Around-the-World Atomic Clocks: Predicted Relativistic Time Gains

Abstract. During October 1971, four cesium beam atomic clocks were flown on regularly scheduled commercial jet flights around the world twice, once eastward and once westward, to test Einstein’s theory of relativity with macroscopic clocks. From the actual flight paths of each trip, the theory predicts that the flying clocks, compared with reference clocks at the U.S. Naval Observatory, should have lost 40 ± 23 nanoseconds during the eastward trip, and should have gained 275 ± 21 nanoseconds during the westward trip. The observed time differences are presented in the report that follows this one.

One of the most enduring scientific debates of this century is the relativistic clock “paradox” (1) or problem (2), which stemmed originally from an alleged logical inconsistency in predicted time differences between traveling and reference clocks after a round trip. This seemingly endless theoretical debate, which has flared up recently with renewed vigor (2, 3), begs for a convincing empirical resolution with macroscopic clocks. A simple and direct experimental test of the clock problem with portable atomic clocks is now possible because of the unprecedented stability achieved with these clocks (4).

In this first of two reports, we present relativistic time differences calculated from flight data for our recent around-the-world flying clock experiments. The theory predicts a detectable effect with cesium beam clocks if they are flown around the world at typical jet aircraft speeds (4). Moreover, it predicts an interesting asymmetry in the time difference between the flying clocks and a ground reference clock, depending on the direction of the circumnavigation (4). Predicted time differences are compared with our observed time differences in the following report.

A brief elementary review of the theory seems appropriate, particularly because of some confusion about the capacity of such experiments to produce meaningful results (5). Special relativity predicts that a moving standard clock will record less time compared with (real or hypothetical) coordinate clocks distributed at rest in an inertial reference space. For low coordinate speeds (\(u^2 \ll c^2\)), the ratio of times recorded by the moving and reference coordinate clocks decreases to (1 - \(u^2/2c^2\)), where \(c\) is the speed of light. Because the earth rotates, standard clocks distributed at rest on the surface are not suitable in this case as candidates for coordinate clocks of an inertial space. Nevertheless, the relative timekeeping behavior of terrestrial clocks can be evaluated by reference to hypothetical coordinate clocks of an underlying nonrotating (inertial) space (6).

For this purpose, consider a view of the (rotating) earth as it would be perceived by an inertial observer looking down on the North Pole from a great distance. A clock that is stationary on the surface at the equator has a speed \(R\Omega\) relative to nonrotating space, and hence runs slow with a corresponding time difference, to a first approximation, is given by

\[ r - r_n = -(2Rtv + v^2/2c^2) \] (1)

Consequently, a circumnavigation in the direction of the earth’s rotation (eastward, \(v > 0\)) should produce a time loss, while one against the earth’s rotation (westward, \(v < 0\)) should produce a time gain for the flying clock if \(|v| \sim R\Omega\).

General relativity predicts another effect that (for weak gravitational fields) is proportional to the difference in the gravitational potential for the flying and ground reference clocks. If the surface value of the acceleration of gravity is \(g\) and the altitude for the circumnavigation is \(h \ll R\), the potential difference is \(gh\), and Eq. 1 then reads

\[ r - r_n = gh/c^2 - (2Rtv + v^2)/2c^2 \] (2)

The \(gh/c^2\) term, which is related to the gravitational “red shift,” predicts a time gain for the flying clock irrespective of the direction of the circumnavigation. For typical aircraft speeds and altitudes, both the gravitational and kinematic terms in Eq. 2 are comparable in absolute magnitude, and \(v^2/2c^2\) is small compared with \(R\Omega v/c^2\). For a westward circumnavigation (\(v < 0\)) both terms are positive and they add to give a large net time gain, but for an eastward circumnavigation (\(v > 0\)) they tend to cancel and produce a net time difference.

Table 1. Predicted relativistic time differences (nsec).

<table>
<thead>
<tr>
<th>Effect</th>
<th>Direction</th>
<th>East</th>
<th>West</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravitational</td>
<td>East</td>
<td>144 ± 14</td>
<td>179 ± 18</td>
</tr>
<tr>
<td></td>
<td>West</td>
<td>-184 ± 18</td>
<td>96 ± 10</td>
</tr>
<tr>
<td>Kinematic</td>
<td>East</td>
<td>-184 ± 18</td>
<td>96 ± 10</td>
</tr>
<tr>
<td></td>
<td>West</td>
<td>-184 ± 18</td>
<td>96 ± 10</td>
</tr>
<tr>
<td>Net</td>
<td>East</td>
<td>-40 ± 23</td>
<td>275 ± 21</td>
</tr>
<tr>
<td></td>
<td>West</td>
<td>-40 ± 23</td>
<td>275 ± 21</td>
</tr>
</tbody>
</table>
ence that may be positive or negative depending on details of the flight.

We can compare the predicted time differences with detection thresholds. If the circumnavigation is nonstop, to a first approximation the trip time \( t_0 = \frac{2\pi R}{v} \). Substitution of this value for \( t_0 \) in Eq. 2 gives

\[
\Delta t = \frac{2R}{c^2} [\sqrt{gh/|v|} - Rtv/|v| - |v|/2] 
\]

This relationship is graphically illustrated in Fig. 1 over the range of ground speeds and altitudes of interest. The area within the hatched lines in Fig. 1 is below detection thresholds estimated from past experience at the U.S. Naval Observatory with portable cesium beam clocks (7). The labeled points correspond to cruising altitudes and ground speeds for the indicated aircraft (8). Figure 1 illustrates that circumnavigations with cesium beam clocks at jet aircraft speeds should produce measurable relativistic time differences. Furthermore, the mere existence of a definite east-west directional asymmetry in the observed time differences would give strong evidence for the validity of the kinetic term in Eq. 2.

These predictions for equatorial circumnavigations will be modified somewhat for our actual flights because commercial around-the-world jet flights, of course, do not follow equatorial paths, nor do they maintain constant altitude, ground speed, or latitude. In this case, however, it is only necessary to integrate the appropriate differential form of Eq. 2 along the actual flight path:

\[
\Delta t = \int \left[ \sqrt{gh/|v|} - Rtv/|v| - |v|/2 \right] \, dt 
\]

This expression contains a slightly modified directionally dependent term, which for nonequatorial flights becomes proportional to both the eastward component of the ground speed, \( v \cos \theta \), and the cosine of the latitude, \( \cos \lambda \) (4, 9). Because only lowest order relativistic time differences can be detected in our experiments, only lowest order terms need be retained in the calculated predictions, and, to this order of approximation, it is immaterial whether the differential time for \( \tau \) or for \( t_0 \) is used in Eq. 4.

The eastward trip began on 4 October 1971 at 19:30 U.T. and lasted 65.4 hours with 41.2 hours in flight. The westward trip began during the following week on 13 October at 19:40 U.T. and lasted 80.3 hours with 48.6 hours in flight. Flight data necessary for numerical evaluation of Eq. 4 were provided by the various flight captains. In most cases they traced their flight path on an appropriate flight map and recorded the time and aircraft ground speed and altitude at various navigation check points along the flight path. This information divided the eastward trip into 125 intervals and the westward trip into 108 intervals. The latitude and longitude for each check point, read directly from the flight maps, combined with the time (U.T.) over each check point permits calculation of an average ground speed, latitude, and eastward azimuth for each interval. The average altitude for each interval was taken as the average of the altitudes at the end points. This information then permits numerical evaluation of the integral in Eq. 4. Table 1 gives the predicted time differences resulting from these calculations.

We conclude this report with a word about uncertainty in these predictions. Possible errors stem from two sources: (i) errors and deficiencies in the flight data, and (ii) theoretical approximations used in the derivation of Eq. 4. We estimate the maximum possible fractional uncertainty from flight data errors to be less than 10 percent for each term of Eq. 4 after numerical integration. This estimate includes both systematic and random errors in the flight data. If it is assumed that the errors from these terms add in quadrature, the maximum fractional uncertainty in the net value for the eastward circumnavigation is about 60 percent, while that for the westward circumnavigation is only 8 percent. These uncertainties are listed in Table 1.

Although neglect of higher order terms (proportional to \( c^{-4}, c^{-6} \), and so forth) in our theoretical approximations is fully justified, small but perhaps not entirely negligible first order effects may arise from the presence of the moon and sun. In fact, the center-of-mass of the earth-moon system, not the center of the earth, is in free fall around the sun, and a more precise calculation should include this effect. It is unlikely, however, that the precision of our experiments permits detection of any effects other than the dominating ones retained in Eq. 4 (10, 11).

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References and Notes
6. It is important to emphasize that special relativity purports to describe certain physical phenomena only relative to (or from the point of view of) inertial reference systems, and the speed of a clock relative to one of these systems determines its timekeeping behavior. (G. Builder, Aust. J. Phys. 11, 279 (1958)). Although inertial systems are highly specialized, they have an objective physical relationship with the universe because they have no acceleration or rotation relative to the universe. The difference in the times indicated by two clocks located at the same place is a physically observable quantity that is invariant. Therefore, a correct derivation of a relativistic time difference after a round trip using a particular (inertial) reference system is independent of that system. This means that a subsequent coordinate transformation into the (noninertial) rest system for the clocks is unnecessary. Appropriate transformations in this case would be those of the general theory. The application of such a transformation would not be incorrect, but one is not required for a correct calculation; it would only unnecessarily complicate the theoretical description.
7. Previous trips with cesium beam clocks (HP-
Around-the-World Atomic Clocks: Observed Relativistic Time Gains

Abstract. Four cesium beam clocks flown around the world on commercial jet flights during October 1971, once eastward and once westward, recorded directionally dependent time differences which are in good agreement with predictions of conventional relativity theory. Relative to the atomic time scale of the U.S. Naval Observatory, the flying clocks lost 59 ± 10 nanoseconds during the eastward trip and gained 273 ± 7 nanoseconds during the westward trip, where the errors are the corresponding standard deviations. These results provide an unambiguous empirical resolution of the famous clock "paradox" with macroscopic clocks.

In science, relevant experimental facts supersede theoretical arguments. In an attempt to throw some empirical light on the question of whether macroscopic clocks record time in accordance with the conventional interpretation of Einstein's relativity theory (1), we flew four cesium beam atomic clocks around the world on commercial jet flights, first eastward, then westward. When we compared the time they recorded during each trip with the corresponding time recorded by the reference atomic time scale at the U.S. Naval Observatory, the flying clocks lost 59 ± 10 nanoseconds during the eastward trip and gained 273 ± 7 nanoseconds during the westward trip, where the errors are the corresponding standard deviations. These results provide an unambiguous empirical resolution of the famous clock "paradox" with macroscopic clocks.

Relativistic effects are expected to cause the relativistic time offsets expected in our experiments are only of the order of 0.1 μsec per day (1, 4), any such time divergences (or rate differences) must be taken into account. A much more serious complication is caused by the fact that the relative rates for cesium beam clocks do not remain precisely constant. In addition to short term fluctuations in rate caused mainly by shot noise in the beam tubes, cesium beam clocks exhibit small but more well defined quasi-permanent changes in rate. The times at which these rate changes occur typically are separated by at least 2 or 3 days for good clocks. Some clocks have been observed in the laboratory to go as long as several months without a rate change (2, 5).

These unpredictable changes in rate produce the major uncertainty in our results. Because of the nature of these changes, however, their effect on the observed time differences can be removed to a large extent in the data analysis. Under normal conditions changes in relative rates occur independently, that is, there are no known systematic correlations between rate changes of one clock and those of another. Consequently, the chance that two or more clocks will change rate by the same amount in the same direction at the same time is extremely remote. Because of the random and independent character of these rate changes, the long-term average rate of an ensemble of clocks is more stable than the rate of any individual member.

Starting at 0 U.T. on 25 September 1971, we recorded more than 5000 time differences during the data period. Figure 1 shows the time difference data relative to MEAN(UUSNO) for the entire data period, which lasted 636 hours. The labels in Fig. 1 are the serial numbers of the corresponding clocks, and the traces give the measured differences in time between the corresponding clock and MEAN(UUSNO). Of course, no comparisons with MEAN(UUSNO) were possible during the trips. Exactly the same electronic arrangement was used for all time intercomparison mea-

| Table 1. Observed relativistic time differences from application of the correlated rate-change method to the time intercomparison data for the flying ensemble. Predicted values are listed for comparison with the mean of the observed values; S.D., standard deviation. |
|-----------------|-----------------|-----------------|-----------------|
| Clock serial No.| Eastward* | Westward |
|-----------------|-----------------|-----------------|-----------------|
| 120             | -57          | 277            |
| 361             | -74          | 284            |
| 408             | -55          | 266            |
| 447             | -51          | 266            |
| Mean ± S.D.     | -59 ± 10     | 273 ± 7        |
| Predicted ± Error est. | ± 40 ± 23 | 275 ± 21      |

* Negative signs indicate that upon return the time indicated on the flying clocks was less than the time indicated on the MEAN(UUSNO) clock of the U.S. Naval Observatory.

Around-the-World Atomic Clocks:

J. C. Hafele

The development of compact and portable cesium beam atomic clocks (3) permits a terrestrial test of relativity theory. However, no two "real" cesium beam clocks keep precisely the same time, even when located together in the laboratory, but generally show systematic rate (or frequency) differences which in extreme cases may amount to time differences as large as 1 μsec per day. Because the relativistic time offsets expected in our experiments are only of the order of 0.1 μsec per day (1, 4), any such time divergences (or rate differences) must be taken into account. A much more serious complication is caused by the fact that the relative rates for cesium beam clocks do not remain precisely constant. In addition to short term fluctuations in rate caused mainly by shot noise in the beam tubes, cesium beam clocks exhibit small but more well defined quasi-permanent changes in rate. The times at which these rate changes occur typically are separated by at least 2 or 3 days for good clocks. Some clocks have been observed in the laboratory to go as long as several months without a rate change (2, 5).

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