Hale Bradt’s *Astronomy Methods* is full of good information on these topics. Karttunen et al. *Fundamental Astronomy*, Ch. 2 is good, but note that they use the South point of the horizon as the origin of Azimuth!

These notes were assembled from several different handouts, compiled over the years, so there is some duplication.

1. **References.** In addition to the references given in Unsöld & Baschek (5th edn., page 532) or BOB (2d edn, page 21): especially Smart & Green, “Spherical Astronomy”), the following may prove useful in some circumstances, e.g. in answering public questions, or in planning your own observations.


   M. Zombeck’s *Handbook of Space Astronomy and Astrophysics* pp 107-113 contains a very condensed summary of some formulae. (Out of print, available on the Web, see NED Level 5).

   As always, the *Astronomical Almanac* and *Explanatory Supplement* should be considered definitive. [page refs. below: 2009 edition]
2. Coordinates and Time

2.1 Mean, Apparent, and Topocentric Position

Mean Positions (catalogue positions) refer to the position of a star as "viewed" from the center of the Sun, referred to the ecliptic and mean equinox at the date of the catalogue (or to the mean equator and mean equinox). (In precise work, a distinction might have to be made between the center of the Sun and the solar system barycenter.)

Things affecting the mean position as time advances are (1) the star’s proper motion and (2) precession. A catalogue of mean positions constructed for another date must take these into account. These must also be accounted for in computing the apparent position for any date other than the catalogue date. The effects of proper motion and precession accumulate from year to year.

The largest stellar proper motion is about 10"/y (Barnard’s Star).

Precession changes the longitude of a star on the ecliptic by 50" per year. In R.A. this can amount to about 46"/y or 3.1 s/y. Approximate and rigorous formulae are given in the Astron. Almanac. See also item 1.8 on this handout.

Apparent Position of a star is its place on the celestial sphere as “seen” from the center of a moving Earth, referred to the instantaneous equator, ecliptic, and equinox.

Effects of observing from the Earth are (3) nutation, which alters the true equator from the mean equator; (4) annual aberration, due to the motion of the Earth with respect to the Sun; (5) annual parallax of the star, due to the displacement of the Earth from the Sun. A sixth effect is (6) relativistic deflection of starlight, depending on the angular distance from the Sun. Nutation, aberration, parallax, and deflection are annually periodic (18.6 y for nutation) and do not accumulate.

The amplitude of nutation is 9.2”; of aberration is 20.5”, of parallax is less than 0.8” for the nearest star; and of deflection is 1.87” at the limb of the Sun, decreasing to 0.1” at 5 degrees from the Sun.

Topocentric Position displaces the observer from the center to the surface of the Earth (or above).

Aberration and parallactic corrections due to the rotation and finite radius of the Earth enter at this stage. Only the Moon and other near-Earth objects are affected significantly by topocentric parallax, with maximum amplitude 8.794”/d(in AU). The topocentric aberration can amount to at most about 0.3” (206265” times v/c). These effects are diurnally periodic. Formulae are given in the Astronomical Almanac.
2.2 Radial Velocities

Not only does the position of an object change due to the Earth’s orbital motion (aberration), but the radial velocity (3-D velocity projected on the line-of-sight vector) will change as well. Catalogues list the heliocentric velocity, but what you observe in the geocentric (actually, topocentric) velocity.

The heliocentric velocity correction is maximal for a point on the ECLIPTIC, as much as 
\[ v_{\text{peri}} = 1.017 \times 29.78 \text{ km s}^{-1} = 30.3 \text{ km s}^{-1}. \]
(The topocentric correction is maximal for observers at the equator and for objects at zero declination, not larger than 0.5 km s\(^{-1}\).) Correction formulae are given in the Astronomical Almanac.

2.3 Heliocentric, Barycentric, Geocentric Time.

A useful recent reference is the paper by Eastman, Siverd & Gaudi (2010; PASP 122, 935).

Section B of the Astronomical Almanac describe several of the timescales used in astronomy. TT (Terrestrial Time) is tied to atomic time (TAI+32.184s). TT is a dynamical time used in theories of planetary motion. Its equivalent at the solar system barycenter is TDB (Barycentric Dynamical Time). They differ periodically due to the changing gravitational potential along the Earth’s orbit. Formula in the Almanac, size of TDT-TDB correction about 2 msec or less.

UTC is broadcast, and differs from UT (Universal Time) by less than one second. The correction can only be determined after the fact, but can be predicted reasonably well. UT is tied to the rotation of the Earth, and can involve leap seconds to allow the Earth to “catch up” in its rotation. It is thus not a basis for dynamical theories, but for observation and timekeeping. The correction from UT to TT is tabulated in the Almanac [part K]; it presently amounts to about 66 s.

The time of flight of photons within the solar system must be taken into account in establishing the time of events, such as stellar pulsations or eclipses in binary star systems (as well as events within the solar system). Geocentric times are normally corrected to heliocentric times or barycentric times (often ignoring the TT-TDB difference discussed above). The Julian Date form is often used, hence GJD (Geocentric JD), HJD (Heliocentric JD) and BJD (Barycentric JD). If this is in the UTC system, often no further notation is given; if this is in the TT system, the notation HJED (Heliocentric Julian Ephemeris Date) is often used, although Eastman et al. prefer BJD\(_{TT}\), for example.

The heliocentric correction HJD−GJD is maximal, up to 1.017 \times 499.00 \text{ s} = 507.33 \text{ s}, for a point on the ecliptic; it’s zero at the ecliptic poles. Formulae in the Almanac.
2.4 Sidereal Time and Hour Angle.

The local apparent hour angle (HA or $h$) of a star is given by the difference between the Local Apparent Sidereal Time and its R.A.: $HA = LAST - RA$. HA increases with the passage of time.

RA is computed from the mean position with due account taken of the effects discussed above (proper motion, precession, aberration, etc.)

LAST is computed from Local Mean Sidereal Time by adding the equation of the equinoxes, which is a nutation-induced term, about 1 s in size, tabulated in the Almanac.

$$LAST = LMST + eqn.eqx.$$  

LMST is computed from the Greenwich Mean Sidereal Time by adding the East longitude. (The East longitude of State College is negative.) $LMST = GMST + \text{East longitude}.$

GMST increases uniformly at the rate of 1.00273791 sidereal days per mean solar day (86400 SI seconds), an excess over 24 sidereal hours of 3m 56.55s (sidereal). Formulae and tabulations of GMST are given in the Almanac [part B]. Thus:

$$HA = (GMST + \text{East longitude} + eqn.eqx.) - RA$$  

2.5 Conversion from Equatorial to Horizon Coordinates.

Use the HA as an intermediary angle (Local Equatorial coordinates). Then either a standard Astronomical Triangle involving the Pole, the Zenith, and the Star can be solved for the zenith distance and (interior) Azimuth angle; or the direction cosines in the Horizon System can be computed by a rotation matrix applied to the direction cosines of the Local Equatorial System.

The altitude of the pole (i.e., the geodetic or geographic or astronomical latitude of the site) must be known; this is not the same as the geocentric latitude, which should be used along with the geocentric distance to compute diurnal parallax and aberration.

2.6 Atmospheric Refraction.

This raises the apparent altitude of an object (decreases its zenith distance), usually without altering the azimuth (unless there are lateral inhomogeneities). It depends on the density of the air, and therefore on temperature and pressure. It also depends on wavelength, being larger for shorter wavelengths (prismatic dispersion). In ”visual” light, it amounts to about 34 arcminutes at the horizon (the Almanac’s adopted value for rising and setting calculations). More details are given in the paper by Filippenko (1982, PASP, 94, 715).
Since the vertical direction is usually not parallel to a line of constant R.A. or Declination, refraction introduces corrections into both the apparent R.A. and the apparent Declination, which vary with H.A.

Differential refraction may be important across a wide-field image, of from one wavelength to another, amounting to several arcsec.

2.7 How to Think about Precession.

Precession is simple in ecliptic coordinates: aside from a small planetary induced wobble in the Ecliptic, the Ecl. Latitude ($\beta$) of a star does not change, and its Ecl. Longitude ($\lambda$) increases steadily at the rate of 5029” per century, as the Equinox slips backwards.

However, we prefer to catalog our stars in Equatorial coordinates so that we may point our telescopes simply.

To transform from one equator and equinox to another, one can imagine a three-part process:
(1) transform from ($\alpha_1$, $\delta_1$) at the first epoch to ecliptic to($\lambda_1$, $\beta_1$), using the old Pole and Equinox.
(2) add the precession in Longitude, keeping $\beta$ fixed.
(3) transform from ($\lambda_2$, $\beta_2$) to ($\alpha_2$, $\delta_2$) at the second epoch, using the new Pole and Equinox.

These three steps can be codified either in a cumbersome notation derived from the spherical triangles; or as a matrix which is the product of three elementary rotation matrices that represent the above steps. The matrix is orthonormal, like any rotation matrix, and its inverse is its transpose.

The precession matrix operates on the direction cosines of the position vector (a unit vector in the equatorial system):

\[
x = \cos(\alpha) \cdot \cos(\delta) \quad (1)
\]
\[
y = \sin(\alpha) \cdot \cos(\delta) \quad (2)
\]
\[
z = \sin(\delta) \quad (3)
\]

It rotates the unit vector about the line of nodes (through the equinoxes), then about the Ecliptic Pole, then about the new line of nodes.
2.8 General Coordinate Transformations on the Sphere.

All coordinate transformation problems on the sphere can be handled either within spherical trigonometry, using the **Laws of Sines and Cosines** (for Angles and Sides), as described in e.g. Kartunnen Ch. 2; or using unit vectors and rotations (direction cosines and matrices). See also items 5 & 6 of this document.

The **elements of the rotation matrices** can be set up easily if the coordinates of **three orthogonal points** are known in both the “old” and “new” systems. These points correspond to the **unit basis vectors** in the *old* system. (In *Equatorial* coordinates, the unit vectors point to the **Vernal Equinox**, the point *(6h,0d)*, and the **North Celestial Pole**.) The rows of the matrix are just the direction cosines of these points, expressed in the *new* system, i.e., relative to the three new unit vectors.

**LEARN** the positions of **key points on the Celestial Sphere** in all the relevant coordinate systems, so that you can construct the rotation matrices quickly!

Certain problems, such as determining the “**parallactic angle**”, are easier expressed in terms of spherical triangles. Others are easier to understand in terms of vectors and matrices. Thus it behooves you to know both ways of thinking. [Of course the parallactic angle, which indicates a direction *along* the sphere rather than *to* the sphere from its *center*, can also be found using matrices, but one must consider differentials of direction cosines and the angles on the sky to which they correspond...]

2.9 Angular Distance between Two Points on the Sphere.

Using direction cosines for two points at *(α₁, δ₁)* and *(α₂, δ₂)*, it is easy to show that the angular distance \( \rho \) between them is given by

\[
\cos(\rho) = \cos(\alpha_1 - \alpha_2) \cdot \cos(\delta_1) \cdot \cos(\delta_2) + \sin(\delta_1) \cdot \sin(\delta_2)
\]

The trigonometric identity for the cosine of the difference of two angles has been used. The limiting cases, for \( \delta_1 = \delta_2 \) and \( \delta_1 = \delta_2 = 0^\circ \) or \( \delta_1 = \delta_2 = 90^\circ \), and for \( \alpha_1 = \alpha_2 \), can be checked as a means to verify the correctness of the formula. [It is always good to check some limiting cases by hand, to verify a newly derived formula.]
3. Motions in the Solar System

1. Earth rotation.
Mean solar day = 86400 SI seconds (unit of UT)
   = 86636.5537 mean sidereal sec = 24h 03m 56.55s
Mean sid. day = mean rotation period = 86164.10 sec of UT
   = 23h 56m 04.1s
ratio is 1.0027378...

Precession: p = 5029.0966"/Julian century (36525 d), 2pi radians in 25770 yr.
   eps = 23d 26' 21".488 (tilt = "obliquity of the ecliptic")
Nutation: forced + free motion; 18.6 y main period, 9."20 amplitude.

Figure of the Earth: Radius: 6378.137 km mean equatorial; flattening 1/298.257
Rotation speed at equator = 2pi R/86164.1s = 0.465 km/s

Julian Date: 0.0 = noon at Greenwich on 1 Jan 4713 B.C.,
on the Julian proleptic calendar (not Gregorian!).

2. Sun’s (apparent) motion: along the zodiac. Traditional Zodiac constellations
   or "signs": Ari, Tau, Gem, Cnc, Leo, Vir, Sco, Sgr, Cap, Aqr, Psc.

   tropical year 365.242190 = 365d 05h 48m 45.2s [equinox]
sidereal year 365.256363 = 365d 06h 09m 09.8s (note: dt = 20m 24.6s, or
   365.25d in 25,770 yr !)
anomalistic yr 365.259635 [perigee/apogee]
eclipse year 346.620074 [nodal passage]

relations: 1/solar day = 1/sid.d - 1/sid.y; or f_3 = f_1 - f_2 (frequency).

Position of Sun at: approx. date Equat. Ecliptic
   Vernal Equinox, 3/21 (0h, 0d), (0d, 0d)
   June Solstice, 6/21 (6h,+eps), (90d, 0d)
   Autumnal Equinox, 9/23 (12h, 0d), (180d, 0d)
   December Solstice, 12/22 (18h,-eps), (270d, 0d).

Earth orbit: e = 0.01671 at 1996 Jan 0.0 TDT; perihelion in early January.
   Consequence: seasons are of unequal length. Known to Hipparcos.
3. Moon: mean lengths of months at 1996.0 (AA96, D2):
   synodic month (phases) 29.530589d
   tropical month (eqx) 27.321582d
   sidereal month (star) 27.321662d
   anomalistic mo. (perigee) 27.554550d --> advances (P > P_sid)
   draconic month (node) 27.212221d --> regresses (P < P_sid)

   inclination of lunar orbit to ecliptic: 5.145 deg;
   eccentricity varies, e^-0.055

4. Planets: see BOB 2d edn. pp. A-1,2;
   or U&B 5th edn. Tables 2.2 & 3.1,
   or Karttunen 5th edn, Tables C.9 ff (Appendices/back matter)
4. The Spherical Triangle

4.1 Parts of the Triangle
Vertices: A, B, C; Angles: A, B, C; Sides: a, b, c (opposite the corresponding angle).
Note: Sides are segments of great circles. Angles are “dihedral” (angle between two planes containing the adjoining sides and passing through the center of the sphere). Angles and sides cannot be larger than 180°. The sum of the angles is between 180° and 540°. The sum of the sides cannot be larger than 360°.

4.2 Law of Sines
\[
\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}
\]

4.3 Law of Cosines for Sides
\[
\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A
\]
and cyclic permutations.

4.4 Law of Cosines for Angles
\[
\cos A = -\cos B \cdot \cos C + \sin B \cdot \sin C \cdot \cos a
\]
and cyclic permutations. Note the minus sign!

4.5 Quadrant Disambiguation. Ambiguities are possible, e.g., \( \cos A > 0 \) does not tell you whether \( A > 0 \) or \( A < 0 \). Use the Sine and Cosine of the angle together to resolve this ambiguity. Or use other information, e.g., if \( h > 0 \) then \( \sin(A \hat{z}) < 0 \) (item 4.6).

4.6 Interior angles only! Be careful to distinguish the Azimuth (measured from the North horizon point, toward the East), with \( \hat{A} \) (“A-twiddle”), which is the interior angle at the Zenith of the usual Astronomical Triangle (vertices: Pole, Zenith, Star). Since the Sine of \( \hat{A} \) is always \( \geq 0 \), you cannot use the Law of Sines (alone) to find \( \hat{A} \) or \( A \).

For hour angle \( h < 0 \), \( \hat{A} = A \) and \( 0 \leq A \leq 180° \). Note that \( \sin A \geq 0 \).

For \( h > 0 \), we have \( 180° \leq A \leq 360° \). Here \( \hat{A} = 360° - A \), and \( 0 \leq \hat{A} \leq 180° \). Thus \( \cos \hat{A} = \cos A \); but \( \sin \hat{A} = -\sin A \geq 0 \).
5. Orthogonal Triads of Unit Vectors

In each coordinate system, we define a right-handed orthogonal set ($\hat{e}_x, \hat{e}_y, \hat{e}_z$) of unit basis vectors, such that $\hat{e}_i \cdot \hat{e}_j = \delta_{ij}$ and $\hat{e}_x \times \hat{e}_y = \hat{e}_z$ (plus cyclic permutations).

A point $\hat{P}$ on the sphere with (generalized) Azimuthal coordinate “Azi” and co-polar distance “alt” will be denoted ($Azi, alt$) or ($A, a$). It can be expressed as a unit vector

$$\hat{P} = x \cdot \hat{e}_x + y \cdot \hat{e}_y + z \cdot \hat{e}_z$$

in any basis. If $\hat{e}_z$ is chosen to point along the pole of the coordinate system, and $\hat{e}_x$ points along the direction to the origin of the Azimuthal coordinate, then $(x, y, z)$ are just direction cosines, $x = \hat{e}_x \cdot \hat{P}$, etc:

$$x = \cos(Azi) \cos(alt)$$
$$y = \sin(Azi) \cos(alt)$$
$$z = \sin(alt)$$

Warning: In Local Equatorial coordinates, $h$ (H.A.=hour angle) increases to the West, the “wrong” way around the Pole; so $y = \sin(-h) \cos(\delta)$. In Horizon coordinates, Azi increases from N towards E, so $x = \sin(Azi) \cos(alt)$, $y = \cos(Azi) \cos(alt)$.

[The indicated changes make these proper right-handed systems.]

<table>
<thead>
<tr>
<th>System</th>
<th>Symbols</th>
<th>$\hat{e}_x$</th>
<th>$\hat{e}_y$</th>
<th>$\hat{e}_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ecliptic</td>
<td>$(\lambda, \beta)$</td>
<td>Aries:($0^h, 0^\circ$)</td>
<td>June Solstice: ($90^\circ, 0^\circ$)</td>
<td>N.Ecl.Pole ($-90^\circ$)</td>
</tr>
<tr>
<td>Equatorial</td>
<td>$(\alpha, \delta)$</td>
<td>Aries:($0^h, 0^\circ$)</td>
<td>6h later:($6^h, 0^\circ$)</td>
<td>N.Cel.Pole ($-90^\circ$)</td>
</tr>
<tr>
<td>Local Equ.</td>
<td>$(h, \delta)$</td>
<td>Meridian/Equ.Pt: ($0^h, 0^\circ$)</td>
<td>East Point: ($-6^h, 0^\circ$)</td>
<td>N.Cel.Pole ($-90^\circ$)</td>
</tr>
<tr>
<td>Horizon</td>
<td>$(A, a)$</td>
<td>East Point: ($-6^h, 0^\circ$)</td>
<td>North Point: ($0^\circ, 0^\circ$)</td>
<td>Zenith ($-90^\circ$)</td>
</tr>
</tbody>
</table>

Relations:
Ecl. → Equ. by rotating through angle $\epsilon \approx 23.4^\circ$ around (common) $\hat{e}_x$ axis.
Equ. → Local Equ. by rotating through angle LAST around (common) $\hat{e}_z$ axis.
Local Equ. → Horizon by rotating through angle $\phi$ around East Point.
[$\epsilon =$ obliquity, $\phi =$ observer’s latitude, LAST = observer’s (local) sidereal time.]

[August 2004]
6. Rotations & Rotation Matrices (review your undergrad mechanics!)

Elementary rotations are through an angle around a single pole. Finite elementary rotations do not commute! Elementary rotations can be expressed by matrices (linear combinations of vectors). The transpose of a (simple) rotation matrix is its inverse. General rotations can be expressed by compounded elementary rotations carried out in the correct sequence, hence by products of simple rotation matrices.

If you can express each orthogonal unit vector of one system in terms of components of the other system, you can construct the rotation matrix that relates the two systems. (See Proof.)

Proof:

Consider a general vector \( \hat{P} = x \cdot \hat{e}_x + y \cdot \hat{e}_y + z \cdot \hat{e}_z \).

But suppose:

\[
\begin{align*}
\hat{e}_x &= R_{11} \hat{e}_{x'} + R_{12} \hat{e}_{y'} + R_{13} \hat{e}_{z'} \\
\hat{e}_y &= R_{21} \hat{e}_{x'} + R_{22} \hat{e}_{y'} + R_{23} \hat{e}_{z'} \\
\hat{e}_z &= R_{31} \hat{e}_{x'} + R_{32} \hat{e}_{y'} + R_{33} \hat{e}_{z'}
\end{align*}
\]

Then

\[
\hat{P} = (xR_{11} + yR_{21} + zR_{31}) \hat{e}_{x'} + (xR_{12} + yR_{22} + zR_{32}) \hat{e}_{y'} + (xR_{13} + yR_{23} + zR_{33}) \hat{e}_{z'}
\]
or

\[
\hat{P} = x' \hat{e}_{x'} + y' \hat{e}_{y'} + z' \hat{e}_{z'}
\]

where \( x' = R_{11}x + R_{21}y + R_{31}z \), etc.

Or

\[
\begin{pmatrix}
x' \\
y' \\
z'
\end{pmatrix} =
\begin{pmatrix}
R_{11} & R_{21} & R_{31} \\
R_{12} & R_{22} & R_{32} \\
R_{13} & R_{23} & R_{33}
\end{pmatrix} \cdot
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
\]

This is general. Elements of the rotation matrix \( R \) are just the components of \( \hat{e}_x, \hat{e}_y, \text{ and } \hat{e}_z \text{ in the primed system} \) (which has basis vectors \( \hat{e}_{x'}, \hat{e}_{y'}, \hat{e}_{z'} \)). \( R_{11} = \hat{e}_x \cdot \hat{e}_{x'} \), etc.

Keep rows and columns straight!

[August 2004]
7. Almanac Formulas for GMST, GAST, etc.

Numerical values are based on the 2009 edition of the Astronomical Almanac.

7.1 Two different ways to do this:
   a. Old way, using the Equation of Equinoxes.
   b. New way, using Earth Rotation Angle (ERA) and Equation of Origins.

   We’ll use the old way. (ERA is proportional to UT1).

7.2 TAI = international atomic time.
   TT = Terrestrial time = TAI + 32.184 s.

   UT1 = TT - Delta(T): "proportional to "Earth Rotation Angle",
       used in computation of GMST, etc.

   Delta(T) tabulated (after the fact) [Almanac page K9].
   Expected/predicted value of Delta(T) for 2009 is ~66 seconds.
   Accounts for long-term slowing of Earth’s rotation (tides).

   UTC = "broadcast time"; offset from TAI by integer number of seconds.
   UTC approximates UT1, accurate to better than one second.

7.3 GMST = Greenwich Mean Sidereal Time
   (hour angle of the Mean Equinox of Date (First Point of Aries)
     at the Greenwich meridian).
   GAST = Greenwich Apparent Sidereal Time (hour angle of true Equinox).

   Mean Equinox varies secularly with Date, owing to precession.
   True Equinox differs from Mean Equinox, owing to nutation (periodic).

   See Almanac page [B6] for more.
7.4 Assuming we have UT1, we can find GMST/GAST by:

a. Table lookup: GMST and GAST at UT1 = 0h are tabulated for each day of year [B13-B20].

Then add Delta(ST) = 1.0027379094 x Delta(UT1) for fraction of day.

or b. Calculate from day of year (d) and hour of day (t) using

\[ \text{GMST} = 6.6527125 + 0.0657098244 \times d + 1.00273791 \times t \] [hours]

where \( d = \) day of year (Jan 1 = 1, etc.) is an integer [tabulated: B4-B5], and \( t \) is in hours.
The above formula is valid for 2009 (only), and it assumes that Delta(T) = TT - UT1 = 66 seconds during 2009.

\[ \text{GAST} = \text{GMST} + \text{EOE} \] (EOE= Equation of Equinoxes: typical magnitude = 1 or 2 seconds).
GAST is used for computing Hour Angles, often can ignore EOE.

7.5 To find UT1 from GMST, tables on [B13-B20] also give UT1 at GMST = 0h. Then add Delta(UT1) = Delta(ST) x 0.9972695663.


7.7 Local hour angle of object \( X \) = Local Sidereal time - \( X \)'s Right Ascension
or: \( \text{LHA}(X) = \text{LST} - \text{RA}(X) \)

\( \text{LAST} = \text{GAST} + \text{East Longitude} = \text{GAST} - \text{West Longitude} \)
8. Almanac Tools

The *Astronomical Almanac* contains tabular material, but also lots of “recipes” related to positional astronomy and “reduction of observations”. You should browse the *Almanac* for the current year (or a recent year), to learn what sorts of things it contains. Copies for recent years are in room 530 (do not remove these reference books from room 530 for more than a few minutes!), or in the PAMS library (2d floor of Davey Lab).

The list below points to a number of specific “tools” that you should know about. The *Almanac* gives very little background information on these, so you need to supply some understanding to make sure you are using the recipes correctly.

The page numbers given in [square brackets] refer to the 2007 edition of the Astronomical Almanac. Exact pagination varies from year to year.

1. **JULIAN DATE**: page [B3]. Definition, and tabulation at beginning of calendar month for current year (more extensive table, pages [K2-4]). Also gives exact dates and Julian Dates of "standard epochs". Also defines MODIFIED JULIAN DATE, and JULIAN EPOCH.

2. **APPROXIMATE REDUCTION for PROPER MOTION**, and for **ANNUAL PARALLAX** [B26]. "Reduction" refers to converting coordinates or other quantities from mean values to apparent values (or vice versa), or updating catalogued coordinates to a new EPOCH, etc. "Approximate" means, for example, a linear approximation, or validity over short intervals of time; user beware!

3. **REDUCTION OF TIME** to solar system BARYCENTRE [B27]. In the paragraph on annual parallax.

4. **APPROXIMATE REDUCTION** for **ANNUAL ABERRATION** [B27]. It should be an exercise for the reader to understand how the various reduction recipes relate to the 3-dimensional motions of objects, and how they represent particular coordinate representations of vector operations such as dot-products and cross-products.

5. **REDUCTION OF RADIAL VELOCITY** to the solar system BARYCENTRE [B27]. In the paragraph on annual aberration.

**Note:** Various reduction formulas make use of the **RECTANGULAR COORDINATES** of the Earth (X,Y,Z) and their time derivatives: See pages [B71-78].
6. DIFFERENTIAL ABERRATION [B27]. You probably don’t care, unless you are responsible for writing telescope control software. (The amplitude of the effect is small.) But note that the KEPLER spacecraft has a big enough field of view that this effect shifts star images by a significant fraction of a pixel (from one side of the field to the other).

7. APPROXIMATE REDUCTION for LIGHT DEFLECTION [B28]. Gives a formula and a short table for the amplitude of deflection of light by the gravitational field of the Sun.

8. LOW-PRECISION FORMULAS for the SUN’S COORDINATES and the EQUATION OF TIME [C24]. To use these, you should understand the definitions of the terms used.

   Similar formulas (with more terms) for the GEOCENTRIC and TOPOCENTRIC COORDINATES of the MOON are given on page [D22].

9. Formula for the ELLIPSE, and a series expansion for the TRUE ANOMALY in terms of the mean anomaly and the eccentricity, [E6]. Also on this page, an example of projecting a planet’s radius vector onto rectangular coordinate axes.