Galaxies are clustered …

• We have already seen that galaxies like to live in groups and clusters
• These groups and clusters like to cluster together to form superclusters, we can even define a supergalactic plane
• On even larger scales we see filaments, voids, sheets and walls in large redshift surveys
• These large scale structures cause peculiar velocities due to their gravitational attraction which cause the redshift-distance relation to deviate from a pure “Hubble flow”
• We can use large galaxy redshift surveys to trace the mass distribution of the universe and measure $\Omega_m$
  – What assumption do we have to make?
• The amount of clustering we observe also provides strong constraints on the amount and type of dark matter in the universe and the energy density ($\Lambda$)
Local Group
Nearby Superclusters
APM survey, Maddox et al, 1/10 of the sky
2MASS survey of the whole sky
6000 brightest galaxies on the sky
\~15000 brightest galaxies on the sky, in galactic coordinates. Solid line defines the supergalactic plane.
Nearby Superclusters on the supergalactic plane
Redshift Surveys

• Started in earnest in the 1980’s, allowed us to measure clustering in THREE dimensions, instead of just two!
• The first large scale redshift survey was the CfA2 (Center for Astrophysics) survey led by Margaret Geller & John Huchra started in 1984 to 1995.
  – Note there was a CfA1 survey 1977-1982, 2500 galaxies with b<14.5 (Huchra, Davis, Latham, & Tonry)
• CfA2 observed 20,000 galaxies brighter than B=15.5 with a 1.5 m telescope. Note, one redshift at a time, this was a massive undertaking!
• Later the Las Campanas Redshift Survey was done in the south (with multiobject spectroscopy), ~25000 galaxies covering 700 square degrees of the sky to r=17.5. Finished in the mid-1990’ s.
Redshift Surveys

• The CfA2 redshift survey revealed surprising amounts of large scale structure (LSS) in the universe

• There are filaments, walls, and voids
  – Voids are “3500-5000 km/s” in diameter or >50h\(^{-1}\) Mpc across
  – The “Great Wall” stretches for 100h\(^{-1}\) Mpc or ¼ of the way across the sky!
  – The universe is like a big sponge … or soapy bubbles

• Note that walls appear thinner in redshift space than they really are – why?

• And clusters (like Coma) appear elongated – this is the “Finger of God” effect. Again, why?
The motivation for the survey was to study the three-dimensional spatial distribution of galaxies, about which the authors note that "although not entirely unexpected, it is striking how strongly clustered our galaxies are in velocity space," as seen in strongly peaked one-dimensional redshift histograms in each field.

The original CfA survey, completed in 1982, contained redshifts for 2,400 galaxies brighter than magnitude 14.5 across the north and south galactic poles, covering a total of 2.7 steradians.

The major aims of the survey were cosmological and included quantifying the clustering of galaxies in three-dimensions. This survey produced large area, moderately deep three-dimensional maps of large-scale structure (see Fig. 8-2), in which one could identify galaxy clusters, voids, and an apparent "filamentary connected structure" between groups of galaxies, which the authors caution could be random projections of distinct structures (Davis et al. 1982).

This paper also performed a comparison of the so-called complex topology of the large-scale structure seen in the galaxy distribution with that seen in N-body dark matter simulations, paving the way for future studies of theoretical models of structure formation.

The second CfA redshift survey, which ran from 1985 to 1995, contained spectra for ∼5,800 galaxies and revealed the existence of the so-called Great Wall, a supercluster of galaxies that extends over 170 h−1 Mpc, the width of the survey (Geller and Huchra 1989). Large underdense voids were also commonly found, with a density 20% of the mean density.

Redshift surveys have rapidly progressed with the development of multi-object spectrographs, which allow simultaneous observations of hundreds of galaxies, and larger telescopes.
The infamous Stickman diagram...

First CfA Strip

$26.5 \leq \delta < 32.5$

$m_3 \leq 15.6$

Coma

Copyright SAO 1998

de Lapparent, Geller, & Huchra et al 1985
Ramella, Geller, & Huchra et al 1992
“Hockey Puck” projection of the CfA2 survey.

CfA2

Max Radius 12000
0 ≤ h < 12000 (km/s)

m_g ≤ 15.5

Copyright 2001 SAO
Las Campanas Redshift Survey
An illustration of the “Fingers of God” (FoG) or elongation of virialized structures along the line of sight, from Tegmark et al. (2004). Shown are galaxies from a slice of the SDSS sample (projected here through the declination direction) in two-dimensional comoving space. The top row shows all galaxies in this slice (67,626 galaxies in total), while the bottom row shows galaxies that have been identified as having “Fingers of God.” The right column shows the position of these galaxies in this space after modeling and removing the effects of the “Fingers of God.” The observer is located at (x,y=0,0), and the “Fingers of God” effect can be seen in the lower left panel as the positions of galaxies being radially smeared along the line of sight toward the observer.
Huge Redshift Surveys

• More recently, two LARGE redshift surveys have been undertaken

• The 2dF (2 degree Field) redshift survey done with the Anglo-Australian telescope
  – ~220,000 galaxies covering 5% of the sky reaching to z~0.3 with B<19.5
  – Spectrograph can measure 400 redshifts at a time!

• The Sloan Digital Sky Survey (SDSS) which uses a dedicated 2.5m telescope at Apache Point Observatory in New Mexico
  – Performed multicolor imaging to r=22.5 AND spectra of galaxies down to r<17.5 reaching to z~0.4, ~500 redshifts at a time!!
  – Photometry of 200 million objects, spectroscopy of ~1 million objects, more than 675,000 galaxies and 185,000 stars
  – Also measured redshifts (90,000) of quasar candidates out to much higher redshifts (Schneider et al.)
2dF Galaxy Redshift Survey

2dF Redshift Survey
Sloan Digital Sky Survey
Pencil Beam Surveys

• To probe structure at higher redshifts is generally done with deep “pencil beam” surveys in small patches of the sky with big telescopes

• Original pencil beam surveys done by David Koo, Richard Kron, & collaborators in early 1990s showed walls showing up at large redshifts
  – The same scale of voids & walls we see locally seems to continue out to z~1

• Even deeper surveys done with Keck of the Hubble Deep Field and several other deep surveys show the same thing
Great Wall

Deep pencil beam surveys, Willmer & Koo 1996
Hubble Deep Field Redshifts, Cohen et al. 2000
We want a way to quantify the amount of structure that we see on various scales. The most common way of doing this is to measure the two-point correlation function $\xi(r)$. We calculate the correlation function by estimating the galaxy distances from their redshifts, correcting for any distortions due to peculiar velocities, and counting the number of galaxies within a given volume. Mathematically, the probability of finding a galaxy within a volume $\Delta V_1$ and a volume $\Delta V_2$ is

\[ \Delta P = n^2[1 + \xi(r_{12})]\Delta V_1\Delta V_2 \]

Where $n$ is the average spatial density of galaxies (number per $\text{Mpc}^3$) and $r_{12}$ is the separation between the two regions.
Measuring clustering …

- \( \Delta P = n^2[1 + \xi(r_{12})] \Delta V_1 \Delta V_2 \)
  - When \( \xi(r) > 0 \), then galaxies are clustered (which is what we see)
  - On scales of \(< 50h^{-1}\) Mpc, we can parameterize the correlation function as a power-law: \( \xi(r) \sim (r/r_0)^{-\gamma} \) where \( \gamma > 0 \)
  - Thus the probability of finding one galaxy within a distance \( r \) of another is significantly increased (over random) when \( r < r_0 \). \( r_0 \) is called the “correlation length”.
  - Note that the 2 point correlation function isn’t good for describing one-dimensional filaments or two-dimensional walls. We need 3 and 4 point correlation functions for those. Much harder!
  - From the SDSS: \( r_0 = 6.1 \pm 0.2 \) h\(^{-1}\) Mpc, \( \gamma = 1.75 \) over the scales 0.1 – 16 h\(^{-1}\) Mpc
Las Campanas Redshift Survey
Measuring clustering …

- The Fourier transform of $\xi(r)$ is the power spectrum
  - $P(k), P(k) = 4\pi \int \xi(r) [\sin(kr)/kr] r^2 dr$
- $k$ is the wavenumber, small values of $k$ correspond to large physical scales
- $P(k)$ has the dimensions of volume. It will be at maximum close to the radius $r$ where $\xi(r)$ drops to zero.
- Roughly speaking the power spectrum is a power-law at large $k$ (small physical scales) and turns over at small $k$ (large physical scales)
- We can combine information from different measurements (redshift surveys, CMB, Ly$\alpha$ forest, weak lensing) to trace $P(k)$ over a large range of physical scales
- The power spectrum provides strong constraints on the amount and type of dark matter and dark energy in the universe
Measuring clustering …

- We would also like to know how well the galaxies trace the mass distribution, or in other words how biased are the galaxies relative to the dark matter
- We generally assume that the two densities are linearly related such that:
  - Let $\delta_x = \delta \rho_x / \rho_x$ be the density fluctuation of a given population
  - Linear biasing for galaxies implies $\delta_{\text{galaxies}} = b \delta_{\text{dark matter}}$
  - Biasing may be a function of scale and of galaxy luminosity
- We can measure relative biasing by measuring the power spectrum of different populations
Biasing in the SDSS, Tegmark et al. (2004)
The spatial distribution of galaxies in the SDSS main galaxy sample as a function of redshift and right ascension projected through 8° in declination and color-coded by restframe optical color. Red galaxies are seen to be more clustered than blue galaxies and generally trace the centers of groups and clusters, while blue galaxies populate further into the galaxy voids (Taken from Zehavi et al. (2011))
The clustering scale length, $r_0$ (left), and slope, $\gamma$ (right), for all, red and blue, galaxies in SDSS as a function of luminosity. While all galaxies are more clustered at brighter luminosities and red galaxies are more clustered than blue galaxies at all luminosities, below $L^*$, the red galaxy clustering length increases at fainter luminosities. The clustering slope for faint red galaxies is also much steeper than at other luminosities (Taken from Zehavi et al. (2011))
BAO at $z \sim 0.3$ from SDSS Eisenstein+ 2005

Magenta is pure CDM, other lines are $\Lambda$CDM with varying $\Omega_m h^2$
Origin of the observed large scale structure in the universe is a central problem in astrophysics.

Understanding the origin of structure allows us to:
- Understand the formation of galaxies and clusters of galaxies
- Identify the amount & type of dark matter
- Probe the events in the early universe shortly after the big bang (inflation)
- Provide a useful cross check on estimates of cosmological parameters

Structure is generally thought to come about due to a well-understood process (gravity) but the effects of the growth of density perturbations are non-linear. We also need to consider gas dynamics and radiative processes.
- Structure formation generally studied via computer simulations
Sloan Digital Sky Survey

-23 < M_r < -22
-22 < M_r < -21
-21 < M_r < -20
-20 < M_r < -19
-19 < M_r < -18
-16 < M_r < -17
-17 < M_r < -16
Origin of structure in the universe

• What do we know about the formation of structure at early times?
  – We see temperature fluctuations in the cosmic microwave background of $\delta T/T \sim 10^{-5}$. Before recombination, radiation and baryons are coupled so the temperature fluctuations tell us about density fluctuations in the baryonic matter.
  – High-redshift objects: We observe galaxies and quasars at $z>5$. A galaxy requires an overdensity of $10^5$ relative to the universe as a whole.

• Can we get to the large density enhancements required for galaxies, clusters, etc by evolving the small fluctuations we see in the CMB?
Origin of structure in the universe

• Where do the density fluctuations come from?
  – We think they came from quantum fluctuations in the scalar field that caused inflation, and were then amplified by the exponential inflation of the universe.

• There are two different types of fluctuations:
  – Isothermal – these affect the matter but not the radiation field
  – Adiabatic – these affect both light and matter together. These can’t happen now, but were probably common prior to recombination. Adiabatic fluctuations in the baryonic matter are what we see in the CMB.
Origin of structure in the universe

• How do fluctuations in density evolve with time?
  – For simplicity, let’s start with flat, matter-dominated universe with $\Omega_m = 1$
  – From the Friedmann equation:

$$H^2 - \frac{8}{3} \pi G \rho = - \frac{k}{R^2} = 0$$

Now consider a small area with a slight overdensity of matter, it will evolve slightly differently:

$$H^2 - \frac{8}{3} \pi G \rho' = - \frac{k}{R^2}$$
Origin of structure in the universe

- Subtract the two equations, and you find:

\[-\frac{8}{3}\pi G (\rho' - \rho) = -\frac{k}{R^2}\]

which can be rewritten as:

\[\rho' - \rho = -\frac{3k}{8\pi G R^2}\]

We can define \(\delta\) as the fractional overdensity:

\[\delta \equiv \left(\frac{\rho' - \rho}{\rho}\right) = -\frac{3k}{8\pi G R^2 \rho}\]
Origin of structure in the universe

\[ \delta \equiv \left( \frac{\rho' - \rho}{\rho} \right) = -\frac{3k}{8\pi G R^2 \rho} \]

But note that:

\[ \delta \sim \frac{1}{R^2 \rho} \sim \frac{1}{R^2 R^{-3}} \sim R \]

But we know \( R \sim (1+z)^{-1} \), so

\[ \delta \sim (1 + z)^{-1} \]
Origin of structure in the universe

\[ \delta \sim (1 + z)^{-1} \]

We can use this to relate density fluctuations at different times (or redshifts):

\[ \frac{\delta_f}{\delta_i} = \frac{(1 + z)_i}{(1 + z)_f} \]

What happens from recombination to \( z=5 \)?

\[ \delta_f = 10^{-5} \left( \frac{1 + 1000}{1 + 5} \right) \sim 0.002 \]

Oops. This isn’t going to work!!
Origin of structure in the universe

\[
\frac{\delta_f}{\delta_i} = \frac{(1 + z)_i}{(1 + z)_f}
\]

We need to start with larger fluctuations in order to get galaxies, etc, today. How can we do this? We need non-baryonic dark matter. Before recombination, the radiation prevented the baryonic fluctuations of collapsing. But non-baryonic matter doesn’t interact with radiation, so fluctuations in non-baryonic dark matter can start growing much earlier!

Standard picture – the fluctuations we see in the CMB (in the baryonic matter) are sitting on top of much stronger fluctuations in the non-baryonic matter. Once recombination occurs, the baryons can fall into the dark matter concentrations to form galaxies ...
Origin of structure in the universe

Once we have an overdensity of $\delta = 1$, the region will “decouple” from the surrounding Hubble flow and collapse due to gravity. The gravitational collapse timescale is on the order of the free-fall time:

$$t_{ff} \approx \sqrt{\frac{1}{G \rho}}$$

Thus, low density lumps collapse more slowly than high density ones.

More massive structures are less dense, take longer to collapse

<table>
<thead>
<tr>
<th>Object</th>
<th>$\rho (M_\odot/pc^3)$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Globular cluster</td>
<td>150</td>
<td>$10^9$</td>
</tr>
<tr>
<td>galaxy</td>
<td>0.2</td>
<td>$10^6$</td>
</tr>
<tr>
<td>cluster</td>
<td>$10^{-5}$</td>
<td>50</td>
</tr>
</tbody>
</table>
Origin of structure in the universe

- Different types of dark matter form structure differently
- Baryonic dark matter is coupled to radiation, does not help in forming structure!
- Non-baryonic dark matter
  - Hot dark matter – particles are relativistic, like neutrinos
  - Cold dark matter – move more slowly (but of course we don’t know for sure that any of these exist!), axions, weakly interacting massive particles (WIMPs) …
  - HDM vs. CDM make very different predictions for the evolution of structure in the universe!
Origin of structure in the universe

- **Hot dark matter**
  - HDM particles are relativistic, their speed means they can escape from small density fluctuations. This removes mass from the fluctuation and essentially smooths out any small fluctuations.
  - For example, large amounts of neutrinos will dissolve away masses fluctuations smaller than $10^{15} \, M_\odot$ before recombination. Thus the baryons won’t collapse into small lumps!
  - Instead, only big lumps survive to collapse. These lumps are on the scale of clusters of galaxies, which have relatively low overdensities. This collapse occurs slowly. After the big structures have collapsed, fragmentation into smaller structures (like galaxies) can occur.
  - Structure forms slowly, “top-down”. Galaxies form late in the universe’s history. This isn’t what we see, hot dark matter doesn’t work!!
Origin of structure in the universe

• Cold dark matter:
  – CDM particles don’t diffuse out of small lumps. So lumps exist on all scales, both large and small.
  – Small lumps collapse first, big things collapse later. The larger overdensities will incorporate smaller things as they collapse.
  – Structure forms early with CDM, and it forms “bottom-up.”
  – Galaxies form before clusters!
  – This more closely matches what we observe, this picture is known as “hierarchical structure formation.”
Origin of structure in the universe

- At what point will an overdensity collapse:
  - From the virial theorem:
    \[ 2K + U = 0, \text{ where } K=\text{kinetic energy}, \ U=\text{potential energy} \]
    The potential energy for a spherical cloud of constant density is:
    \[
    U = -\frac{3}{5} \frac{GM^2}{R}
    \]
    The kinetic energy of N atoms is:
    \[
    K = (N)\left(\frac{3}{2}kT\right)
    \]
    So the virial theorem, becomes:
    \[
    3NkT = \frac{3}{5} \frac{GM^2}{R}
    \]
Origin of structure in the universe

\[ 3NkT = \frac{3}{5} \frac{GM^2}{R} \]

If the equality holds, the system is in virial equilibrium. If the left side is bigger, thermal (kinetic) energy wins over gravity and the system expands. If the right side is bigger, gravity wins and the system collapses!

We’re interested in when the system will collapse, so that means:

\[ 3NkT < \frac{3}{5} \frac{GM^2}{R} \]
Origin of structure in the universe

\[3N kT < \frac{3}{5} \frac{GM^2}{R}\]

We can rewrite this in terms of density, by noting that:

\[N = \frac{M}{m}\]

\[R = \left(\frac{3M}{4\pi \rho}\right)^{1/3}\]

Substituting and doing the math, gives us:

\[M > \left(\frac{5kT}{Gm}\right)^{3/2} \left(\frac{3}{4\pi \rho}\right)^{1/2}\]
Origin of structure in the universe

At what point will an overdensity collapse?

This limit is known as the Jean’s mass, the cloud will collapse if its mass is bigger than the Jean’s mass.

\[ M_J = \left( \frac{5kT}{Gm} \right)^{3/2} \left( \frac{3}{4\pi\rho} \right)^{1/2} \]

What is the Jean’s mass at recombination (z~1100)?

The temperature is \(~3000K\) and the density (baryonic) is:

\[ \rho_B = \rho_B,0 R^{-3} = \rho_B,0 (1 + z)^3 \]
\[ \rho_B \sim (3 \times 10^{-31})(1 + 1100)^3 \sim 4 \times 10^{-22} \text{ gm/cm}^3 \]

Plugging in the numbers gives us \( M_J = 2 \times 10^6 M_\odot \). This is about the size of a globular cluster. Overdensities smaller than this could not collapse. Larger masses could, but more massive (and less dense) objects will take longer to collapse.
Origin of structure in the universe

Standard picture – the fluctuations we see in the CMB (in the baryonic matter) are sitting on top of much stronger fluctuations in the non-baryonic matter. Once recombination occurs, the baryons can fall into the dark matter concentrations to form galaxies …

How can we test these theories?
   The best way is to perform numerical simulations of the evolution of the structure.

Observational constraints –
   Existence of galaxies at high redshift.
   Existence of massive clusters at $z \sim 1$.
   Observed power spectrum.
Figure 6. A schematic representation of a “merger tree” depicting the growth of a halo as the result of a series of mergers. Time increases from top to bottom in this figure and the widths of the branches of the tree represent the masses of the individual parent halos. Slicing through the tree horizontally gives the distribution of masses in the parent halos at a given time. The present time $t_0$ and the formation time $t_f$ are marked by horizontal lines, where the formation time is defined as the time at which a parent halo containing in excess of half of the mass of the final halo was first created.
The VIRGO Collaboration 1996
Millennium Simulation
10,077,696,000 particles
Springel et al. 2005

Red = Millennium simulation
Blue = 2dFGRS observations
Green = DM corr. function
Points = Millennium simulation

Dashed = 2dFGRS/SDSS observations

Springel et al. 2005