Generalized Insurer Bargaining

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Abstract

Bargaining between intermediaries and multiple upstream providers, such as insurers bargaining with hospitals, is an active area of research. We show that the common Nash-in-Nash bargaining solution, while useful for estimation, can predict Nash overpricing: prices may exceed the treatments value for a particular hospital. We consider several alternatives. Our suggested approach is based on repeated interaction. When used for estimation, it maintains many attractive features of the Nash-in-Nash. The two models differ in important ways. In particular, mergers that would be approved using Nash-in-Nash may be rejected using the general model.

Keywords: Multilateral bargaining; Health economics; Intermediaries.

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1 Introduction

In most economic settings, agents care about prices and costs. Consumers care about the prices of the goods they buy, firms about their production costs, and so on. There are, however, settings in which decision makers ignore prices when taking their decisions. Examples of such price-insensitivity include: (1) A patient seeking medical treatment that is covered by medical insurance; (2) a university faculty member reading a scholarly article does not pay for the journal subscription or the office equipment used; (3) a military commander choosing which (or how much) ammunition to use in battle is unlikely to heavily consider procurement costs.

A common feature of all the examples above is the vertically-structured economy in which an upstream intermediary (e.g., an insurance company, purchasing department or Department of Defense) bargains prices with suppliers (e.g., hospitals, equipment wholesalers, military contractors), under the assumption that the negotiated prices will have minimal to no effect on the end users’ decisions.

For concreteness—and in order to compare our findings to those of the health economics literature—we focus on the case of an insurer who bargains with hospitals for per-treatment prices, and whose goal is to maximize the consumers’ (i.e., patients’) expected surplus. However, our results apply to any environment which is characterized by the aforementioned vertical structure and price-insensitivity.

We start by considering the case where the insurer Nash bargains with each hospital separately, when the prices with all other hospitals are being taken as given. This approach originated in Horn and Wolinsky (1988). In the sequel, we follow the term coined in Collard-Wexler et al. (2014) and call it Nash-in-Nash (NiN). This model has been used in several empirical applications to the health care industry (Gaynor and Town, 2011; Gowrisankaran et al., 2015; Ho and Lee, 2017b).

In the NiN model, the insurer bargains with each hospital, holding the contracts with all other hospitals fixed. Price is determined by comparing the network’s surplus with and without the hospital. Consider, for start, the case of two hospitals, \(A\) and \(B\). Consider bargaining with \(A\), given the negotiated price with hospital

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1 Crawford and Yurukoglu (2012) and Lee et al. (2013) apply a similar framework to the TV market. However, in contrast with healthcare markets, the end-user in the TV industry is offered a menu of networks and prices, designed by the intermediary (i.e. the cable company). The TV industry differs in two other important ways: Supplier payment in the TV market is in general based on the distributor (“insurer”) revenue from bundles that include the supplier rather than actual consumption and supplier costs decrease with user consumption (due to advertising revenue).
The network’s surplus without \( A \) depends not only on \( B \)’s price but also on \( A \)’s patients that would now use \( B \) and their value for \( B \). Namely, some of \( A \)’s would-be patients remain in the program even if \( A \) is not included in the network. Since patients are price-insensitive, they go to hospital \( B \) even if their value for \( B \) is lower than \( B \)’s price. Thus, these patients may create a negative surplus when \( A \) is not in the network. When \( A \) joins the network, its marginal value per patient (given \( B \)’s price) may be higher than its actual per-patient value, since its addition to the network prevents the aforementioned negative-surplus creation. We show that in any network with two or more hospitals, if some patients remain with the insurance plan even when their most-preferred hospital is out of the network, prices for a hospital exceed patient valuations for a sufficiently large hospital bargaining power. We call this phenomenon Nash overpricing. In Theorem 1, we show that if hospitals’ bargaining power is strong enough, Nash overpricing will occur for every hospital in the network, which we refer to as complete Nash overpricing.

We next consider two possible extensions to the traditional NiN framework which allow insurers to exclude hospitals from their networks before or after bargaining. Proposition 1 finds that excluding a subset prior to bargaining has no qualitative effect. Proposition 2 finds that exclusion after negotiation alleviates complete Nash overpricing, although the possibility of Nash overpricing for a subset of hospitals remains. In the hospital insurance context, ex-post exclusion from the network may be impractical for insurers as it will be difficult to separate inclusion in the network from price negotiations. As a practical matter, applying ex-post exclusion in empirical work requires that analysts consider counterfactual optimal networks for each possible disagreement which may quickly become intractable. Moreover, the equilibrium mapping for a given network, may be non-differentiable, making it difficult to apply in empirical work.

To address Nash overpricing, we propose a tractable variant of the NiN model in which the aforementioned price inflation does not occur for any number of hospitals. We first consider a sequential variant, which is identical to NiN, except that

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2Recently, Ho and Lee (2017a) have considered a model where hospitals are exclude from negotiation prior to bargaining to explain the existence of so-called narrow networks.

3Counterfactual post-negotiation optimization has been considered by Crawford and Yurukoglu (2012) when cable companies construct channel bundles following negotiations with content provider. However to our knowledge the consideration of optimal networks following hospital-insurer negotiations has not been considered.
negotiations happen sequentially, rather than simultaneously. A straightforward backward induction argument shows that this approach rules out complete Nash overpricing: Given that prices with hospitals \(\{1, \cdots, J-1\}\) have been determined, the insurer faces a standard 2-player bargaining problem with hospital \(J\), in which his payoff (according to the Nash bargaining solution) is positive. In the negotiation with hospital \(J-1\) this is taken into account, and so negotiations with hospital \(J-1\) are also a standard 2-player bargaining problem in which the insurer’s payoff is positive, and so on.

Surprisingly, insurer’s surplus in the sequential model is independent of the order of negotiations. However, hospitals care about the order of negotiations in the sequential Nash model. For example, in a two hospital setting, the first hospital’s price is lower than the second hospital’s price. We believe that this result can be generalized to any number of hospitals; namely, that prices increase monotonically in the hospital’s position in the order of negotiations. However, we are able to prove this monotonicity only for the case of two hospitals or \(J\) symmetric hospitals.

Since the order of negotiations matters, and since no particular order can be justified over another, we consider a third model, in which this drawback is taken care of: the Repeated Nash model (RN model for short). This model is as follows. Time is discrete, and runs as \(t = 1, 2, \cdots\). In the beginning of every period, the insurer approaches the \(J\) hospitals simultaneously, and offers each hospital a price. Each hospital accepts or rejects its offered price, these responses being simultaneous. Every accepted price is contracted with the hospital for the period, and every rejection leads to a particular bilateral bargaining problem (on which we elaborate later) between the insurer and the rejecting hospital. If no rejections occur then play proceeds to the next period, in which the insurer approaches the hospitals again with price offers. If at least one price is rejected, the game shift to an absorbing “punishment phase,” in which one of the rejectors is punished. The punishment payoffs equal the payoffs of the sequential Nash model, in which the order of negotiations is such that the punished hospital is placed first.

We characterize the insurer-surplus-maximizing equilibrium of the RN model. If the hospitals’ discount factor is sufficiently large, the surplus for the insurer is positive for any bargaining power and there is no Nash overpricing. As the discount factor tends to zero the RN prices converge to the NiN prices. Thus, when the hospitals’ discount factor is low, negotiations according to the sequential Nash model
are preferred, from the insurer’s point of view, to those of the RN model.

Our proposed RN model is based on the assumption that the insurer can commit (at least temporarily) not to reopen negotiations with a hospital. Our impression is that insurers have sufficient mechanisms to make such commitment. For example, a negotiating unit that is limited by capacity to only negotiate with a small fraction of the industry’s providers at a time would accomplish this.

To gain insight into this question, we interviewed David Klein, a former CEO of a Blue Cross and Blue Shield in upstate New York – [Klein (2015)]. Klein confirmed that insurers lack the resources (negotiators, actuaries, accountants, etc.) to negotiate with all their hospitals at the same time. Moreover, as a negotiating tactic, insurers avoid simultaneous negotiations even if those were possible as those are likely to increase the hospitals’ negotiating power and ability to coordinate. Instead, the insurer seeks to negotiate with each hospital once every several years and generally does not reopen negotiations.

Estimation based on RN can be viewed as a generalization of NiN that does not increase the complexity of the estimation procedure. Appendix A provides a detailed derivation and estimation procedure and Appendix D reports the results of a Monte Carlo simulation. As the discount factor decreases, estimation based on the RN model approaches the NiN model.[4] Estimates of hospital bargaining power and costs will be biased when using the NiN model if the true underlying model is RN.

Extending the NiN model to RN requires adding one parameter, the discount factor ($\delta$). This new parameter, in combination with the standard bargaining power parameter ($\beta$) determines how the hospital’s contribution to average patient surplus is split between the hospital and the insurer. Adding a hospital to the network increases average surplus in a direct way (providing service to patients) and through complementarities (more effective sorting of patients into hospitals). NiN uses one bargaining power parameter ($\beta$) to determine the extent each hospital captures the value created in both of these channels.

The bargaining model directly relates to the important question of merger evaluation. Section 4.1 shows that mergers that the NiN model suggests would decrease

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[4] While the estimation (conditional on prices) is generalized, the RN model with myopic hospitals does not converge to NiN. In particular if hospitals are completely myopic, the RN model converges to the sequential model which does not predict unique equilibrium prices.
prices may actually increase prices according to the RN model. In particular, the NiN model would suggest that a merger between two hospitals that are high priced and relatively substitutes may actually decrease prices due to the elimination of the dynamic identified in section 1. In contrast, the RN model would predict that such mergers will increase prices.

While our exposition uses the hospital-insurer bargaining context, the analysis is also relevant to other important settings. In particular, large organizations (firms, hospitals, military, etc.) often have dedicated “purchasing departments” that negotiate with multiple “approved vendors”, while the actual product choice is done by professionals (e.g. doctors, engineers, military commanders) based on the products’ features on a case-by-case basis. Such situations map directly to our model. The objective of the “purchasing department” is to maximize the surplus generated to the firm from the acquisitions. In these settings the eventual customer is either unable or unlikely to change an intermediary when a specific supplier quits the network and the intermediary’s success relies on maximizing the value for its customers.

The next subsection reviews the related bargaining literature in general, and in the specific health economics aspect; Section 2 lays out the bargaining environment. The three multilateral bargaining models and formal results are provided in Section 3. Implications, including merger evaluation, estimation, and extensions are discussed in Section 4. Appendices A and D detail the estimation procedure and Monte Carlo results. Section 5 concludes.

1.1 Literature Review

In terms of the theoretical framework, our paper belongs to a strand of literature that concerns bilateral bargaining in vertically-structured markets. In this regard, the main reference to our work is that of Horn and Wolinsky (1988), who apply the asymmetric Nash bargaining solution to determine input prices in a duopoly model in which two firms acquire inputs from a common upstream supplier. More recently, hospitals often form a Group Purchasing Organization (GPO) which negotiates on behalf of multiple hospitals with the same supplier. GPOs have been studied extensively, mostly in the supply chain literature. See Cleverley and Nutt (1984), Schneller (2000) and Scanlon (2002) for reviews of GPO effectiveness.

Iozzi and Valletti (2014) considered a similar setting, in which each of $N$ identical downstream firms buys its input from a single supplier at a first stage, which is followed by competition (among the firms who made a non-zero input purchase) at a second stage. As in Horn and Wolinsky (1988), Iozzi and Valletti (2014) assume that each of the $N$ input prices is determined through bilateral Nash bargaining. Iozzi and Valletti (2014) consider the case where the second-stage competition is either a Bertrand competition or Cornout competition.

Binmore et al. (1986) and Davidson (1988) provide extensive-form models that result in the Nash solution, but are based on different underlying assumptions than those we study here. Collard-Wexler et al. (2014) provide a similar model motivated by recent applications. However, Collard-Wexler et al. (2014) depart from the empirical literature by assuming that insurers and hospitals negotiate a lump sum payment rather than fee-per-service also guarantees positive surplus, though raises additional concerns. Our critique of the NiN approach is generically independent of the underlying extensive form game that is assumed to generate the equilibrium.

Studying bargaining between cable operators and broadcasters, Crawford and Yurukoglu (2012) use a variation of renegotiation-proofness to refine the equilibrium. They allow the operator – the intermediary – to remove broadcasters from the network after the negotiation is complete. In our setting, this would imply the insurer can independently remove hospitals from their network after negotiations are completed. As we show in Section 3.2, this guarantees the insurers at least zero utility from any equilibrium network. The corresponding estimation model is less tractable and it would still be possible for a specific hospital to charge more than the per-patient surplus it generates in equilibrium.

The central difference between our work and the aforementioned papers is in the bargaining stage. Both in Horn and Wolinsky (1988) and Iozzi and Valletti (2014), the upstream-downstream negotiations are modeled as a collection of (simultaneous) independent bargaining problems. In our model, by contrast, there are significant interdependencies between the problems. Indeed, the main contribution of our analysis is the realization that the interdependencies are so strong that even

Iozzi and Valletti (2014) also distinguish between (1) the case where negotiation breakdown between the upstream monopolist and a particular firm $i$ is observable by $i$’s rivals and (2) the case where it is not observable. Our analysis does not rely on downstream (insurer) competition. However, if such competition exists, we follow the literature and assume that the negotiation outcome (i.e. insurer network structure) is common knowledge.
the most basic intuitive properties of a bargaining solution may be violated. Crucially, the objective of the upstream entity of our model—the insurer, or, to use a more general term, the collective bargainer—is not a linear revenue-function; instead, the collective bargainer’s payoff (the expected welfare of the mass population it represents) is sensitive to the composition of the downstream market.

From the theoretical bargaining literature, the central work about interdependencies among bargaining problems is by Bennett (1997). She considers the case where \( n \geq 3 \) players form (possibly several) coalitions, bargaining takes place within each coalition, and the interdependencies among the different bargaining problems are reflected in the endogenous determination of the outside options: the disagreement payoff of player \( i \) in the bargaining problem in which he participates is calculated on the basis of what would have happened had that player belonged to another coalition (i.e., what payoff he would have obtained then). There are two important differences between Bennett’s work and ours. First, in her work each player can participate in at most one coalition. In our work, by contrast, the collective bargainers bargain with each hospital separately; that is, he simultaneously belongs to several coalitions. Secondly, at the conceptual level, one of the main issues in Bennett (1997) is that of coalition formation. In our work, by contrast, this is not an issue, due to exogenously given vertical structure: it is a priori clear what are the possible coalitions to look at, and, moreover, it is clear what is the coalition-configuration to focus on—the one corresponding to the full hospital-network.

Various studies consider sequential bargaining. Section 6 in Horn and Wolinsky (1988) considers the sequential offer equilibrium of the game there. Noe and Wang (2004) shows, in a general setting, the benefit of hiding the order of negotiations, a possibility we do not consider here. Gal-Or (1999) considers both simultaneous (NiN) and (one-shot) sequential bargaining between hospitals and insurers. However, the analysis there considers the insurer’s incentive to exclude a hospital from the network and the implications for industry consolidation.

Our approach of embedding the one-shot game in a repeated framework resembles that of Rey and Tirole (2013), who use the properties of the repeated game in a patent pool formation. The bargaining setting and potential inefficiencies there are however very different.

Our application of the Nash bargaining solution in the context of health economics closely follows the handbook chapter by Gaynor and Town (2011), which
builds on numerous works from the health economics literature. Regarding the Nash bargaining that takes place between an insurance provider and a hospital, Gaynor and Town write (p. 530) “if the net surplus from the hospital-insurer match is not greater than zero, then bargaining does not take place and the hospital is not in the insurer’s network.” One informal way to interpret our Nash overpricing result is that the aforementioned circumstances are actually plausible in many settings, if one accepts the simultaneous Nash model as appropriate.

2 The environment

An insurer bargains with \( J \geq 2 \) hospitals on behalf of a unit measure of heterogeneous patients. The utility of patient \( i \) from being treated at hospital \( j \) is denoted by \( u_{ij} \). We let \( j = 0 \) denote the patient’s option to either forego treatment or utilize out-of-network services, which we normalize as providing net zero benefit: \( u_{i0} = 0 \). Under the full network, which comprises of all \( J \) hospitals, the quantity of patients treated by hospital \( j \) is:

\[
q_j = \int 1[u_{ij} \geq u_{ik}, \forall k \in \{0, \ldots, J\}] di.
\]

It is assumed that \( q_j > 0 \) for all \( j = 1, \ldots, J \). Taking expectation over patients \( (i) \), the expected value for a patient who chooses to go to hospital \( j \) is:

\[
v_j = E[u_{ij}|u_{ij} \geq u_{ik}, \forall k \in \{0, \ldots, J\}].
\]

If a hospital, say \( j \), is not part of the network, then it is not available for consumers to seek treatment. This event only affects those consumers who prefer to be treated at \( j \), who then go to their next preferred hospital. The hospital choice of patients that chose hospital \( l \neq j \) with \( j \) in the network does not change. Thus, the number of additional consumers for hospital \( k \) when hospital \( j \) is dropped from the network and those patients’ expected utility are:

\[
q_{k,-j} = \int 1[u_{ij} > u_{ik}, \forall \ell \in \{0, \ldots, J\} \setminus \{j\}] di
\]

\[
v_{k,-j} = E[u_{ik}|u_{ij} > u_{ik}, \forall \ell \in \{0, \ldots, J\} \setminus \{j\}].
\]
The hospitals are *symmetric* if \( q_j, v_j, q_{k,-j} \) and \( v_{k,-j} \) are independent of \( k \) and \( j \).

Hospitals can treat patients with a marginal cost of \( c_j < v_j \). Therefore, under the full network hospital \( j \)’s profit is:

\[
\pi_j = (p_j - c_j)q_j.
\]

If hospital \( j \) is out of the network, it receives zero profit. However, if another hospital, say \( k \), is out of the network, \( j \) receives more patients and its profits become:

\[
\pi_{j,-k} = (p_j - c_j)(q_j + q_{j,-k}).
\]

We assume the following regarding substitution between hospitals:

**Assumption 2.1.** (i) For all \( j = 1, \ldots, J, \sum_{k \neq j, k>0} q_{k,-j} > 0 \) and (ii) For all \( j, k \), if \( q_{j,-k} > 0 \) then \( v_j > v_{j,-k} \).

The first part of Assumption 2.1 is that at least some patients have a second choice hospital they prefer over the outside option. The second part requires that patients whose first choice of a hospital is \( j \) on average value hospital \( j \) more than patients for whom \( j \) is the second choice. Both assumptions are de-facto standard in current applications and are natural when differentiation between hospitals is mostly horizontal and there are frictions in patient insurance choice.

The insurer maximizes patient surplus\[^8\] Therefore, the insurer’s value given a price vector \( p = (p_1, \ldots, p_J) \) is:

\[
F(p) = \sum_{j=1}^{J} (v_j - p_j)q_j.
\]

Similarly, the insurer’s surplus from the network without hospital \( j \) given remaining hospital prices is,

\[
F_{-j}(p) = \sum_{k \neq j} [(v_k - p_k)q_k + (v_{k,-j} - p_k)q_{k,-j}].
\]

As common in the literature, whenever the insurer bargains with a single hospital, the two bargain over the price per admitted patient, the Nash bargaining solution

\[^8\] We discuss the tradeoffs and implications of considering a short-term profit maximizing insurer in section 4.4.
applies and the hospital’s bargaining power parameter is \( \beta \in (0, 1) \). The models below consider three extensions to multi-lateral bargaining. While we assume a common bargaining power parameter to all hospitals, allowing for a different parameter to each hospital is technically tedious but straightforward and has no qualitative effect.

3 Models of Multilateral Bargaining

3.1 Nash-in-Nash

We start by considering the Nash in Nash (hereafter NiN) solution concept when applied to the model presented above. Namely, we assume that prices between the insurer and each hospital are set following the Nash bargaining solution between the two parties, holding all other prices fixed. We first state the general result and then provide an illustrative two-hospital example.

Letting \( p_{-j} \) denote the vector of prices for all hospitals except \( j \), the NiN solution requires that all prices satisfy:

\[
 p^N_j = \max_p [F(p) - F_{-j}(p)]^{(1-\beta)} \cdot [q_j(p_j - c_j)]^\beta.
\] (3.1)

The solution to the NiN model is the vector of prices \( (p^N_1, \ldots, p^N_J) \) such that each price \( p^N_j \) solves the above maximization, when the other prices are being taken as given. It is not hard to verify that the solution is:

\[
 p^N_j = \beta v_j - \frac{\beta}{q_j} \sum_{k \neq j} (v_{k,-j} - p^N_k) q_{k,-j} + (1 - \beta) c_j
\] (3.2)

Equation (3.2) can be mapped to equation (9.7) in GT, the differences being merely notational. Based on this equation, we prove our first result:

**Theorem 1.** (Complete Nash overpricing) There exists a \( \bar{\beta} < 1 \), such that if the hospital’s bargaining power parameter satisfies \( \beta \in (\bar{\beta}, 1) \), then each of the NiN

\(^9\)We removed the \( h \) subscript in GT 9.7, used our notation \( q_{k,-j} \), and removed additional fixed cost elements (hospital’s outside option and insurer-hospital fixed costs).
prices exceed the value of service in the corresponding hospital. That is,

\[ p^N_j > v_j \quad \forall j = 1, \cdots, J. \]

**Proof.** Consider equation (3.2):

\[ p^N_j = \beta v_j - \frac{\beta}{q_j} \sum_{k \neq j}(v_{k,-j} - p^N_k)q_{k,-j} + (1 - \beta)c_j \]

Suppose that \( q_{k,-j} = 0 \) for all distinct \( k \) and \( j \). Then \( p^N_j \to v_j \) as \( \beta \to 1 \).

For every distinct \( k \) and \( j \) define \( \epsilon_{kj} \) by:

\[ p^N_k = v_{k,-j} + \epsilon_{kj}. \]

By assumption, \( v_k > v_{k,-j} \) for all \( j \) and \( k \). Therefore, there exists an \( \epsilon > 0 \) such that for all sufficiently large \( \beta \)'s it holds that \( \epsilon_{kj} > \epsilon \). Wlog, suppose that \( \beta \) is sufficiently large such that the above holds. Let \( \theta_j = \sum_{k \neq j} \epsilon_{kj} \). If \( \beta \) is sufficiently large, the following must hold for all \( j \):

\[ \frac{\beta}{q_j} \epsilon \theta_j > (1 - \beta)(v_j - c_j). \] (3.3)

Now, increase \( \{q_{k,-1}\}_{k>1} \) from zero to their true values. These \((J - 1)\) coefficients only appear in the formula for \( p^N_1 \), so this increase only affects \( p^N_1 \), not any other price. It increases this price by \( \frac{\beta}{q_1} \sum_{k \neq 1} \epsilon_{k1}q_{k,-1} > \frac{\beta}{q_1} \epsilon \theta_1 \). In view of (3.3), it follows that \( p^N_1 > v_1 \). Repeating this process for \( j = 2, \cdots, J \) completes the proof.

The source of overpricing is the fact that when a hospital, say \( j \), leaves the network, some of its patients stay in the network; that is, \( \sum_{k \neq j} q_{k,-j} > 0 \). These patients, being indifferent to the costs of service, go to some other hospital which they value less than patients who choose this hospital initially (i.e. \( v_k \)). If \( p_k \to v_k \), the average value of service for a treatment in hospital \( k \) now drops below the hospital’s per-capita price.

**Example 1. Two symmetric hospitals**

An insurer negotiates with two hospitals, \( A \) and \( B \). The cost of serving a patient is zero. The market has four types of patients, \( \{ab, a0, ba, b0\} \). Patients of type \( ab \)
(resp. \(ba\)) have a value of \(u^h = 10\) from being served by hospital \(A\) (resp. \(B\)) and \(u^l = 5\) from being served by the other hospital. Patients of type \(a0\) (resp. \(b0\)) have a value of \(u^h = 10\) from hospital \(A\) (resp. \(B\)) but would choose the outside good (0) if hospital \(A\) (resp. \(B\)) leaves the network.

In other words, the \(ab\) and \(a0\) types prefer \(a\) over the alternatives but disagree on their second choice hospital (\(B\) or out-of-network). The \(ab\) and \(ba\) types (equivalently \(a0\) and \(b0\)) disagree on whether their first option is \(A\) or \(B\). With both hospitals in the network, the insurer expects a unit mass of patients for each hospital. For symmetry between hospitals, there are \(\alpha\) patients of type \(ab\) and the same for \(ba\), and \((1 - \alpha)\) patients of each of types \(a0\) and \(b0\).

Thus:

\[
F(p) = 20 - p_A - p_B \; \text{; and } \; F_{-A}(p) = \alpha \cdot (5 - p_B) + (10 - p_B).
\]

Let \(\beta \in (0, 1)\) denote a hospital’s bargaining power parameter. Under Nash bargaining, if we hold \(p_B\) fixed then \(p_A\) solves:

\[
\max_p (F - F_{-A})^{(1-\beta)} \cdot p^\beta
\]

The price response is given by:

\[
p^N_A = \beta \cdot \left[ 10 \cdot (1 - \alpha) + \alpha \cdot (5 + p^N_B) \right].
\]

Equation (3.5) shows that \(A\) obtains a fraction \(\beta\) of the surplus it generates. The \((1 - \alpha)\) new patients each account for 10 utils. The \(\alpha\) patients that instead would have went to \(B\) only gain 5 directly from going to their preferred hospital, but also save the payment of \(p_B\).

Of these two consumer segments \((1 - \alpha\) and \(\alpha)\), prices may surpass value only because of the second \((\alpha)\) group. In particular, whenever \(p_B > 5\), the insurer actually generates negative surplus (net of prices) serving these patients without \(A\) in the network. The surplus that \(A\) generates to these patients is then higher than it’s ex-post per-patient value of 10. If the hospitals’ bargaining power is sufficiently high so that hospitals obtain most of the surplus they generate, price will then be higher than the ex-post value.

Formally, prices are obtained by solving (3.5) and the symmetric equation for
Figure 3.1: Symmetric example for $\alpha = 0.25, 0.5$ or $0.75$

$p_B$. This gives:

$$p_j^N = 5\beta \frac{2 - \alpha}{1 - \beta \alpha},$$

(3.6)

for both $j \in \{A, B\}$. Recalling that with both hospitals in the network the value of each service is 10, the following is easy to verify:

$$p_j^N \leq 10 \iff \beta \leq \frac{2}{2 + \alpha},$$

(3.7)

That is, for this example, $\bar{\beta}$ from Theorem 1 equals $\frac{2}{2 + \alpha}$.

Figure 3.1 illustrates the resulting price per patient. Even if just a quarter of the patients stay with the insurer even if their favored hospital leaves the network, prices exceed value when $\beta \geq 0.88$. As the share of patients that would not adjust their insurer choice increases, prices significantly increase and may well be more than double the value per patient. The example is easily generalized for any $u_h$ and $u^l$. In particular, letting $u^l = \lambda u_h$ for any $\lambda \in (0, 1)$ the insurer generates negative surplus per patient (i.e., $p_j^N > u_h$) iff

$$\beta \geq \frac{1}{1 + \alpha(1 - \lambda)}.$$  

(3.8)

In particular, for any $\alpha, \lambda$ both in $(0, 1)$, there is some $\bar{\beta} < 1$ such that for any $\beta > \bar{\beta}$ the price of each service is higher than its value.
3.2 Two-Stage NiN

The NiN model takes as given that the insurer bargains with the hospitals observed in the network. However, suppose that before or after bargaining, the insurer may choose to remove some hospitals from the network altogether.

First, consider the *ex-ante exclusion NiN* case in which a group of $J$ hospitals is selected from a finite pool of potential hospitals, and only then, in a second stage, the NiN interaction occurs simultaneously with all hospitals. The solution concept for this model is subgame perfect equilibrium; namely, the insurer selects a group of hospitals such that its surplus will be maximized, given the second-stage NiN bargaining.

**Proposition 1.** *In the ex-ante exclusion NiN model there exists a $\beta^* < 1$ such that if $\beta \in (\beta^*, 1)$ then in any subgame perfect equilibrium the selected network consists of a single hospital.*

*Proof.* Let $\mathcal{X}$ denote the hospital pool. By Theorem 1, for each subset of hospitals $X \subset \mathcal{X}$ with at least two hospitals, there is a discount factor $\beta_X < 1$ such that if $\beta > \beta_X$ the insurer makes a negative surplus in the NiN model in which the hospital network is $X$. Since the insurer makes a positive surplus when it Nash bargains with any single hospital, the result follows by taking $\beta^*$ to be the maximum of the $\beta_X$’s. \qed

Proposition 1 does not suggest small networks are to be expected in insurance markets. In fact, Wedig (2013) finds that consumers insurance choice reflects significant welfare costs for having access to a smaller hospital network. Rather, Proposition 1 only implies that this specific two-stage structure cannot satisfactory resolve the price inflation due to applying NiN, and in particular complete Nash overpayment.

An alternative refinement is *ex-post exclusion NiN*, suggested in Crawford and Yurukoglu (2012). Here, the insurer may remove hospitals from the network after the negotiation is complete. Ex-post exclusion guarantees the insurer at least zero surplus and thus avoids complete Nash overpricing. Moreover, the insurer’s outside option during the negotiation is bounded at zero, limiting each hospital’s ability to overprice.

However, this solution has two drawbacks. First, the equilibrium correspondence is non-differentiable, making it less tractable for estimation. Second, some
Nash overpricing may persist. That is, it is still possible that for sufficiently high $\beta$, a hospital’s price exceeds the value of service to patients choosing it in equilibrium. Both problems are illustrated in the proof of the following Proposition.

**Proposition 2.** The NiN model with ex-post upstream selection always provides the insurer non-negative surplus. However, there exists a $\beta^* < 1$ such that if $\beta \in (\beta^*, 1)$ a subset of the hospitals’ prices $p_j$ may exceed the value of service in the corresponding hospital ($v_j$).

**Proof.** That insurer surplus is non-negative is immediate. The remainder is proved by the following modification of example [1]. Assume only two types of patients $ab, ba$, with the same $u^h = 10$ and $u^l = 5$ values as above. However, in contrast with the previous example, assume a unit measure of patients, with $s$ of the patients type $ab$ and $1 - s$ of the patients of type $ba$.

The insurer’s value from a full network given prices $p_A, p_B$ is:

$$F(p) = \max\{0, 10 - p_As - p_Bs, 10s + 5(1 - s) - p_A, 10(1 - s) + 5s - p_B\}$$

Without $A$, the insurer’s value given $p_B$ is:

$$F_{-A}(p) = \max\{0, 10(1 - s) + 5s - p_B\}$$

The Nash Bargaining price responses are:

$$p_A(p_B) = \begin{cases} \beta(p_B + 5) & p_B \leq 10 - 5s \\ \frac{\beta(10 - p_B(1-s))}{s} & p_B \geq 10 - 5s \end{cases}$$

$$p_B(p_A) = \begin{cases} \beta(p_A + 5) & p_A \leq 5(1 + s) \\ \frac{\beta(10 - p_A s)}{1-s} & p_A \geq 5(1 + s) \end{cases}$$

The pair $\hat{p}_A = 10 \frac{\beta}{s(1+\beta)}$ and $\hat{p}_B = 10 \frac{\beta}{(1-s)(1+\beta)}$ is an equilibrium if $\beta$ and $s$ are such that $\hat{p}_A > 5(1 + s)$ and $\hat{p}_B > 10 - 5s$. Both $\hat{p}_j$ increase with $\beta$ and are continuous for $\beta \geq 0$.

Assume $\beta = 1$. Then for $s \in (0.5 - \xi, 0.5 + \xi)$, with $\xi = 1 - \frac{\sqrt{5}}{2}$ we have $\hat{p}_A = \frac{5}{s} > 5(1 + s)$ and $\hat{p}_B = \frac{5}{1-s} > 10 - 5s$. Moreover, for any $s \neq 0.5$, in this equilibrium, $\max_j p_j > 10$.

By continuity, there is a $\hat{\beta} < 1$ such that for any $\beta > \hat{\beta}$ the equilibrium is sustained for $s \in (0.5 - \xi, 0.5 + \xi)$ for some $\xi \in (0, \xi)$.

This proves the proposition. For completeness, we consider the case that $s \notin$
For sufficiently high $\beta$, the equilibrium derived from the responses $p_j(p_{-j}) = \beta(5 + p_{-j})$ obtains prices higher than 10 and thus violates the required conditions. Without loss of generality, consider the case $s > 0.5 + \xi$. Then the equilibrium must satisfy $p_A \leq 5(1 + s)$ and $p_B \geq (10 - 5s)$. The resulting equilibrium is given by:

$$p_A = \beta \frac{10 - 5\beta(1 - s)}{s + \beta^2(1 - s)}; \quad p_B = \beta(p_A + 5)$$

Prices are again continuous in $\beta$ and $\lim_{\beta \to 1} p_A = 5 + 5s$ and $\lim_{\beta \to 1} p_B = 10 + 5s > 10 = v_B$.

Interestingly, in the case considered in the proposition, the hospital with the smaller share and thus less value to the network obtained the higher margin in equilibrium.

### 3.3 Sequential Nash bargaining

The difficulty arising from NiN lies in the hypothetical “disagreement event” associated with each negotiation. For example, in the two-hospital case, when $A$ is out of the network (i.e., there is disagreement with $A$) its entry-contribution exceeds its true value because its absence from the market would generate an adverse shift in patients going to hospital $B$ which would be welfare reducing at $B$’s price. Avoiding this outcome crucially depends on constructing a bargaining mechanism where the impact of disagreement on other bargaining outcomes is a part of the analysis. This is the focus of the present Section.

One way is to assume that once there is a negotiation breakdown with some hospital, other hospitals observe the breakdown and negotiation continues accordingly. This can be achieved by sequential bargaining: the hospitals are ordered in a (commonly known) sequence; if negotiation breaks down with some hospital early in that sequence all subsequent negotiations assume the rejecting hospital is not in the insurer’s network. Such sequential bargaining have been frequently considered in theoretical work, including section 6 in [Horn and Wolinsky](1988).

It is easy to see that the insurer’s surplus is positive under sequential negotiations. Consider again the two-hospital case with $A$ going first and $B$ going second.
Given any outcome of negotiations with \( A \), the bargaining problem with \( B \) must result in a non-negative addition to the insurer’s overall surplus (or else the insurer will not sign a deal with \( B \)). Also, one can map any possible outcome in the negotiations with \( A \), say \( o \), to the subsequent bargaining problem that will be played with \( B \), say \( P(o) \). Since, as we just observed, the insurer’s surplus in \( P(o) \) is non-negative given any possible \( o \), the bargaining problem with \( A \) boils down to a standard Nash bargaining problem in which both parties make positive profits. This idea generalizes to any length of hospital-sequence.

The sequential order of negotiations does not affect the insurer’s surplus, but it does affect negotiated prices and hospital payoffs. For example, in the two-hospital case where \( A \) is the first in the sequence, disagreement with \( A \) automatically makes \( B \) a monopolist. In contrast, disagreement with \( B \) cannot have such a favorable effect on \( A \)’s bargaining position, since it can only happen after the interaction with \( A \) has concluded. Specifically, either (i) a deal with \( A \) has already been signed and so \( A \)’s price is fixed, or (ii) \( A \) has dropped out, and is no longer in the network. We show that if there are only two hospitals, or if there are \( J \) symmetric hospitals, then a hospital earlier in the sequence has a lower payoff than all those that follow.

In terms of formalism, the economic environment that we consider is the same as in the previous Section, the only difference being that the Nash products now reflect the order of negotiations. We call this model the sequential Nash model.

Since it is instructive and fairly simple, we start by presenting the specific case of two hospitals and then turn to the general case.

### 3.3.1 Two hospitals

Without loss of generality, assume the insurer negotiates first with hospital \( A \). Consider the insurer’s bargaining with hospital \( B \) given that bargaining with hospital \( A \) has concluded. Since \( p_A \) has already been determined, this bilateral negotiation is effectively equivalent to the NiN bargaining setup. Without hospital \( B \), the insurer’s surplus is determined by his agreement with hospital \( A \). The insurer’s surplus with only \( A \) is:

\[
F_{-B} = q_A(v_A - p_A) + q_{A,-B}(v_{A,-B} - p_A).
\]
For any price $p_B$ the insurer’s value with both hospitals in the network is

$$ F = q_A(v_A - p_A) + q_B(v_B - p_B). $$

Thus the additional surplus accruing to the insurer from adding hospital $B$ to a network with only hospital $A$ is:

$$ F - F_{-B} = q_B(v_B - p_B) - q_{A,-B}(v_{A,-B} - p_A). $$

The Nash product is $[F - F_{-B}]^{1-\beta} \cdot [q_B(p_B - c_B)]^\beta$. Maximizing it gives the price:

$$ p_B = \beta \frac{q_B v_B - q_{A,-B}(v_{A,-B} - p_A)}{q_B} + (1 - \beta)c_B. \quad (3.9) $$

Substituting this $p_B$ into the expression for $F$ we obtain:

$$ F = q_A(v_A - p_A) + q_B v_B - q_B \left[ \beta \frac{q_B v_B - q_{A,-B}(v_{A,-B} - p_A)}{q_B} + (1 - \beta)c_B \right] $$

$$ = q_A(v_A - p_A) + q_B(v_B - c_B)(1 - \beta) + \beta q_{A,-B}(v_{A,-B} - p_A). $$

$$ (3.10) $$

Now consider bargaining with $A$. Without $A$, the insurer will bargain with $B$, when the insurer’s outside option is zero. Maximizing the Nash product for this problem gives:

$$ F_{-A} = (1 - \beta)(q_B(v_B - c_B) + q_{B,-A}(v_{B,-A} - c_B)). $$

Therefore,

$$ F - F_{-A} = q_A(v_A - p_A) + \beta q_{A,-B}(v_{A,-B} - p_A) - (1 - \beta)q_{B,-A}(v_{B,-A} - c_B). $$

Maximizing the Nash product $[F - F_{-A}]^{1-\beta} \cdot [q_A(p_A - c_A)]^\beta$ gives:

$$ p_A = \beta \frac{q_A v_A + \beta q_{A,-B} v_{A,-B} - (1 - \beta)q_{B,-A}(v_{B,-A} - c_B)}{q_A + \beta q_{A,-B}} + (1 - \beta)c_A. \quad (3.11) $$

Equipped with all these formulas, we can turn to the results.

**Proposition 3.** *In the sequential Nash model with two hospitals, the insurer’s surplus is independent of the order of negotiations.*
Proof. Consider the case where hospital $A$ is first and $B$ is second. Plugging $p_A$ from (3.11) into the expression for the surplus, (3.10), gives the surplus:

$$(1 - \beta)q_A(v_A - c_A) + (1 - \beta)q_B(v_B - c_B) + \beta(1 - \beta)q_{A,-B}(v_{A,-B} - c_A) + \beta(1 - \beta)q_{B,-A}(v_{B,-A} - c_B).$$

Clearly, the same expression obtains if $B$ goes first.

Denote by $\pi_j^1$ the profit of hospital $j$ if it is the first hospital in the sequence, and denote by $\pi_j^2$ its profit if it is second.

Proposition 4. Consider the sequential Nash model with two hospitals. There exists a $\beta^{**} < 1$ such that if $\beta \in (\beta^{**}, 1)$ then $\pi_j^1 < \pi_j^2$ for both $j = A, B$.

Proof. Wlog, consider hospital $A$. Since the quantities are unaffected by price and by the order of negotiations (hospital $A$ serves $q_A$ patients under the solution), it is enough to show that for all sufficiently large $\beta$’s we have $p_A^1 < p_A^2$, where $p_A^1$ is the price corresponding to $\pi_A^1$. It is enough to verify that that’s the case when $\beta = 1$.

Setting $\beta = 1$ in (3.11) gives:

$$p_A^1 = \frac{q_A v_A + q_{A,-B} v_{A,-B}}{q_A + q_{A,-B}}. \quad (3.12)$$

Setting $\beta = 1$ in the analog of (3.9) gives:

$$p_A^2 = \frac{q_A v_A - q_{B,-A}(v_{B,-A} - p_B^1)}{q_A}. \quad (3.13)$$

We argue that

$$\frac{q_A v_A + q_{A,-B} v_{A,-B}}{q_A + q_{A,-B}} < \frac{q_A v_A - q_{B,-A}(v_{B,-A} - p_B^1)}{q_A}. \quad (3.14)$$

Simplifying this expression we get:

$$q_A q_{A,-B}(v_{A,-B} - v_A) < -q_{B,-A}(v_{B,-A} - p_B^1)(q_A + q_{A,-B}).$$

The LHS is negative since $v_{A,-B} < v_A$. Therefore, it is enough to prove that the RHS is positive, or that $p_B^1 > v_{B,-A}$. Clearly, it is enough to show that $p_A^1 > v_{A,-B}$. This follows immediately from (3.12), since $v_A > v_{A,-B}$.

\qed
3.3.2 An arbitrary number of hospitals

In the case of \( J \geq 3 \) hospitals we add the following assumption: We assume that patients leave the insurer if their top two options leave the network. To see the importance of this assumption, suppose that the hospitals are ordered from 1 to \( J \), and consider negotiations with hospital \( j - 1 \). If there is disagreement in these negotiations, then the insurer moves on to bargain with hospital \( j \). Now, in order to formulate the Nash product for these negotiations, it is important to know what happens in case there is disagreement with \( j \); in particular, we need to know what would happen to the patients who had \( j - 1 \) at their top choice, and would choose \( j \) if \( j - 1 \) is out of the network. Our assumption allows us to ignore these patients; that is, to assume that they leave the network.\(^{10}\)

The following is a generalization of Proposition 1.

**Proposition 5.** In the sequential Nash model with \( J \) hospitals, the insurer’s surplus is independent of the order of negotiations.

The following is a generalization of Proposition 2, under the restriction to symmetric hospitals. In its statement, \( \pi^l \) is the profit of a hospital if it is in the \( l \)-th position in the order of negotiations (there is no need in a hospital-index, since the proposition only speaks of symmetric hospitals).

**Proposition 6.** Consider the sequential Nash model with \( J \) symmetric hospitals. There exists a \( \tilde{\beta} < 1 \) such that if \( \beta \in (\tilde{\beta}, 1) \) then \( \pi^l \) is strictly increasing in \( l \).

The proofs involve some tedious algebra, relegated to the appendix.

The reliance of sequential bargaining on an “order of negotiations” is a significant drawback. In practice, it is unlikely that the order is fixed exogenously or known by the hospitals. Moreover, for empirical work, the ordering is almost certainly unknown although it has significant implications on prices and hospital profits. We therefore consider a third model, which is designed to address this issue. In this new model, the NiN and Sequential Nash models serve as building blocks.

\(^{10}\)Incidentally, in our interview, \( [\text{Klein} (2015)] \) without this assumption being mentioned at all, indicated that as an insurer, he expected backlash but not significant loss of business if one hospital left the network, but a drastic reaction and loss of business if two (or more) hospitals leave.
3.4 Repeated Nash bargaining

Suppose that the insurer and hospitals bargain in every period $t$, where $t = 1, 2, \cdots$. In the beginning of each period $t$, the insurer makes a price offer to each hospital, these offers being simultaneous. Denote these price offers by $(p_1(t), \cdots, p_J(t))$. Then, each hospital $j$ accepts or rejects $p_j(t)$, these responses being simultaneous. Every price that is accepted is implemented in that period. If all prices are accepted, then all are implemented, and play moves on to period $t + 1$, in the beginning of which the insurer offers prices, $(p_1(t + 1), \cdots, p_J(t + 1))$. If, on the other hand, hospital $j$ rejects $p_j(t)$ then the price for this hospital is determined by maximizing a Nash product of a one-shot (i.e., NiN) problem, in which the prices of all other hospitals are taken to be $p_{-j}(t)$. Price offers and responses are publicly observable.

Consider a period in which a price offer is rejected for the first time. Let $R$ denote the set of hospitals who rejected their offer. The following happens: a hospital in $R$, call it $j^*$, is selected randomly and with equal probabilities, and then an ordering of hospitals is selected (randomly and with equal probabilities) in which $j^*$ is placed first. From period $t + 1$ onwards each hospital $j$ receives the periodic payoff $z_j$, where $(z_1, \cdots, z_J)$ is the vector of hospital payoffs in the sequential Nash model under the aforementioned order.

We assume that all hospitals share a common discount factor, $\delta \in (0, 1)$, and each hospital’s objective is to maximize its discounted expected profits, from each period onwards. We call this model the repeated Nash model (RN for short).

An RN equilibrium is a vector of prices, $(p_1^*, \cdots, p_J^*)$, such that each hospital $j$ prefers acceptance of $p_j^*$ to its rejection, given that (a) no rejection by any hospital has happened in the past, (b) no other hospital rejects its current offer, (c) no hospital will reject its offer in the future.

Our goal is to find the insurer-surplus-maximizing RN equilibrium.

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11 This is also what happens if more than one hospital rejects the offer. We believe this slight inconsistency is not an issue, since the solution to the model (see below) only requires consideration of at most a single rejection.

12 We model explicitly offers and responses. Following a rejection, only payoffs are specified, not the non-cooperative play that generates them. Any such play that results in the sequential game payoff for the rejecting hospital may be assumed. The RN equilibrium corresponds to a “Nash reversion” equilibrium of that unspecified extensive form.
A punishment price—the post-rejection price that is supposed to deter a hospital from rejecting the equilibrium price offer—is the price this hospital obtains in the sequential Nash model, where this hospital is placed first in the order of negotiations. We denote this price by $p^F_j$, where the superscript indicates that the hospital’s position is “first.”

**Lemma 1.** In the RN model:

$$p_j^F = \beta q_j v_j + \sum_{k \neq j} [\beta q_k v_k - (1 - \beta) q_k (v_k - j - c_k)] q_j + \beta \sum_{k \neq j} q_k v_k - j$$

(3.13)

*Proof.* Set $i = 1$ in equation (B.6). \qed

Note that, as opposed to NiN prices, the prices of hospitals $k \neq j$ do not appear in the formula for $p^F_j$. That $p^F_j$ is a low price is formalized in the following result. All remaining proofs for this section are in the appendix.

**Proposition 7.** In the RN model, if hospitals have sufficiently large bargaining power the punishment price converges to the hospital’s per patient value if it was the only hospital in the network: For every $\varepsilon > 0$ there is a $\tilde{\beta} < 1$ such that for all $\beta \in (\tilde{\beta}, 1)$, $|p_j^F - \frac{q_j v_j + \sum_{k \neq j} q_k v_k - j}{q_j + \sum_{k \neq j} q_k v_k - j}| < \varepsilon$.

By assumption, a hospital standalone value is smaller than its value to the patients for whom it is the top choice. Hence the following result is immediate:

**Corollary 1.** In the RN model, if $\beta$ is sufficiently large then the profit for a punished hospital is strictly lower than the average per-patient value that the hospital provides under the full network.

Next, we describe what happens if hospital $j$ rejects a proposal. Recall that in this case the insurer and hospital $j$ engage in NiN bargaining where all other prices are fixed at equilibrium level. With $p^*$ denoting the equilibrium prices, the price of the aforementioned “deviating” (i.e., rejecting) hospital, $p^D_j$, is given by:

$$p_j^D = \beta q_j v_j - \sum_{k \neq j} (v_k - j - p^*_k q_k - j q_j + \sum_{k \neq j} q_k v_k - j)$$

(3.14)

The hospital’s price in all periods after deviation is the punishment price $p_j^F$. This
gives rise to the following incentive compatibility (IC) constraint for hospital $j$:

$$(1 - \delta)p_j^D(p^*_{-j}) + \delta p_j^F \leq p_j^*.$$ (3.15)

As surplus from the network is independent of prices, finding the surplus-maximizing RN equilibrium corresponds to minimizing prices $p_j$ subject to these $J$ constraints. As the insurer’s objective and all constraints are linear in prices, all IC must bind. This gives us $J$ linear equations and $J$ unknowns; the solution of this problem provides the equilibrium prices of the RN model:

**Proposition 8.** In the surplus-maximizing equilibrium of the RN model, hospital prices are given by equations (3.14) and (3.15) as an equality.

Note that when $\delta \sim 0$, the RN prices are approximately the NiN prices, in which case the insurer’s surplus is negative (given that the hospitals’ bargaining power is sufficiently large). Thus, the RN model is only appealing if the hospitals are sufficiently patient. If they are not, there is no point in considering the RN model. Recall, however, that positive surplus can always be guaranteed—no matter the hospitals’ patience level—in the sequential Nash model.

If hospitals are very patient, prices converge to the punishment levels: $p^* \rightarrow p^F$ as $\delta \rightarrow 1$. Since the punishment levels provide each hospital with less profit than the value it generates (Corollary 1), the insurer is guaranteed a positive surplus.

**Corollary 2.** In the RN model, there is $\bar{\delta} < 1$ such that for any hospitals’ discount factor $\delta \geq \bar{\delta}$ the insurer’s per-period surplus is strictly positive for any $\beta$.

Figure 3.2 compares the prices as a function of $\beta$ for Example 1. The three panels correspond to the different bargaining models: NiN, sequential Nash, and RN. Note that for this example, a price exceeding 10 (the dashed line) implies that the insurer receives negative surplus from contacting with the hospital. This occurs in the NiN model when hospitals have large bargaining power and there are a large number of “switching” consumers. Moving from NiN to the sequential model, we see that the insurer is now guaranteed positive surplus. However, as $\beta \rightarrow 1$ the hospitals are able to capture all the surplus. In particular, the second hospital in the negotiation obtains prices that are higher than its value, while the first hospital obtains prices that are lower. Moving to the RN model shifts the prices for both hospitals to the “first hospital” price, leaving the insurer a strictly positive margin.
even when the hospitals have all the bargaining power. If hospitals are sufficiently patient the RN prices converge to the “first hospital” prices. Note that even as $\beta \to 1$ the insurer still retains some surplus in this case.

Example 2. Logit Demand: We illustrate the difference between the NiN and RN models and their predictions using a simple Logit demand model. Suppose there are $n = 11$ hospitals, each with the same base utility $v = 1.86$ for a patient, with the net utility of out-of-network treatment normalized to zero. Figure 3.3 plots the predicted equilibrium prices for all valid $\beta$ values from the NiN and RN models for selected $\delta$ (0.3, 0.5, 0.8, 1). Appendix C provides the detailed derivations. The dashed line represents the average valuation of a patient in the network (4.27).

The NiN price (top curve) becomes very sensitive to $\beta$ at high values of hospital bargaining power. Moreover, the NiN price “takes off” once $\beta > 0.6$ and the NiN price exceeds average patient surplus whenever $\beta > .85$.

In contrast under RN, even a small amount of hospital patience ($\delta = .3$) is sufficient to eliminate the possibility of market breakdown. More generally, the effect of $\beta$ is more contained with patient hospitals. As hospital patience increases, the effect of hospital bargaining power on price decreases (i.e., $\partial^2p/\partial\beta\partial\delta < 0$). At the limit of perfectly patient firms ($\delta = 1$) even with full bargaining power ($\beta = 1$) each hosp-

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Figure 3.2: Prices for the symmetric example from section 1 for $\alpha = 0.25, 0.5$ or 0.75. The left panel is for the 2-hospital symmetric NiN model (as in section 1). The middle panel is the average price paid by the insurer using the sequential model. The right panel shows the limit of the price as $\delta \to 1$ for the repeated sequential model. Note that price in the left panel goes up to 25, as opposed to only 10 in the other panels.

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13The example values were chosen to be similar to the values estimated in Gowrisankaran et al. (2015). In particular, the number of hospitals and the outside good share here is similar to those reported there.

14Note that because patients go to their preferred hospital and have logit demand average valuation is $\log(11e^{xp(v)} + 1) = v + \log(11 + \exp(-v)) = 1.86 + 2.41$. 

25
Figure 3.3: Price in the Logit example for \( n = 11, v = 1.86 \) as a function of the hospital bargaining power \( \beta \). The dashed line is the full network value per patient, \( \log(n \cdot e^v + 1) \). The higher, thin, line is the NiN price. The other, thick lines are the RN solution with discount factors, for highest line to lowest, of 0.3, 0.5, 0.8 and 1.

tal only captures its value to the average patient in the market, \( p^R \approx v = 1.86 \) while the insurer captures all the gains due to patients’ ability to choose their preferred hospitals (2.41). This illustrates our intuition that the introduction of \( \delta \) provides the model flexibility to distinguish between the hospital’s ability to capture the direct surplus it generates (\( \beta \)) and the surplus generated through complementarities between hospitals (\( \delta \)).

Finally, the effect of hospital patience on price decreases as \( \delta \) rises (i.e., \( \partial^2 p / \partial \delta \partial \delta < 0 \)). If the hospitals are at least as patient as usually assumed (\( \delta \geq .8 \)), the RN model is relatively insensitive to small changes in \( \delta \) than at lower discount levels.

4 Discussion

4.1 Economic Implications

Theorem \[ \] implies that using simultaneous Nash-in-Nash bargaining to analyze policy relevant questions has the potential to produce counterintuitive implications. For example, one (perverse) implication is that under the simultaneous Nash-in-Nash bargaining model, profit maximizing insurers may well generate more surplus than surplus maximizing ones. In particular, surplus maximizing insurers may
generate negative surplus, while profit-maximizing insurers will generate positive surplus (subject to imperfections of consumer insurer choice).

This insight is also important for other settings to which our theory applies. To be specific, consider a purchasing department that contracts with various competing providers over inputs to be used by several downstream “user” departments (e.g., a university’s purchasing department negotiating with several laptop makers). If the department is evaluated as a profit center based on some transfer price (or willingness to pay) from each of the downstream departments and the downstream users must make their purchases through the upstream purchasing department, then the organization will be subject to the same problem as the surplus maximizing insurer and may well generate negative surplus.

Bargaining models are frequently used to predict the change in prices resulting from a merger of hospital networks. Whether a NiN or RN bargaining model is used can lead to different predictions on how prices will respond to a merger.

Such differences can occur even when all of the parameters (demand, cost, and bargaining power) of the model are known. The differences between the models are most stark in the case where \( p_j > v_{k-j} \). Here, a merger between hospitals \( j \) and \( k \) will lead to a price decline under NiN, as shown in the two hospital example in section \( \text{1} \). In other words, using the NiN model for counterfactuals would support approving the merger by a regulatory authority concerned with maximizing surplus. In contrast, the RN model for the same specification will likely predict that prices increase as a result of the merger. In particular, in the two-hospital example, merger would lead to a monopoly and the insurer would no longer be able to use the order of negotiations as a threat.

The example is straightforward to generalize following the same arguments as in Proposition \( \text{1} \). If hospitals’ bargaining power is high enough, any merger simulation would result in lower prices:

**Corollary 3.** In the NiN model, there is a \( \bar{\beta} < 1 \) such that for any \( \beta > \bar{\beta} \), surplus from a enrollee-surplus-maximizing insurer is maximized if and only if all hospitals merge.

Finally, use of a mis-specified NiN model can bias structural estimation out-
comes of bargaining power and costs. Biased estimates of these parameters are likely to affect regulators perceptions of merger effects. For example, a merger between two hospitals with very low $\beta$ cannot have significant negative price implications, as any cost savings from the merger will also be passed through to the patients (who are the insurer’s agents). On the other hand, an over-estimate of $\beta$ is likely to lead regulators to doubt hospitals arguments that a merger is necessary to counteract insurer bargaining power. Biased estimates of costs can affect not just regulators understanding of bargaining but also the scope of possible cost reductions as a result of the merger. For example, if the estimated costs are much lower than they really are, as found in Appendix D for the Monte Carlo simulation with $\beta = \delta = .8$, it is difficult to accept any cost-saving arguments in favor of the merger, while those may well exist. Moreover, the upward bias in markups in this example may lead regulators to assume a hospital merger would only enhance what the model indicates is a significant degree of market power.

4.2 Implications for Empirical Estimation

The Nash-in-Nash (NiN) framework has served as a workhorse model for the empirical investigation of multilateral bargaining, in part due to it’s empirical tractability (cf. Draganska et al. (2009); Crawford and Yurukoglu (2012); Gowrisankaran et al. (2015); Ho and Lee (2017b)). This tractability stems from the fact that, given observed prices and estimated demand elasticities, it is possible to recover implied costs from the NiN model by solving a linear system of equations. This permits straightforward estimation of NiN bargaining parameters and costs via generalized method of moments. In Appendix A, we show that the repeated sequential Nash (RN) model is also tractable for estimation. RN represents a generalization of NiN with one additional parameter—the bargaining discount factor $\delta$, and the system of equations that define costs under RN remains linear (see equation A.7 in Appendix A). Thus, there is no increase in computational complexity to estimate the RN model over NiN.

A second question is whether using the more complicated RN model is likely to result in different conclusions than the simpler NiN approach. To investigate this, we conduct a Monte Carlo investigation where we estimate both models for a variety of parameter values of the RN model. We summarize the conclusion of this
exercise here, while full details of the Monte Carlo setup and analysis are available in Appendix D.

First, we consider the case when the correct model is NiN, i.e., when $\delta = 0$. The following is proved in the appendix:

**Proposition 9.** Hospital costs and bargaining parameter estimates implied by NiN model estimation are equivalent to those implied by RN model estimation constrained to $\delta = 0$.

In this case, as we would expect, both estimation approaches work well, but the NiN approach is more precise. However, when we raise $\delta$ to 0.8, so that the NiN model is misspecified, we find that estimated bargaining parameters can be biased either upwards or downwards, and that these biases also corrupt estimates of hospital costs. The bias in parameter estimates grows more pronounced as hospital bargaining power increases. Finally, we also examine the precision with which $\delta$ itself can be estimated. Here the results are mixed. When hospitals have little bargaining power, NiN and RN prices are very similar (they are equal if $\beta = 0$) and the estimates of $\delta$ tend to be very imprecise. However, for higher values of $\beta$ we show that the RN model is able to reject NiN for reasonably sized datasets.\(^\text{16}\)

All in all the simulations suggest that the RN model can separately identify the bargaining and discount parameters. Using RN estimation in place of NiN does not reduce the tractability or performance of the estimation. If hospital bargaining power is limited, both models provide similar predictions (this is intuitive, since little bargaining power implies hospitals price close to cost). If hospital bargaining power is significant, the additional flexibility of the RN model improves the accuracy of the bargaining and cost estimates in cases where NiN is misspecified.

### 4.3 The role of substitutes in NiN vs. RN

The main difference between the NiN and RN frameworks lies in the role of alternative hospitals as substitutes. In the NiN framework, each hospital’s price increases in the surplus the hospital provides and decreases in the amount of surplus other hospitals could provide patients who would prefer $j$ were it in network. That is, when negotiating with one hospital, the other hospital prices serve an important

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\(^{16}\)The Monte Carlo featured in Appendix A uses 1000 markets of four hospitals each. We have also verified that NiN is rejected with the same specification using only 100 markets.
role by setting the disagreement outcome of the insurer. Therefore, the insurer benefits from lower prices not just directly (i.e. paying less) but also indirectly due to the improved bargaining position with other hospitals. However, because the NiN framework takes all rival prices as given, it does not incorporate the effect of one hospital’s price on negotiations with other hospitals, leading to the overpayment result in Theorem \[1\].

The RN framework, incorporates the effect of negotiated prices on other negotiation in (3.13). Most clearly, other prices do not enter (3.13) as they will be negotiated in the future. The insurer’s overall surplus in the event of agreement is not simply his surplus in the resulting network, but instead reflects how agreement affects his position in future negotiations with other hospitals.

This has three parts: First, in the numerator, \( \beta q_{j,-k} v_{j,-k} \) reflects the increase in the insurer’s surplus from having hospital \( j \) to serve as a disagreement point in negotiations with all hospitals. Note that the impact of other hospitals being dropped does not appear at all in (3.2). This term is weighted by \( \beta \) to reflect the fact that a better bargaining position with other hospitals is increasingly important to the insurer as other hospitals bargaining power increases. Second, the term \((1 - \beta) q_{k,-j}(v_{k,-j} - c_k)\) is the insurer’s share of the additional surplus generated by having both hospitals in the network over just hospital \( k \), it represents patients who prefer hospital \( j \), who will no longer be bargained over in future negotiations once it is clear that \( j \) is in network. This term does have an analogue in (3.2), \( q_{k,-j}(v_{k,-j} - p_k) \) however now, the insurer’s surplus reflects it’s share \((1 - \beta)\) of the total surplus effect —\( c_k \) instead of \( p_k \)— of agreement on future negotiation with \( k \). The final difference is the fact the insurer internalizes the impact of \( p_1 \) on future negotiations. This is reflected in the extra term in the denominator is \( q_j + \beta \sum_{k \in J, k \neq j} q_{j,-k} \) instead of just \( q_j \).

### 4.4 Extensions

Our analysis included two qualitative assumptions about consumer and insurer preferences. First, we assumed that the consumer does not internalize the price \( p_j \) that the insurer pays the hospital for treatment. Effectively, this means that either co-payments are equal across hospitals or that, to a reasonable approximation, consumers are insensitive to copays. Accounting for patient price sensitivity is pos-
sible, but significantly complicates the notation since consumer utility will be a function of prices faced by patients. However, this would not affect any qualitative results unless co-payments from second choice hospitals are higher than the value for consumers.

Moreover, evidence suggest that patient sensitivity to hospital prices is economically very small. [Gowrisankaran et al., 2015] (GNT hereafter) estimate a statistically significant but economically limited effect of price on patient choice of 0.0008 per out-of-pocket dollar. With a co-pay of 20%, a minute’s driving distance difference between the hospitals is equivalent to $835 difference in the hospital’s price. In GNT’s sample, out of 13 hospitals, the mean price for the lowest costing six was within $800 of each other ($9,545 to $10,273); the mean price for the next six was again within $800 of each other ($11,420 to $12,112); the most expensive hospital has a much higher mean price of $13,270. In other words, the price difference between hospitals in the same price group has no effect and the price difference between the most and least expensive hospital has a smaller impact on demand than a five minute distance difference. Moreover, for the typical out of pocket limit of insured patients, only the cost of the first day (in the year) matters.

In practice, Reinhardt (2006) summarizes (pp. 64-67) in rather colorful language that currently in the US it is effectively impossible for patients to compare prices across hospitals due to lack of price transparency and the practice of many separate charges for the same treatment episode.

Second, we assume that the insurer acts as an agent for patients and therefore is concerned with total patient surplus less costs rather than plan profits, outside the scope of the current paper. Insurers that maximize only total premium less total costs are not directly affected if patients use a hospital that provides less value than reflected in the premium, as long as the patients continue to pay the premium. Whether the network formation decisions of most insurers are better modeled as maximizing enrollee-surplus or short-term net profits, and how to model patient insurer choice is an open empirical question, with recent advances, such as Ho and Lee (2017b). Only the qualitative results in Section 3.1 depend on this assumption. All other results and formulations can be directly translated to profit maximizing insurers by replacing the patient values ($v$) with the appropriate primitive for the insurer’s objective.\footnote{An additional reason to consider surplus maximization is that about one in two Americans}

\[\text{\footnotesize 17}\]
In addition, our model can be extended to capture several technical complications typical to related papers (e.g. Gaynor and Town (2011) and Gowrisankaran et al. (2015)). In particular, other papers allow for multiple diagnosis and set price separately for each diagnosis. In addition, Gaynor and Town (2011) allow both the hospital and the insurer to have a fixed cost associated with adding the hospital to the network. All these can be added without qualitative effects.

5 Conclusion

This paper has focused on the analysis of multilateral bargaining settings in which the total surplus from a suppliers’ network is non-linear in the (equilibrium) quantity sold by each supplier. This is particularly common when an intermediary (insurer or purchasing department) bargains with suppliers (hospitals or input producers) on behalf of downstream users.

We showed that if the intermediary cannot commit to a specific network, suppliers can charge unit prices that surpass the unit value because the intermediary must also consider the potential negative surplus from directing its users to their second-best suppliers.

This dynamic has significant normative implications. First and foremost, “forcing” the intermediary to commit not to reopen a failed negotiation guarantees that prices are below the users’ values. If this is not enforceable, encouraging profit (rather than surplus) maximization and reducing frictions in the downstream markets become very important.

Our proposed repeated model describes the outcome that would result under a moderate (and reasonable) commitment power of the insurer—power which is sufficiently great to enforce sequential negotiations (and Nash reversion). The RN model can be used for estimation for both surplus and profit maximizing insurers.

Our analysis has several implications for empirical work. First, it identifies the important implications of the implicit “no-commitment” assumption common to covered in the private market are covered by a self-insured employer. (We thank Henry Mak for suggesting this.) In those plans, the insurer negotiates prices with providers but does not collect the enrollee premiums. Instead, the insurer is typically paid based on costs or volume of claims and enrollees. We are not aware of a model of insurer competition in the self-insured market. However, if employers consider health benefits a service to employees rather than a profit generator, surplus maximization is a more appropriate proxy than than profit maximization, with insurers competing in their ability to create value for the employer’s plan.
the existing models. To the extent that this assumption is maintained, other assumptions – positive surplus, monotonic prices – and expectations – inefficient mergers increase prices – should be doubted. Second, it provides an alternative estimation approach based on the commitment assumption.

References


A Estimating the RN model

Estimation of patient hospital demand is independent of the bargaining problem. Therefore, we assume that the demand system is estimated prior to estimating costs and bargaining parameter. This means that we can treat consumer expected valuations and choices conditional on the hospital network as known. The remaining parameters to estimate are the bargaining parameters \((\beta, \delta)\) and hospital costs. We assume that hospitals face an constant marginal cost per patient which is a function of a set of observable cost shifters, \(z_j\) and a hospital specific error term,

\[ c_j = \lambda z_j + \omega_j \]

where \(\lambda\) is a vector of cost parameters to estimate. To estimate \((\beta, \delta, \lambda)\) we rely on the pricing equations derived in the previous section. Specifically, similar to

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For example, demand could be estimated following [Capps et al., 2003]. In principle, it would be more efficient to estimate demand and supply jointly in a simultaneous equations framework, which is conceptually straightforward although computationally more demanding.
estimation of the NiN model, marginal costs can be derived as a function of demand and bargaining parameters, allowing us to recover \( \omega_j \) as a function of \((\beta, \delta, \lambda)\). We then jointly estimate demand and cost parameters via non-linear generalized method of moments (GMM).

Recall that the NiN price \( p^N \) negotiated if the hospital rejects the initial take-it-or-leave it offer is determined using treating all other prices as given. Letting \( p^* \) denote the observed (and RN equilibrium) prices, equation (3.14), the NiN prices satisfy:

\[
p^D = \beta \theta^D + \beta \Gamma^D p^* + (1 - \beta) c
\]

(A.1)

Where, \( \theta^D \) is a vector and \( \Gamma^D \) is a matrix defined by:

\[
\theta^D_j = \frac{v_j - \sum_{\ell \neq j} v_{\ell,-j} q_{\ell,-j}}{q_j}
\]

\[
\Gamma^D_{j,l} = \frac{q_{l,-j}}{q_j}
\]

Equation (A.1) proves Proposition 9: The price vector on the right hand side is the observed prices. In particular, \( p^N = p^* \) if and only if the NiN model is correct.

Prices under RN are convex combination of NiN prices and each hospital’s “punishment” price from reverting to the sequential model with that hospital negotiating in the first (least-favorable) position, which we derived in (3.13). Rewriting this formula in terms of markups and using matrix notation obtains,

\[
p^F = \theta^F(\beta) + (I + \Psi^F(\beta)) \cdot c
\]

(A.2)

The matrix \( \Psi^F(\beta) \) accounts for the impact of disagreement with hospital \( j \) on the cost of treating patients,

\[
\Psi^F_{j,-j} = \beta, \quad \Psi^F_{j,k \neq j} = \beta (1 - \beta) \frac{q_{k,-j}}{q_j + \beta \sum_{k \neq j} q_{j,-k}}.
\]

From this formula we see that a unit increase in hospitals cost causes price to go up by \( 1 + \beta \). And is increasing in the costs of rival hospitals in proportion to substitution to those hospitals when \( j \) is dropped from the system. That is, in the sequential game, \( j \)’s bargaining position is enhanced when it’s patients are likely to substitute to high cost hospitals. This feature of the sequential model will be inherited by the RN solution but is absent from the NiN solution, which considers only the importance of hospital \( j \)’s provision of surplus holding other hospital prices.
fixed.

The vector $\theta^F(\beta)$ is,

$$\theta^F_j(\beta) = \beta q_j v_j + \sum_{k \neq j} [\beta q_{j,-k} v_{j,-k} - (1 - \beta) q_{k,-j} v_{k,-j}]$$

The first term of the numerator represents the hospitals contribution to surplus, the remaining terms are adjustments to hospital $j$’s bargaining position based on cross-hospital substitution. If hospital $j$ is rewarded to the extent that it can serve as a substitute for hospital $k$ in the event that bargaining with $k$ fails. On the other hand, $j$’s bargaining position is reduced if other hospitals are strong substitutes in the event of its own disagreement.

Next, construct the RN Estimator by merging the NiN and punishment price equations. Following proposition 8 the observed price in the RN model is given by

$$p^* = \delta p^F + (1 - \delta)p^D \quad (A.3)$$

Using equations (A.1) and (A.2) we have an expression that is linear in prices and costs,

$$p^* = \delta \theta^F(\beta) + \delta(I + \Psi^F(\beta))c + (1 - \delta)\beta(\theta^D + \Gamma^D p^*) + (1 - \delta)(1 - \beta)c \quad (A.4)$$

To solve for either $p^*$ or $c$, rewrite (A.4) as:

$$0 = \Psi(\beta, \delta)c + \theta(\beta, \delta) + \Gamma(\beta, \delta)p^* \quad (A.5)$$

Here, the terms multiplying the cost vector $c$ and price vector $p$ are aggregated into the matrices $\Psi$ and $\Gamma$, and the constant terms are aggregated into the vector $\theta$:

$$\Psi(\beta, \delta) = I(1 - \beta(1 - \delta)) + \delta \Psi^F(\beta, \delta)$$

$$\Gamma(\beta, \delta) = (1 - \delta)\beta \Gamma^D - I \quad (A.6)$$

$$\theta(\beta, \delta) = \delta \theta^F(\beta) + (1 - \delta)\beta \theta^D$$

To back out costs from the demand system and bargaining parameters, solve
\( (A.5) \) for costs,

\[
c(\beta, \delta) = -\Psi(\beta, \delta)^{-1}(\theta(\beta, \delta) + \Gamma(\beta, \delta)p^*).
\] (A.7)

Proposition 9 is now immediate by observation: The NiN model is a special case of the RN model with the value for \( \delta \) set to zero.

Estimation of the supply side parameters using RN therefore is similar to the existing methods with the additional parameter \( \delta \) and a slightly more complicated non-linear function for costs. Specifically, given a set of bargaining parameters the structural error in costs is,

\[
\omega_j(\alpha_0, \alpha_1, \beta, \delta) = c_j(\beta, \delta) - \gamma z_j.
\] (A.8)

To construct the moments, define \( h_{j,n} \) as a vector of instruments for hospital \( j \) in market \( n \). The data moments are:

\[
g_{NJ}(\alpha_0, \alpha_1, \beta, \delta) = \frac{1}{NJ} \sum_{n,j} h_{n,j} \omega_{n,j}(\alpha_0, \alpha_1, \beta, \delta).
\] (A.9)

The GMM estimator is:

\[
\arg\min_{\alpha_0, \alpha_1, \beta, \delta} g_{NJ}(\alpha_0, \alpha_1, \beta, \delta)' W g_{NJ}(\alpha_0, \alpha_1, \beta, \delta).
\] (A.10)

Where \( W \) is a symmetric positive definite weight matrix. We use the standard 2-step GMM to derive the optimal weight matrix for each dataset.

The observed cost shifters \( z_j \), are available as instruments, but we clearly need two additional instruments to identify the two bargaining parameters. Candidates are most likely to come from the exogenous variation in the demand system, which generates variation in the substitutability of hospitals (e.g., \( v_{j,k} \) and \( q_{j,k} \)) that represent the key primitives in the matrices defined in \((A.6)\). An example of such an instrument could be the distances between hospitals, or the relative weights of different types of observable consumers which vary across markets. In the Monte Carlo analysis below, we will assume the existence of an observable demand shifter which will serve as instruments.

To conclude this section, note that, due to (9) the simultaneous NiN model nests the RN model by fixing \( \delta = 0 \), allowing us to test NiN against a more general
alternative. To facilitate comparison with the earlier literature, the following remark establishes the connection between our notation and that of GNT.

**Remark 1.** To facilitate comparison with existing work, in particular Gowrisankaran et al. (2015), rewrite (3.14) as

\[
p_N^j - c_j = \beta \left( v_j - p_N^j - \frac{\sum_{i \neq j} q_{i,-j} (u_{i,-j} - p_i)}{q_j} \right)
\]

(A.11)

Next, let \( \xi_j \) equal to the right hand side of equation (A.11) for hospital \( j \) and define \( \Lambda \) as a diagonal matrix with elements \( \Lambda_{j,-j} = -\frac{1}{\xi_j q_j} \). Then,

\[
-\Lambda (p_N^j - c_j) = q_j
\]

(A.12)

\( \Lambda \) is provided from the first stage estimates and observed prices. Note that this is exactly equation 13 in GNT, with the only differences that \( \Lambda \) in GNT has off-diagonal elements to account for cross hospital effects for hospitals within the same hospital system and that we omit \( \Omega \) in GNT which captures patient price sensitivity of demand.

**B Relegated proofs from the main text**

**Preliminary Lemma**

We first prove the following lemma. Suppose that the insurer is bargaining with hospital \( j \) and that:

1. The insurer’s profit without \( j \) in the network is \( V_0 \).
2. The hospital’s unit cost is \( c \)
3. The hospital and the insurer expect the hospital’s quantity to be \( q \)
4. Adding \( j \) to the network at unit price \( p \) increases the insurer’s profit by \( K - p \cdot y \). Note that \( y \) may equal \( q \) but does not have to.

**Lemma 2.** If there is a price \( p \) that solves the bargaining game defined above, then it is:

\[
p = \beta \frac{K}{y} + (1 - \beta)c.
\]

(B.1)
The insurer’s profit is \( V_0 + (1 - \beta)(K - cy) \) and the hospital’s profit per unit is \( \beta \cdot \frac{K - cy}{y} \).

**Proof.** Suppose the hospital and the insurer expect the hospital’s quantity to be \( q \). Then the bargaining problem is,

\[
\max_p [K - py]^{(1 - \beta)} [q (p - c)]^\beta \tag{B.2}
\]

The interior solution must satisfy:

\[
(1 - \beta) y [K - py]^{-\beta} [q (p - c)]^\beta = \beta \cdot q \cdot [K - py]^{(1 - \beta)} [q (p - c)]^{(\beta - 1)} \tag{B.3}
\]

Simplifying:

\[
p \cdot [(1 - \beta) y + \beta y] = \beta K + c \cdot (1 - \beta) y \tag{B.4}
\]

Simplifying again obtains the desired price. the insurer and hospital profits are straightforward. \( \square \)

**Lemma 3.** The expected value for the insurer after negotiating with \( i \) of \( J \) hospitals and obtaining prices \( \{p_1, \ldots, p_i\} \), denoted \( V(\{p_1, \ldots, p_i\}; \{i + 1, \ldots, J\}) \), is given by:

\[
V(\{p_1, \ldots, p_i\}; \{i + 1, \ldots, J\}) = \sum_{j=1}^{i} \left[ q_j (v_j - p_j) + \beta \sum_{k=i+1}^{J} q_{j,k} (v_{j,k} - p_j) \right] \\
+ (1 - \beta) \sum_{j=1}^{i} q_j (v_j - c_j) + \beta \sum_{j=1}^{i} q_{j,k} (v_{j,k} - c_j) 
\]

\[
\tag{B.5}
\]

The price for insurer number \( i \) in the order of negotiations is given by:

\[
p_i = \beta q_i v_i + \beta \sum_{k=i+1}^{J} q_{i,k} v_{i,k} - \sum_{j=1}^{i-1} q_{i,j} (v_{i,j} - p_j) - (1 - \beta) \sum_{k=i+1}^{J} q_{k} (v_{k} - c_k) \\
+ (1 - \beta) c_i 
\]

\[
\tag{B.6}
\]

**Proof.** With a slight abuse of notation, we use \( J \) to refer to the number of hospitals, the set of hospitals and the last hospital in the negotiation.

The proof is by backward induction. Suppose the hospitals are ordered 1..\( J \).
The value after \( J \) negotiations is: 
\[
V\left(\{p_1, \ldots, p_J\}; \emptyset\right) = \sum_{j=1}^{J} q_j (v_j - p_j). 
\]
When negotiating with the last hospital, the insurer’s outside option is 
\[
V\left(\{p_1, \ldots, p_{J-1}\}; \emptyset\right) = \sum_{j=1}^{J-1} q_j (v_j - p_j) + q_{J-1} \cdot (v_{J-1} - p_j). 
\]

The gain from bargaining with \( J \) is therefore 
\[
W_J\left(\{p_1, \ldots, p_{J-1}\}; \emptyset\right) = V\left(\{p_1, \ldots, p_J\}; \emptyset\right) - V\left(\{p_1, \ldots, p_{J-1}\}; \emptyset\right) 
= q_J \cdot (v_J - p_J) - \sum_{j=1}^{J-1} q_{J-1} \cdot (v_{J-1} - p_j). 
\]

Applying Lemma 2, \( p_J = \beta \left( v_J - \frac{\sum_{j=1}^{J-1} q_{J-1} (v_{J-1} - p_j)}{q_J} \right) + (1 - \beta) c_J. \)

Then we can place \( p_J \) to obtain the value after negotiating only with the \( J - 1 \) hospitals:
\[
V\left(\{p_1, \ldots, p_{J-1}\}; \{J\}\right) = \sum_{j=1}^{J-1} q_j (v_j - p_j) + q_J v_J - q_J p_J 
= \sum_{j=1}^{J-1} q_j (v_j - p_j) + \beta q_{J-1} (v_{J-1} - p_{J-1}) + (1 - \beta) q_J (v_J - c_J). 
\]

We now have a formulation for both \( p_J \) and the insurer’s value given the first \((J - 1)\) prices. Moving to the second to last bargaining game, if the bargaining succeeds and price is \( p_{J-1} \) the value to the insurer is given in \( V\left(\{p_1, \ldots, p_{J-2}\}; \{J\}\right) \) above. If the bargaining fails, we can then continue to the \( J \) negotiation step with only the first \((J - 2)\) hospitals, accounting for the patients that would have went to hospital \((J - 1)\) but instead go to either an earlier hospital or to hospital \( J \). The resulting value to the insurer is
\[
V\left(\{p_1, \ldots, p_{J-2}\}; \{J\}\right) = \sum_{j=1}^{J-2} q_j (v_j - p_j) + q_{J-1,j} \cdot (v_{J-1,j} - p_j) + \beta q_{J-1,j} (v_{J-1,j} - p_j) 
+ (1 - \beta) \left( q_J (v_J - c_J) + q_{J-1,j} \cdot (v_{J-1,j} - c_J) \right). 
\]
The gain from bargaining with $J - 1$ is therefore:

$$W_{J-1} \left( \{p_1, ..., p_{J-1}\}; \{J\} \right) = V \left( \{p_1, ..., p_{J-1}\}; \{J\} \right) - V \left( \{p_1, ..., p_{J-2}\}; \{J\} \right)$$

$$= q_{J-1} \cdot (v_{J-1} - p_{J-1}) + \beta q_{J,J-1} (v_{J,J-1} - p_{J-1}) - (1 - \beta) q_{J-1,J} (v_{J-1,J} - c_J)$$

$$- \sum_{j=1}^{J-2} q_{j-1,j} \cdot (v_{j-1,j} - p_j).$$

Applying Lemma 2

$$p_{J-1} = \frac{\beta q_{J-1,v_{J-1}} + \beta q_{J,J-1} v_{J,J-1} - (1 - \beta) q_{J-1,J} (v_{J-1,J} - c_J) - \sum_{j=1}^{J-2} q_{j-1,j} (v_{j-1,j} - p_j)}{q_{J-1} + \beta q_{J,J-1}} + (1 - \beta) c_{J-1}$$

Placing $p_{J-1}$ in $V \left( \{p_1, ..., p_{J-1}\}; \{J\} \right)$:

$$V \left( \{p_1, ..., p_{J-2}\}; \{J - 1, J\} \right) = \sum_{j=1}^{J-2} \left[ q_j (v_j - p_j) + \beta \sum_{k=J-1}^{J} q_{j,-k} (v_{j,-k} - p_j) \right]$$

$$+ (1 - \beta) \sum_{j=J-1}^{J} \left[ q_j (v_j - c_j) + \beta \sum_{k=J-1,k \neq j}^{J} q_{j,-k} (v_{j,-k} - c_j) \right].$$

To generalize, suppose that for a “future hospitals” list of length $J - i$, the expected value for the insurer given prices $\{p_1, ..., p_i\}$ is given by:

$$V(\{p_1, ..., p_i\}; \{i + 1, ..., J\}) = \sum_{j=1}^{i} \left[ q_j (v_j - p_j) + \beta \sum_{k=i+1}^{j} q_{j,-k} (v_{j,-k} - p_j) \right]$$

$$+ (1 - \beta) \sum_{j=i+1}^{j} \left[ q_j (v_j - c_j) + \beta \sum_{k=i+1,k \neq j}^{j} q_{j,-k} (v_{j,-k} - c_j) \right].$$

Then:

$$V(\{p_1, ..., p_{i-1}\}; \{i + 1, ..., J\}) = \sum_{j=1}^{i-1} \left[ q_j (v_j - p_j) + q_{j,-i} (v_{j,-i} - p_j) + \beta \sum_{k=i+1}^{j} q_{j,-k} (v_{j,-k} - p_j) \right]$$

$$+ (1 - \beta) \sum_{j=i+1}^{j} \left[ q_j (v_j - c_j) + q_{j,-i} (v_{j,-i} - c_j) + \beta \sum_{k=i+1,k \neq j}^{j} q_{j,-k} (v_{j,-k} - c_j) \right].$$

So the bargaining with $i$ is over:

$$V(\{p_1, ..., p_i\}; \{i + 1, ..., J\}) - V(\{p_1, ..., p_{i-1}\}; \{i + 1, ..., J\}) = q_i (v_i - p_i) + \beta q_i (v_i,-k) (v_{i,-k} - p_i) - \sum_{j=1}^{i-1} q_{j,-i} (v_{j,-i} - p_j) - (1 - \beta) \sum_{j=i+1}^{j} q_{j,-i} (v_{j,-i} - c_j).$$

Again applying Lemma 2 obtains equation (B.6).
We can now derive $V(\{p_1, \ldots, p_{i-1}\}; \{i, \ldots, J\})$ to complete the recursion. This can be done in two ways (that of course provide the same answer as above): either place $p_i$ in $V(\{p_1, \ldots, p_i\}; \{i+1, \ldots, J\})$ or apply Lemma 2. Here are the steps using the first approach:

$$V(\{p_1, \ldots, p_{i-1}\}; \{i, \ldots, J\}) = \sum_{j=1}^{i-1} \left[ q_j(v_j - p_j) + \beta \sum_{k=i+1}^{J} q_j,_{-k}(v_{j,,-k} - p_j) \right]$$

$$+ q_i v_i + \beta \sum_{k=i+1}^{J} q_i,_{-k} v_{i,,-k} - (q_i + \beta \sum_{i+1}^{J} q_i,_{-k}) p_i$$

$$+(1 - \beta) \sum_{j=i+1}^{J} \left[ q_j(v_j - c_j) + \beta \sum_{k=i+1, k \neq j}^{J} q_j,_{-k}(v_{j,,-k} - c_j) \right]$$

Placing $p_i$ the second line changes to:

$$(1 - \beta)q_i(v_i - c_i) + (1 - \beta)\beta \sum_{k=i+1}^{J} q_i,_{-k}(v_{i,,-k} - c_i) + \beta \sum_{j=1}^{i-1} q_j,_{-i}(v_{j,,-i} - p_j) + \beta(1 - \beta) \sum_{j=i+1}^{J} q_j,_{-i}(v_{j,,-i} - c_j)$$

Moving the $\beta$ term to the first line we get and the $(1 - \beta)$ term to the third line:

$$V(\{p_1, \ldots, p_{i-1}\}; \{i, \ldots, J\}) = \sum_{j=1}^{i-1} \left[ q_j(v_j - p_j) + \beta \sum_{k=i}^{J} q_j,_{-k}(v_{j,,-k} - p_j) \right]$$

$$\beta(1 - \beta) \sum_{k=i+1}^{J} q_i,_{-k}(v_{i,,-k} - c_i) + \sum_{j=i+1}^{J} q_j,_{-i}(v_{j,,-i} - c_j)$$

$$+(1 - \beta) \sum_{j=i}^{J} q_j(v_j - c_j) + \beta \sum_{k=i+1, k \neq j}^{J} q_j,_{-k}(v_{j,,-k} - c_j)$$

Collecting the $\beta(1 - \beta)$ terms in the second and third lines obtains the formulation in equation (B.5).

Proposition 5

In the sequential Nash model with $J$ hospitals, the insurer’s surplus is independent of the order of negotiations.

Proof. Observing equation (B.5), the expected value for the insurer before any ne-
gotiations is:

\[ V(\emptyset, \{1, \ldots, J\}) = (1 - \beta) \sum_{j=1}^{J} q_j(v_j - c_j) + \beta \sum_{k \neq j} q_{j-k}(v_{j-k} - c_j) \] .

By assumption, \( v_j > v_{j-k} > c_j \) for all \( j \) and so the sum is positive.  \[ \square \]

**Proposition 6**

Consider the sequential Nash model with \( J \) symmetric hospitals. There exists a \( \bar{\beta} < 1 \) such that if \( \beta \in (\bar{\beta}, 1) \) then \( \pi^l \) is strictly increasing in \( l \).

**Proof.** For \( J \) symmetric hospitals, using equation (B.6) we get:

\[
\lim_{\beta \to 1} p_j = \frac{q_j v_j + \sum_{k=j+1}^{J} q_{k,j} v_{k,j}}{q_j + \sum_{k=j+1}^{J} q_{k,j}} + \frac{\sum_{k=1}^{j-1} q_{k,j} (p_k - v_{j,k})}{q_j + \sum_{k=j+1}^{J} q_{k,j}}.
\]

The first fraction increases if \( j \) increases as \( v_j > v_{k,j} \). For the second fraction to increase, a sufficient condition is that \( \lim_{\beta \to 1} p_k \geq v_{j,k} \).

This is immediate by induction:

\[
\lim_{\beta \to 1} p_1 = \frac{q_1 v_1 + \sum_{k=2}^{J} q_{k,1} v_{k,1}}{q_j + \sum_{k=2}^{J} q_{k,1}} \in (v_{k,1}, v_1).
\]

For any \( j > 1 \), if for all \( k < j \) it holds that \( p_k > v_{j,k} \), then \( p_j - v_{j,k} \) is even larger than it is for \( p_1 \) which completes the proof.\(^{19}\) \[ \square \]

**Proposition 7**

Assume the repeated sequential model. If hospitals have sufficiently large bargaining power, the punishment price converges to the hospital’s per patient value if it was the only hospital in the network: For every \( \varepsilon > 0 \) there is a \( \bar{\beta} < 1 \) such that for all \( \beta > \bar{\beta} \), \( |\hat{p}_j - F_j\hat{p}| < \varepsilon \)

\(^{19}\)In fact, for \( J \) symmetric hospitals, suppose that \( q_{i,j} = q_j \cdot \alpha \) then using software we find that for every \( j > 3 \):

\[
p_{j+1} - p_j = (p_j - p_{j-1}) \cdot \frac{1 + \alpha(J - (j + 1))}{1 + \alpha(J - (j - 2))} \tag{B.7}
\]
Proof. Using equation \([B.6]\):

\[
\lim_{\beta \to 1} p_1 = \frac{q_1 v_1 + \sum_{k=2}^{J} q_k v_{k,1}}{q_1 + \sum_{k=2}^{J} q_k} = \frac{F_{jh}}{q_j^{jh}}
\]

\[\square\]

**Proposition 8**

In the equilibrium of the repeated sequential model, hospital prices are given by equation \([3.15]\):

\[
p_j = \delta \hat{p}_j + (1 - \delta) \frac{W_j}{q_j}.
\]

**Proof.** Given \(J\) hospitals in the network, the insurer’s problem is:

\[
\min_{p_1, \ldots, p_J} \sum_{j=1}^{J} q_j p_j
\]

s.t. \(\forall j: p_j q_j - W_j (1 - \delta) \geq \delta \hat{p}_j q_j\)

As \(W_j\) is a linear combination of all the prices, the problem is a standard constrained linear optimization problem with \(j\) unknowns and \(j\) constraints and thus all constraints bind. \[\square\]

**Proposition 9**

Hospital costs and bargaining parameter estimates implied by NiN model estimation are equivalent to those implied by RN model estimation constrained to \(\delta = 0\).

**Proof.** Placing \(\delta = 0\) in \([A.3]\) obtains \(p^* = p^D\), where \(p^D\) is given in \([A.1]\). For any vector of observed prices \(p^*\): \(p^D = p^N\) by construction. \[\square\]
C Logit Example - Technical Details

This provides the technical details for the Logit example in section 2. For completeness, we add a taste shock variance $\sigma$. The consumption utility with $n$ hospitals is:

$$U(n) = \sigma \log(n \cdot e^{u_\sigma} + \frac{u_0}{\sigma}) = \sigma \log((n + e^{\frac{u_0-v}{\sigma}})e^{\frac{v}{\sigma}}) = v + \sigma \log(n + e^{\frac{u_0-v}{\sigma}})$$

The share of the outside option is $s_0(n) = \frac{e^{u_\sigma}}{e^{u_\sigma} + n \cdot e^{\frac{v}{\sigma}}}.

The share of each hospital is $s(n) = \frac{e^{\frac{v}{\sigma}}}{e^{\frac{v}{\sigma}} + n \cdot e^{\frac{v}{\sigma}}}.$

The insurance surplus is $V(n) = v + \sigma \log(n + e^{\frac{u_0-v}{\sigma}}) - s(n) \sum_{j=1}^{n} p_j.$

The added value from each hospital is

$$V(n) - V(n-1) = \sigma \log \left( \frac{n + e^{\frac{u_0-v}{\sigma}}}{n + e^{\frac{u_0-v}{\sigma}} - 1} \right) - s(n) \sum_{j=1}^{n} p_j + s(n-1) \sum_{j=1}^{n-1} p_j$$

$$= \sigma \log \left( \frac{n + e^{\frac{u_0-v}{\sigma}}}{n + e^{\frac{u_0-v}{\sigma}} - 1} \right) + (s(n-1) - s(n)) \sum_{j=1}^{n-1} p_j - s(n)p_n$$

(C.1)

The bargaining problem with each hospital solves

$$\max_{p_n} (V(n) - V(n-1))^{1-\beta} \cdot p_n^\beta$$

And the simultaneous NiN solution for all hospitals is given by

$$p_n = \beta \cdot \left[ \frac{\sigma}{s(n)} \log \left( \frac{n + e^{\frac{u_0-v}{\sigma}}}{n + e^{\frac{u_0-v}{\sigma}} - 1} \right) + \frac{s(n-1) - s(n)}{s(n)} \sum_{j=1}^{n-1} p_j \right]$$

(C.2)

$$= \beta K(n, v, u_0, \sigma) + \beta s(n-1) \sum_{j=1}^{n-1} p_j$$
Where we set \( K(n, v, u_0, \sigma) \equiv \frac{\sigma}{s(n)} \log \left( \frac{n + e^{u_0 - v}}{n + e^{u_0 - \sigma}} \right) > 0 \) and use the relation:

\[
\frac{s(n-1) - s(n)}{s(n)} = \frac{s(n-1)}{s(n)} - 1
\]

\[
= \frac{e^{u_0} + (n - 1)e^{\sigma}}{e^{\sigma} + (n - 1)e^{\sigma}} - 1
\]

\[
= \frac{e^{u_0}}{e^{\sigma} + (n - 1)e^{\sigma}} = s(n-1) \tag{C.3}
\]

The unique solution is symmetric and so can be derived by summing up all the equations for the \( n \) hospitals. Letting \( S(n) \equiv n \cdot s(n) = 1 - s_0(n) \) denote the total share of all the hospitals in the market:

\[
p^N = \beta K(n, v, u_0, \sigma) + \beta S(n - 1)p^N
\]

\[
= \frac{\beta}{1 - \beta S(n - 1)} K(n, v, u_0, \sigma) \tag{C.4}
\]

For the RN model, we have that

\[
p^* = (1 - \delta)p^D + \delta p^F. \tag{C.5}
\]

Determining \( p^D \) given \( p^* \) using the same procedure as for \( p^N \) above obtains:

\[
p^D = \beta(K + S(n - 1)p^R) \tag{C.6}
\]

Simultaneously solving the two equations above yields:

\[
p^D = \beta \frac{K + \delta S(n - 1)p^F}{1 - \beta \cdot S(n - 1) \cdot (1 - \delta)} \tag{C.7}
\]

\[
p^* = \frac{\beta K(1 - \delta) + \delta p^F}{1 - \beta \cdot S(n - 1) \cdot (1 - \delta)}
\]

To determine the punishment price \( p^F \), set \( v_{j,-i} = v \) and \( q_{j,-i} = S(n - 1) \) in equation (B.6):

\[
p^F = \beta v \frac{1 + (n - 1)s(n - 1)(2\beta - 1)}{1 + \beta(n - 1)s(n - 1)} = \beta v \frac{1 + S(n - 1)(2\beta - 1)}{1 + S(n - 1)\beta} \tag{C.8}
\]
For the numerical example, we set the outside option share at 0.014 (consistent with Gowrisankaran et al. (2015), table 2), and there are \( n = 11 \) hospitals. The outside option market share implies \( v = 1.86 \). Using this, we calculate the outside option share with one less hospital, \( s_0(10) = 0.015 \) and for \( \sigma = 1 \), \( K(11, 1.86, 0, 1) = 9 \cdot \log(\frac{11 + e^{-1.86}}{10 + e^{-1.86}}) = 0.845 \). Placing in \( p^N \) above obtains
\[
p^N = \frac{\beta}{1 - 0.985 \cdot 0.845}
\]

The punishment price \( p^F \) using the example numbers is:
\[
p^F \approx 3.72 \cdot \frac{\beta^2}{1.015 + \beta}
\] (C.9)

Placing \( p^F \) in equation (C.7) completes the example.

D Monte Carlo Details and Results

To better understand the properties of the RN model and the implication of choosing a particular bargaining model in empirical work, we conduct a simple Monte Carlo study. The goal of this study is (1) to establish that it is possible to empirically distinguish the RN model from the NiN model, (2) understand the implications of imposing NiN when the true model is RN and (3) understand the effect on precision of estimating \( \delta \) when in fact the NiN model holds.

We adopt the following demand system. Assume each market has four hospitals and a unit mass of consumers divided into three types. The mass of consumers from each type \( t \), is \( m_t \). The utility for consumer \( i \) of type \( t \) from hospital \( j \) is given by
\[
u_{i,j,t} = v + x_{j,t} + \varepsilon_{ij}.
\] (D.1)

In equation (D.1), \( v \) is a constant parameter, \( x_{j,t} \) is type’s \( t \)'s hospital-specific value for hospital \( j \) and \( \varepsilon_{ij} \) is an idiosyncratic taste shock which we assume is distributed type-1 extreme value. In addition to \( \varepsilon_{ij} \), we draw for each market the relative type shares \( m_t \) and the hospital-specific value per type \( x_{j,t} \sim U[0, 2] \) However, we assume these values are observable to the econometrician as they are analogous to the distribution of consumer demographics whose variation can be observed across
markets. Our demand system has no parameters to estimate as we assume these can be consistently recovered using standard demand estimation techniques.

The cost per patient for hospital $j$ is given by

$$c_j = \alpha_0 + \alpha_1 z_j + \omega_j.$$  \hfill (D.2)

In (D.2), $\alpha_0$ and $\alpha_1$ are parameters to be estimated, $z_j \sim U[0,1]$ is a randomly drawn hospital-specific observed cost shifter and $\omega_j \sim U[-0.5,0.5]$ is an unobserved random i.i.d. hospital cost shock.

We set $\alpha_0 = .75$ and $\alpha_1 = 1$ and consider several bargaining and discount parameters. We report results for $\beta = .2, .5, \text{ or } .8$ and $\delta = 0$ (i.e. NiN) or .8. Results for higher (and more commonly used) discount factors are similar to the $\delta = .8$ case.

In each simulation we construct a dataset of 1000 markets and first solve for each market the correct price given the parameters. We then estimate the parameters $(\alpha_0, \alpha_1, \beta, \delta)$ using GMM twice for each dataset: according to the RN model and the NiN model ($\delta = 0$). We use equation (A.4) to isolate $c(\beta, \delta)$ and calculate $\omega(\alpha_0, \alpha_1, \beta, \delta) = c(\beta, \delta) - \alpha_0 + \alpha_1 z_j$.

We repeat the simulation 1000 times for each parameter set. Figure [D.1] reports the empirical distribution of each estimated parameter in the different models using both NiN and RN. The quantiles of these distributions are reported in Appendix D.

In all parameterizations, the true value of all parameters are within the inter quartile range estimated by the RN model. Estimating the RN model does not reject the true parameters even if the true model is NiN, despite the NiN being the boundary for RN. The RN model estimate for $\beta$ is especially sharp in all parameterizations.

The main difficulty of the RN model is to estimate the discount factor $\delta$ when hospital bargaining power $\beta$ is low. In such cases, the equilibrium price is close to costs regardless of the discount factor. As a result, RN may not reliably identify $\delta$ when hospital bargaining is very low. Nevertheless, in all but the parametrization ($\beta = .5, \delta = .8$) the median (across simulations) of the estimated value for $\delta$ is less than .02 away from the true value.

The additional discount parameter increases the flexibility of the model and this translates into higher variability in the empirical distribution of the estimated
Figure D.1: Distribution of the estimates in the Monte Carlo simulations for $\beta = 0.2$, 0.5 or 0.7, $\delta = 0$ or .8 with cost parameters $\alpha_0 = .75$ and $\alpha_1 = 1$. The horizontal dotted lines identify the true parameter. Only the RN model is relevant for estimating $\delta$. All estimated values using NiN for $\alpha_0$ when $\beta = \delta = .8$ are between $-0.5$ and $-1.47$, falling outside the plot’s range.
Figure D.2: Median value of the GMM objective using the RN model with $\beta$ set exogously between .05 and .95. In the true model, $\beta = .5$. The lower line is for data generated using $\delta = 0$ (i.e. NiN) and the upper line is for data generated using $\delta = .8$. The dotted lines indicate the 25th and 75th percentile.

parameters $\beta, \alpha_0$ and $\alpha_1$. However, because $\beta$ enters the estimating equation non-linearly for any given cost and discounting parameters, small changes in $\beta$ have a significant effect on the GMM objective. As a result, the range for $\beta$ is very small in all models.

Figure [D.2] illustrates the identification of $\beta$ further. The figure documents the result of additional Monte Carlo simulations for the RN model using two parametrizations $\beta = 0.5$ and $\delta = 0$ (i.e. NiN), and $\beta = 0.5$ and $\delta = 0.8$. In both cases, we simulate 250 datasets of 250 markets each. We estimate each dataset holding $\beta$ fixed at values between 0.05 and .95. All other parameters ($\delta, \alpha_0, \alpha_1$) are estimated independently for each $\beta$ value in each simulation. The figure reports the median and inter-quartile range of the GMM objective value for each $\beta$. The GMM value function is minimized around the true beta with significant slope.

In addition to $\beta$, the additional discount parameter could increase the variability of the estimate for the cost parameters. The extent of this effect depends on the
quality of the cost data. If the cost shift data has enough variability (as \( z \) in our case) the estimation of the cost coefficient (\( \alpha_1 \)) is almost identical between the RN and NiN models. However, if the cost estimation relies on structural assumptions (as \( \alpha_0 \)) the additional flexibility naturally increases the variability of the estimate. This is because a higher discount factor can rationalize a different cost.

If the NiN assumption is correct (\( \delta = 0 \)), the NiN model identification works extremely well. However, as would be expected, if the NiN assumption is incorrect, (in our context, i.e., the true \( \delta \) is large), the NiN model fails to correctly estimate costs. In all three cases of high \( \delta \), the full support of the NiN model estimate for the cost intercept does not include the true value and is significantly biased. The same applies to the margins estimated by the NiN model. These are either too low (if \( \beta \) is low) or too high (if \( \beta \) is high).

All in all the simulations suggest that the RN model can separately identify the bargaining and discount parameters. The main concern is that using the NiN model when hospitals have bargaining power but do not behave according to the NiN assumptions can drastically bias cost and margin estimation. Using RN estimation in place of NiN does not reduce the performance of the estimation. If hospital bargaining power is limited, both models provide similar predictions (hospitals price close to cost). If hospital bargaining power is significant, the additional flexibility of the RN model improves the accuracy of the cost estimates.

The following tables present the quantiles of the distribution of the RN and NiN estimators for several specifications. Additional results are available upon request.
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RN Estimator

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NiN Estimator

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RN Estimator

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NiN Estimator

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Table D.1: Distribution of RN and NiN Estimators, $M = 1000$.

Table D.2: Distribution of RN and NiN Estimators, $M = 1000$. 53
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Table D.3: Distribution of RN and NiN Estimators, $M = 1000$.

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Table D.4: Distribution of RN and NiN Estimators, $M = 1000$. 

54
### Table D.5: Distribution of RN and NiN Estimators, $M = 1000$.

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### Table D.6: Distribution of RN and NiN Estimators, $M = 1000$.

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<td>-----------</td>
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**RN Estimator**

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<th>$E[p - c]$</th>
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**NiN Estimator**

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Table D.7: Distribution of RN and NiN Estimators, $M = 1000$.

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**RN Estimator**

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**NiN Estimator**

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Table D.8: Distribution of RN and NiN Estimators, $M = 1000$.  

56
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Table D.9: Distribution of RN and NiN Estimators, $M = 1000$. 