathcal{X} \) with at least two hospitals, there is a discount factor \( \beta_{X} < 1 \) such that if \( \beta > \beta_{X} \) the insurer makes a negative surplus in the NiN model in which the hospital network is \( X \). Since the insurer makes a positive surplus when it Nash bargains with any single hospital, the result follows by taking \( \beta^* \) to be the maximum of the \( \beta_{X} \)’s. \( \square \)

Proposition 5 implies that this two-stage structure cannot resolve the overpricing problem satisfactorily: within the two-stage framework, the threat of overpricing leads to a monopoly.

6.2 Ex post exclusion

An alternative to ex ante exclusion is ex-post exclusion NiN, suggested in Crawford and Yurukoglu (2012). Here, the insurer may remove hospitals from the network after the negotiation is complete. Ex-post exclusion guarantees the insurer at least zero surplus—it is also possible to “undo” contracts—and thus avoids complete Nash
overpricing. However, some Nash overpricing may persist. That is, it is still possible that for a sufficiently large $\beta$ some hospital’s price exceeds the value of service. This is illustrated in the following example.

Example: Partial Nash overpricing

Consider two hospitals and two types of patients $ab, ba$, with the same $u^h = 10$ and $u^l = 5$ as in Example 1. However, in contrast to Example 1, assume a unit measure of patients, with $s$ of the patients type $ab$ and $1-s$ of the patients of type $ba$.

The insurer’s value from a full network given prices $p_A, p_B$ is:

$$F(p) = \max\{0, 10 - p_A s - p_B (1 - s), 10 s + 5 (1 - s) - p_A, 10 (1 - s) + 5 s - p_B\}$$

Without $A$, the insurer’s value given $p_B$ is:

$$F_{-A}(p) = \max\{0, 10 (1 - s) + 5 s - p_B\}$$

The Nash bargaining price responses are:

$$p_A(p_B) = \begin{cases} \beta (p_B + 5) & p_B \leq 10 - 5 s \\ \beta \frac{10 - p_A (1 - s)}{s} & p_B > 10 - 5 s \end{cases} ; \quad p_B(p_A) = \begin{cases} \beta (p_A + 5) & p_A \leq 5 (1 + s) \\ \beta \frac{10 - p_A s}{1 - s} & p_A > 5 (1 + s) \end{cases}$$

The pair $\hat{p}_A = 10 \frac{\beta}{s(1 + \beta)}$ and $\hat{p}_B = 10 \frac{\beta}{(1 - s)(1 + \beta)}$ is a solution if $\beta$ and $s$ are such that $\hat{p}_A > 5 (1 + s)$ and $\hat{p}_B > 10 - 5 s$. Both $\hat{p}_j$ increase with $\beta$ and are continuous for $\beta \geq 0$.

Assume $\beta = 1$. Then for $s \in (0.5 - \hat{\xi}, 0.5 + \hat{\xi})$, with $\hat{\xi} = 1 - \frac{\sqrt{5}}{2}$ we have $\hat{p}_A = \frac{5}{s} > 5 (1 + s)$ and $\hat{p}_B = \frac{5}{1 - s} > 10 - 5 s$. Therefore, for any such $s$ which is different from 0.5 one of the prices will exceed 10. By continuity, this is true also for all sufficiently large $\beta$’s below 1.

7 Appendix B: Proofs

Lemma 1. Consider Nash bargaining between the insurer and a hospital, under the following assumptions:
1. The insurer’s profit without the hospital is \( V_0 \).

2. The hospital’s unit cost is \( c \) and its bargaining power parameter is \( \beta \).

3. If the hospital joins the network it serves a population of mass \( q \).

4. Adding the hospital to the network at unit price \( p \) increases the insurer’s profit by \( K - p \cdot y \).

Then the price is:

\[
p = \frac{\beta K}{y} + (1 - \beta)c. \tag{10}
\]

The lemma’s proof boils down to a simple maximization of a Nash product, and is therefore omitted.

In what follows we consider sequential negotiations between the insurer and the hospitals, where the hospitals are order in a particular, commonly known order. Given the order, each hospital has a position in it—the first in line, the second in line, etc. In addition to that, the hospitals have names—hospital 1, hospital 2, etc.—and each name is associated with particular model-parameter-values, such as \( q_1, q_2 \), and so on. We use the term **canonical order** to denote the order where the two label-systems coincide; that is, hospital 1 is placed first in the canonical order, hospital 2 is second, and so on.

**Lemma 2**: Consider sequential negotiations according to the canonical order, \( \{1, \cdots, J\} \). Given this order, denote the insurer’s surplus after bargaining and signing contracts with hospitals \( \{1, \cdots, i\} \) for prices \( (p_1, \cdots, p_i) \), and facing “future hospitals” \( \{i + 1, \cdots, J\} \), by \( V(p_1, \cdots, p_i; \{i + 1, \cdots, J\}) \). This surplus is given by:
\[
V(p_1, \cdots, p_i; \{i + 1, \cdots, J\}) = \\
\sum_{j=1}^{i} [q_j(v_j - p_j) + \beta \sum_{k=i+1}^{J} q_{j,-k}(v_{j,-k} - p_j)] + \\
+ (1 - \beta) \sum_{j=i+1}^{J} [q_j(v_j - c_j) + \beta \sum_{k=i+1, k \neq j}^{J} q_{j,-k}(v_{j,-k} - c_j)].
\]

The price obtained by hospital \(i\) in these negotiations is:

\[
p_i = \beta q_i v_i + \beta \sum_{k=i+1}^{J} q_{i,-k} v_{i,-k} - \sum_{j=1}^{i-1} q_{j,-i}(v_{j,-i} - p_j) - (1 - \beta) \sum_{j=i+1}^{J} q_{j,-i}(v_{j,-i} - c_j) + \\
+ (1 - \beta)c_i
\]

Before we turn to the proof, it is worthwhile to consider the equation for the above-mentioned value function \(V\). The RHS is composed of two terms, one corresponding to the already-contracted-with hospitals and one corresponding to the “future hospitals,” and the first term is such that for each element in the summation (each \(j = 1, \cdots, i\)) we have the “direct value” from the hospital plus \(\beta\) times the value that this hospital generates assuming that all “future hospitals” drop out. It will be useful to bear this meaning in mind later on in the proof.

**Proof of Lemma 2**: Clearly, \(V(p_1, \cdots, p_J; \emptyset) = \sum_{j=1}^{J} q_j(v_j - p_j)\). When negotiating with hospital \(J\), given that contracts with all previous hospitals have been signed, the insurer’s outside option is the value \(V(p_1, \cdots p_{J-1}; \emptyset) = \sum_{j=1}^{J-1} [q_j(v_j - p_j) + q_{j,-J}(v_{j,-J} - p_j)]\). Thus, the gain from adding \(J\) is:
\[ W_J(p_1, \ldots, p_J; \emptyset) \equiv V(p_1, \ldots, p_J; \emptyset) - V(p_1, \ldots, p_{J-1}; \emptyset) = q_J(v_J - p_J) - \sum_{j=1}^{J-1} q_{j,-J}(v_{j,-J} - p_j) = q_J v_J - \sum_{j=1}^{J-1} q_{j,-J}(v_{j,-J} - p_j) - p_J q_J, \]

where the "K" and "y" are the notations of Lemma 1. Applying this lemma we obtain the price \( p_J \):

\[
p_J = \beta q_J v_J - \sum_{j=1}^{J-1} q_{j,-J}(v_{j,-J} - p_j) + (1 - \beta)c_J. \tag{11}
\]

Having obtained the price and surplus for the last bargaining problem in the sequence, we turn to the next-to-last bargaining. The value of these negotiations is

\[
V(p_1, \ldots, p_{J-1}; \{J\}) = \sum_{j=1}^{J-1} q_j(v_j - p_j) + q_J(v_J - p_J). \tag{8}
\]

Note that we slightly abuse notation to have \( p_J \) on the RHS is legitimate even though it does not appear as an argument of the \( V \) function on the LHS, because of (11)—namely, \( p_J \) is pinned down by the previous prices (and the other model parameters). Substituting \( p_J \) into the expression gives:

\[
V(p_1, \ldots, p_{J-1}; \{J\}) = \sum_{j=1}^{J-1} q_j(v_j - p_j) + \beta q_{j,-J}(v_{j,-J} - p_j) + (1 - \beta)q_J(v_J - c_J). \tag{12}
\]

The outside option in the next-to-last negotiations has the value \( V(p_1, \ldots, p_{J-2}; \{J\}) \).

It follows from (12) that this value is:

\[
V(p_1, \ldots, p_{J-2}; \{J\}) = \sum_{j=1}^{J-2} [q_j(v_j - p_j) + \beta q_{j,-J}(v_{j,-J} - p_j)] + (1 - \beta) [q_{J,J}(v_J - c_J) + q_{J,-J,J}(v_{J,-J} - c_J)]. \tag{12}
\]

\[8\]The value when the prices up to and including \( p_{J-1} \) have been contracted, and the insurer expects \( p_J \) to be contracted next is \( \sum_{j=1}^{J} q_j(v_j - p_j) \).
Here, $L$ is the counterpart of $q_j(v_j - p_j)$ from (12), and $M$ is the counterpart of $q_j(v_j - c_j)$ (expected value minus expected cost at the final hospital).\footnote{This is true because (12) holds also for a sequence of length $J' = J - 1$.}

The gain from the $(J-1)$-th bargaining is $W_{J-1}(p_1, \cdots, p_{J-1}; \{J\}) = V(p_1, \cdots, p_{J-1}; \{J\}) - V(p_1, \cdots, p_{J-2}; \{J\})$, or:

$$W_{J-1}(p_1, \cdots, p_{J-1}; \{J\}) =$$

$$= q_{J-1} v_{J-1} + \beta q_{J-1, -J} v_{J-1, -J} - \sum_{j=1}^{J-2} q_{j, -(J-1)} (v_{j, -(J-1)} - p_j) - (1 - \beta) q_{J, -(J-1)} (v_{J, -(J-1)} - c_J) - \sum_{j=1}^{J-2} q_{j, -(J-1)} (v_{j, -(J-1)} - p_j) - (1 - \beta) q_{J, -(J-1)} (v_{J, -(J-1)} - c_J) - p_{J-1} (q_{J-1} + \beta q_{J-1, -J}).$$

By Lemma 1,

$$p_{J-1} =$$

$$= \beta q_{J-1} v_{J-1} + \beta q_{J-1, -J} v_{J-1, -J} - \sum_{j=1}^{J-2} q_{j, -(J-1)} (v_{j, -(J-1)} - p_j) - (1 - \beta) q_{J, -(J-1)} (v_{J, -(J-1)} - c_J) + q_{J-1} v_{J-1} + \beta q_{J-1, -J} v_{J-1, -J} - p_{J-1} (q_{J-1} + \beta q_{J-1, -J}).$$

It follows from equation (12) that:\footnote{This is simply writing separately the $(J - 1)$-th term from the first summation, leaving the first $J - 2$ elements in the sum.}

$$V(p_1, \cdots, p_{J-2}; \{J-1, J\}) = \sum_{j=1}^{J-2} [q_j (v_j - p_j) + \beta q_{j, -J} (v_{j, -J} - p_j)] + (1 - \beta) q_{J, -J} (v_J - c_J) +$$

$$+ q_{J-1} v_{J-1} + \beta q_{J-1, -J} v_{J-1, -J} - p_{J-1} (q_{J-1} + \beta q_{J-1, -J}).$$

Combining this with the formula for $p_{J-1}$ gives:
\[ V(p_1, \cdots, p_{J-2}; \{J-1, J\}) = \]
\[ = \sum_{j=1}^{J-2} [q_j(v_j - p_j) + \beta \sum_{k=J-1}^{J} q_{j-k}(v_{j-k} - p_j)] + (1-\beta) \sum_{j=J-1}^{J} [q_j(v_j - c_j) + \beta \sum_{k=J-1, k\neq j}^{J} q_{j-k}(v_{j-k} - c_j)]. \]

Now assume that given the contracted prices \( \{p_1, \cdots, p_i\} \), and "future hospitals" \( \{i+1, \cdots, J\} \), the insurer's payoff is:

\[ V(p_1, \cdots, p_i; \{i+1, \cdots, J\}) = \]
\[ = \sum_{j=1}^{i} [q_j(v_j - p_j) + \beta \sum_{k=i+1}^{J} q_{j-k}(v_{j-k} - p_j)] + (1-\beta) \sum_{j=i+1}^{J} [q_j(v_j - c_j) + \beta \sum_{k=i+1, k\neq j}^{J} q_{j-k}(v_{j-k} - c_j)]. \]

(13)

As we have shown above, this assumption is indeed correct given a fixed \( J \) and \( i \in \{J-2, J-1\} \). Basically, the same arguments can be applied given that the hospital sequence is of length \( J' = J - 1 \): the formula still holds with \( J' \) replacing \( J \), \( i = J' - 1 \), and also for \( i = J' - 2 \) provided that this is a positive integer. But, one has to be careful in the application and note the role of our assumption that when \( i \) drops out, all of its consumers that choose \( k \neq i \) as their second choice leave the network if the second choice drops out as well. This is what makes the application work, and hence (13) implies:

\[ V(p_1, \cdots, p_{i-1}; \{i+1, \cdots, J\}) = \]
\[ = \sum_{j=1}^{i-1} [q_j(v_i - p_j) + q_{j-i}(v_{j-i} - p_j) + \beta \sum_{k=i+1}^{J} q_{j-k}(v_{j-k} - p_j)] + \]
\[ + (1-\beta) \sum_{j=i+1}^{J} [q_j(v_j - c_j) + \beta \sum_{k=i+1, k\neq j}^{J} q_{j-k}(v_{j-k} - c_j)]. \]

Note that, like in the explanation that preceded the proof, each element in the first summation has a direct benefit component and an additional component, when
in writing down these components we have invoked the abovementioned assumption regarding what happens when \( i \) drops out.

Thus, the gain from bargaining with \( i \) is:

\[
W_i(p_1, \ldots, p_i; \{i+1, \ldots, J\}) = V(p_1, \ldots, p_i; \{i+1, \ldots, J\}) - V(p_1, \ldots, p_{i-1}; \{i+1, \ldots, J\}),
\]

or:

\[
q_i(v_i - p_i) + \beta \sum_{k=1}^{J} q_{i-k} (v_{i-k} - p_i) - \sum_{j=1}^{i-1} q_{j-i} (v_{j-i} - p_j) - (1 - \beta) \sum_{j=i+1}^{J} q_{j-i} (v_{j-i} - c_j) =
\]

\[
= q_i v_i + \beta \sum_{k=1}^{J} q_{i-k} v_{i-k} - \sum_{j=1}^{i-1} q_{j-i} (v_{j-i} - p_j) - (1 - \beta) \sum_{j=i+1}^{J} q_{j-i} (v_{j-i} - c_j) -
\]

\[
\sum_{k=i+1}^{J} q_{i-k}.
\]

Applying Lemma 1 we obtain:

\[
p_i = \beta \frac{q_i v_i + \beta \sum_{k=i+1}^{J} q_{i-k} v_{i-k} - \sum_{j=1}^{i-1} q_{j-i} (v_{j-i} - p_j) - (1 - \beta) \sum_{j=i+1}^{J} q_{j-i} (v_{j-i} - c_j)}{q_i + \sum_{k=i+1}^{J} q_{i-k}} + (1 - \beta)c_i \quad (14)
\]

With (13) and (14) established, the proof is completed. \( \square \)

**Proof of Proposition 3:** It follows from (13) that the insurer’s value, before he approaches the first hospital in the canonical order, is:

\[
V(\emptyset; \{1, \ldots, J\}) = (1 - \beta) \sum_{j=1}^{J} [q_j (v_j - c_j) + \beta \sum_{k \neq j} q_{j-k} (v_{j-k} - c_j)],
\]

and the RHS is independent of the order. \( \square \)
Proof of Proposition 4: Consider \( J \) symmetric hospitals and let \( q \equiv q_j, \hat{q} \equiv q_{j-k}, \)
v \equiv v_j and \( \hat{v} \equiv v_{j-k} \) (recall that symmetry means independence of these quantities
of \( j \) and \( k \)). Let \( p_i^* \equiv \lim_{\beta \to 1} p_i \), where \( p_i \) is given by (14). Setting \( \beta = 1 \) at (14) gives:

\[
p_i^* = \frac{qv + \hat{q}\hat{v}(J - i)}{q + \hat{q}(J - i)} + \frac{\hat{q} \sum_{j=1}^{i-1} (p_j^* - \hat{v})}{q + \hat{q}(J - i)}.
\]

Claim 1: \( A \) is increasing in \( i \).

Proof of Claim 1: The sign of \( \frac{\partial A}{\partial i} \) is the same as the sign of \(-\hat{q}\hat{v}[q + \hat{q}(J - i)] + \hat{q}[qv + \hat{q}\hat{v}(J - i)]\), and the latter is positive if and only if \( v > \hat{v} \), which is true.

Claim 2: \( B \) is increasing in \( i \).

Proof of Claim 2: It is enough to prove that \( p_j^* > \hat{v} \). This is true for \( j = 1 \) in virtue of \( v > \hat{v} \), and the fact that it is true for all \( j' < j \) implies that it is true also for \( j \). \( \square \)

References