**The Ionization Parameter**

A convenient way of thinking about the ionization state of a medium is through a variable called the **ionization parameter**. Consider the equation of ionization balance for species \( i \),

\[
N_i^0 \int_{\nu_i}^{\infty} \frac{L_\nu}{4\pi r^2 h\nu} a_{\nu_i} d\nu = N_i^+ N_e \alpha_i \quad (25.01)
\]

The ratio of number of particles in the ionized state, to that in the non-ionized state is

\[
\frac{N_i^+}{N_i^0} = \frac{1}{4\pi r^2 N_e} \int_{\nu_i}^{\infty} \frac{L_\nu}{h\nu} \left( \frac{a_{\nu_i}}{\alpha_i} \right) d\nu \quad (25.02)
\]

Equation (25.02) is good anywhere, for all species. Note, however, that the only species-dependent term in the equation is \( a_{\nu_i}/\alpha_i \) and the lower limit of the integration. (We could, in fact, move \( a_{\nu_i} \) outside the integral by taking the photon-weighted average of it, \( \text{i.e.,} \)

\[
\bar{a}_{\nu_i} = \frac{\int_{\nu_i}^{\infty} \frac{L_\nu}{h\nu} a_{\nu_i} d\nu}{\int_{\nu_i}^{\infty} \frac{L_\nu}{h\nu} d\nu} \quad (25.03)
\]

The recombination coefficient, \( \alpha_i \), of course, is independent of photon frequency.) Now let us define a species-independent ionization parameter, such that

\[
\Gamma = \frac{1}{4\pi r^2 N_e c} \int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu \quad (25.04)
\]

(The value of \( c \) in the denominator is there just to make \( \Gamma \) dimensionless.) If we are now talking about species with ionization energies close to that of hydrogen, then \( \nu_i \sim \nu_0 \), and

\[
\frac{N_i^+}{N_i^0} = \left( \frac{\bar{a}_{\nu_i}}{\alpha_i} \right) c \Gamma \quad (25.05)
\]
Note what $\Gamma$ is. The integral in (25.04) is the number of ionizing photons coming from the source. When divided by $4\pi r^2$, this becomes the surface density of ionizing photons. In the case where $N_e \sim N_p$, this density of photons is then compared with the density of material. In other words, $\Gamma$ describes how many ionizing photons there are per atom.

$$\Gamma = \frac{Q(H^0)}{4\pi r^2 c N_H} \approx \frac{\langle N_{ph} \rangle}{\langle N_H \rangle}$$  \hspace{1cm} (25.06)

In the transition region surrounding an AGN, $\Gamma$ will drop rapidly. The large variation in $\Gamma$ will therefore result in a large range in ionization states. Typical AGN have gas both in high and low states of ionization.
The Broad Line Region

The extremely broad lines seen in quasars only occur for permitted (and occasionally semi-permitted) transitions. In addition to the lines of hydrogen and helium, the commonly seen lines include

1) Permitted lines of lithium-like ions (from s-shell electrons). These include C IV $\lambda 1549$, N V $\lambda 1240$, O VI $\lambda 1035$, Mg II $\lambda 2798$, and Si IV $\lambda 1400$.

2) Semi-forbidden lines of beryllium-type ions (from a $2s2p$ orbital to the $2s^2$ ground state). These are C III] $\lambda 1909$, N IV] $\lambda 1488$, and O V] $\lambda 1216$.

3) Permitted lines of Fe II (there are many, many of these)

Forbidden lines are never seen in broad line regions.

- The absence of forbidden lines, along with the presence of semi-forbidden lines places a constrain on the density in the broad-line region. For example, the collisional de-excitation of O$^{++}$ in its $^1D_2$ state becomes more important than radiative transitions downward when

$$8.629 \times 10^{-6} \left( \frac{N_e}{T_e^{1/2}} \right) \left( \frac{\Omega(3P,^1D)}{\omega_{1D}} \right) > A_{5007} + A_{4959} \quad (25.07)$$

When you plug in the numbers (and make a reasonable guess for the temperature), then this critical value occurs when $N_e > 7 \times 10^5$ cm$^{-3}$. So the density must be greater than this. On the other hand, the C III] $\lambda 1909$ line has a $A$-value that is 3,500 times greater than that of the oxygen lines, and its critical density is $\sim 10^{10}$ cm$^{-3}$. Thus, the density of the broad line region must be $10^8$ or $10^9$ cm$^{-3}$. 
There is no good way to estimate the electron temperature in the broad line region. There are, however, a couple of ways to place limits on the temperature. According to the Saha equation, the ratio of Fe III to Fe II due to collisional ionization is

\[
\frac{N(\text{Fe III})}{N(\text{Fe II})} = \frac{2(2\pi m_e kT_e)^{3/2}}{h^3} \frac{u_{\text{Fe III}}}{u_{\text{Fe II}}} e^{-\Delta E/kT_e}
\]

(25.08)

where the \( u \) values are the partition functions of the ions. The ionization potential of Fe II is \( \Delta E = 16.18 \text{ eV} \). If the density of the broad-line region is \( \sim 10^9 \text{ cm}^{-3} \), then almost all the Fe II would be collisionally ionized to Fe III if \( T_e \sim 35,000 \text{ K} \). Thus, the electron temperature must be less than this.

A better limit on the temperature can be found from the presence of C III\( \lambda 1909 \), but not C III\( \lambda 977 \). These are the two lowest (excited) states of C\( ^{++} \): \( \lambda 1909 \) comes from a \( ^3P_0 \) triplet state (both electrons same spin), while the \( \lambda 977 \) line comes from the \( ^3P_0 \) singlet (opposite spin) state. (Note that the \( \lambda 977 \) is permitted because the electrons do have opposite spins; the \( \lambda 1909 \) is semi-forbidden, because the excited electron has to change its spin in the decay to the s-shell.) Electrons will enter both \( P \) states via collisions from the ground state, with the ratio

\[
\frac{N_{3P_0}}{N_{1P_0}} = \frac{\Omega(1S_0,^3P_0)}{\Omega(1S_0,^1P_0)} e^{-\Delta E/kT_e}
\]

(25.09)

where \( \Delta E \) is the energy difference between the two states. Since collisional de-excitation is not important, each collision upward should generate a photon with an energy appropriate for the transition. The observed ratio of \( \lambda 1909 \) to \( \lambda 977 \) is

\[
\frac{j_{\lambda 1909}}{j_{\lambda 977}} = \frac{\Omega(1S_0,^3P_0)}{\Omega(1S_0,^1P_0)} \frac{\nu_{\lambda 1909}}{\nu_{\lambda 977}} e^{-\Delta E/kT_e} \gtrsim 20
\]

(25.10)

This observed ratio therefore implies \( T_e \lesssim 15,000 \text{ K} \).
We can estimate the mass and size of the broad line region in a manner similar to that of the narrow line region. Recall that the H$\beta$ luminosity from recombination is given by

$$L(\text{H}\beta) = N_e N_p \alpha_{\text{H}\beta}^e f \nu_{\text{H}\beta} \cdot fV$$  \hspace{1cm} (21.09)$$

while the total mass of ionized gas is

$$\mathcal{M} = (N_p m_H + N_{He} m_{He}) fV \sim 1.3 N_p m_H fV$$  \hspace{1cm} (21.10)$$

When we combine these two equations, the total mass becomes

$$\mathcal{M} = 1.3m_H \left\{ \frac{L(\text{H}\beta)}{N_e \alpha_{\text{H}\beta}^e \nu_{\text{H}\beta}} \right\}$$  \hspace{1cm} (21.11)$$

A typical luminosity for H$\beta$ is $10^9 \ L_\odot$. If we adopt a density of $N_e \sim 10^9 \ \text{cm}^{-3}$, the total mass of broad line becomes $\mathcal{M} \sim 40 \mathcal{M}_\odot$. This is extremely small. Moreover, when we calculate the volume of this gas in the usual way,

$$V = \frac{4}{3} \pi R^3 = \frac{\mathcal{M}}{1.4 N_p m_H f}$$  \hspace{1cm} (21.11)$$

and plug in some numbers, the size becomes

$$R \sim 0.015 f^{-1/3} \left( \frac{10^9}{N_e} \right)^{2/3} \ \text{pc}$$  \hspace{1cm} (25.11)$$

Note that this is an extremely small region. The filling factor only enters as the cube-root, so invoking small values of $f$ does not make that much difference. Neither does changing the electron density: a factor of 10 decrease only increases the size by a factor of four. The broad line region is therefore only a few tenths of a light year! This is unresolvable with modern telescopes.
The Unique Problems of Broad Line Regions

Figuring out the physics of broad line clouds is an extremely difficult problem in astrophysics, as many of the approximations and assumptions that work for H II regions begin to break down. Specifically,

1) Many of the photons emanating from the ionizing source have extremely high energy. For these photons, the cross section for hydrogen absorption is small; instead, metals such as C,N,O, and Ne dominate the opacity. Moreover, if you ionize K-shell electrons from these elements, the resulting decays may ionize another electron, thereby releasing two electrons from one photon.

2) Ionizations from high-energy photons release electrons with very large velocities. The thermalization timescale of these electrons can be longer than the timescale for a collisional excitation or hydrogen ionization. Thus the use of $T_e$ to describe the distribution of electron speeds may be incorrect.

3) Electrons which have been photo-ionized with high energy photons can have velocities high enough to collisionally ionize hydrogen. In this case, two photo-electrons can come from one photon. Alternatively, the high-energy electron can excite a ground state hydrogen electron. The net result of this is two Ly$\alpha$ photons from a single ionizing photon.

4) Because the transition region for an H II region ionized by a power law is large, there will be a significant amount of neutral hydrogen mixed in with the free electrons. In this case, collisional cooling with hydrogen will be significant. (It’s still a big jump to excite a ground state hydrogen electron to $n = 2$, but there’s alot of neutral hydrogen lying around.) These collisions will produce an excess of Ly$\alpha$ photons.
5) Because of the relatively large percentage of neutral hydrogen, and the high density of the material, the optical depth at Ly\(\alpha\) is huge, \(\tau \sim 10^6\)！ At this opacity, the number of times a Ly\(\alpha\) photon will be absorbed and re-emitted before escaping will be \(\gtrsim 850,000\) times. Now note: each Ly\(\alpha\) absorption will cause a hydrogen atom to be in the \(n = 2\) state for \(t \approx 1/A \approx 10^{-9}\) sec. So, on its way out, each Ly\(\alpha\) photon will cause a “mean” hydrogen atom to be in the \(n = 2\) state for

\[
T_{n=2} = 850,000 \times t = 1.4 \times 10^{-3} \text{ sec}
\]

(25.12)

This will cause a substantial population of hydrogen atoms to be in the \(n = 2\) state. Consequently,

- Upward collisions from \(n = 2\) will occur, creating excess Balmer photons. Since the easiest collision (energy-wise) is \(n = 2\) to \(n = 3\), H\(\alpha\) will be artificially enhanced.

- Since the \(n = 2\) state of hydrogen is populated, absorptions from this state can occur. Radiative transfer of Balmer lines can no longer be ignored.

- Ly\(\alpha\) photons will be able to ionize hydrogen in the \(n = 2\) state. This will destroy a Ly\(\alpha\) photon.

- Collisonal excitation for \(n = 2\) will occur; in these cases, a Ly\(\alpha\) photon will not be produced.