DIRECT MEASUREMENT OF THE ELECTRON MOMENTUM PROBABILITY DISTRIBUTION IN ATOMIC HYDROGEN

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We report here the direct measurement of the momentum distribution of electrons in the ground state of atomic hydrogen using the noncoplanar symmetric (e, 2e) technique.

Ever since Schrödinger’s first paper in 1926 [1], in which he used his equation to solve the problem of the hydrogen atom, this problem has been of central importance in the teaching of quantum mechanics. This is so not only because the Schrödinger equation can be solved exactly in this case, but also because the solutions form the basis for approximate solutions for other atoms and molecules.

The solution of the Schrödinger equation for the hydrogen atom is usually carried out in configuration space, and the wave functions \( \psi_{nlm}(r) \) which are produced are interpreted as probability amplitudes. The probability of the electron being at a certain point \( r \) with respect to the centre of mass origin is given simply by \( |\psi(r)|^2 \).

Although the probability amplitudes are not observables, the absolute squares of these amplitudes are, \( |\psi_{nlm}(r)|^2 \) being the probability of finding the electron (and the charge it carries) at various positions \( r \) in the hydrogen atom. However, although \( |\psi(r)|^2 \) is stated to be an observable, it has never been directly observed, and the standard texts all show only calculated values of \( |\psi_{nlm}(r)|^2 \).

If instead of solving the Schrödinger equation for the hydrogen atom in configuration space we solve it in momentum space, the absolute square of the wave function \( |\psi_{nlm}(P)|^2 \) would give the momentum probability distribution of the electron in the state defined by the quantum numbers \( n, l \) and \( m \). We report here a measurement of this distribution for \( n = 1 \) using the noncoplanar symmetric (e, 2e) technique.

In a noncoplanar symmetric electron impact ionizing collision, the two outgoing electrons have equal energies \( (E_A = E_B = E/2) \) and are emitted at equal polar angles \( \theta \) to the incident direction. \( \theta \) is kept fixed at 45°, the angle for a free electron—electron collision, and the out-of-plane azimuthal angle \( \phi \) is varied in order to vary the ion recoil momentum \( q \),

\[
q = [(2k_A \cos \theta - k_0)^2 + 4k_A^2 \sin^2 \theta \sin^2(\phi/2)]^{1/2},
\]

where \( k_A = k_B \) and \( k_0 \) are respectively the magnitudes of the momentum of the emitted and incident electrons.

This kinematical arrangement maximises the momentum transfer \( K = k_0 - k_A \) from the incident to the struck electron ensuring close electron—electron collisions. At high enough energies the electron waves are well described by plane waves, and the momentum of the struck bound electron must be equal and opposite to the ion recoil momentum \( q \). The reaction can then be accurately described by the plane wave impulse approximation [2,3]. In atomic units the cross section for hydrogen is [4]

\[
\frac{d^5 \sigma}{dE_A d\Omega_A d\Omega_B} = (2\pi)^4 \frac{k_A k_B}{k_0} |T_M|^2 |\phi_{ls}(q)|^2,
\]

where \( T_M \) is the antisymmetrized two-electron (Mott scattering) \( T \) matrix and \( \phi_{ls}(q) \) is the momentum space wave function for the ground state of atomic hydrogen. It is spherically symmetric and in atomic units.
\( \phi(q) = \frac{(2^{3/2}/\pi) (1 + q^2)^{-2}}{1 + q^2} \). \hspace{1cm} (3)

Therefore, the noncoplanner symmetric (e, 2e) cross section is given by

\[ \sigma(q) = 2^7 \pi^2 k_A^2 k_0^{-1} |T_M|^2 (1 + q^2)^{-4} \]. \hspace{1cm} (4)

\( |T_M|^2 \), the half-off-shell Mott-scattering cross section can easily be calculated [3]. In symmetric geometry \( |k_0 - k_A| = |k_0 - k_B| = K \) and

\[ |T_M|^2 = \frac{1}{4\pi^4} \left( \frac{e^2}{2\eta} \right) \frac{K}{1 - K^4} \] \hspace{1cm} (5)

where

\[ \eta = |k_A - k_B|^{-1} \]. \hspace{1cm} (6)

In the noncoplanner symmetric geometry \( K \) is constant and \( |T_M|^2 \) is essentially independent of \( \phi \), i.e. \( q \), over the region of interest \( (q < 2 \text{ au}) \). Therefore the noncoplanner symmetric (e, 2e) cross section at fixed incident energy \( E_0 \) and summed final energies \( E = E_0 - 13.6 \text{ eV} \), should be directly proportional to the square of the momentum space wave function [eq. (3)].

The atomic hydrogen source was a dc discharge tube similar to that discussed previously [4,5]. The details of the noncoplanner symmetric coincidence spectrometer are given elsewhere [6] and only a brief description will be given here. Two cylindrical-mirror electron energy analyzers are mounted at polar angles of 45° relative to the incident electron beam on two concentric turntables, which are rotated by a stepping motor under computer control to vary the out-of-plane azimuthal angle \( \phi \). Collimating apertures placed before retarding lenses define the angular field of view of the electron detectors \( (\Delta \theta = 1.5^\circ, \Delta \phi = 2^\circ) \). The three element retarding lenses form an image on the entrance apertures of the cylindrical mirror analyzers. After traversing the cylindrical-mirror analyzers the electrons are detected by channel electron multipliers. The angle \( \phi \) is varied about 0° in both directions in order to check that the coincidence count rate is symmetric about the coplanar position; the singles counts are independent of \( \phi \). The interaction region is defined by the intersection of the collimated electron beam and the atomic hydrogen beam.

The electron beam is 1 mm in diameter and passes 3 mm from the edge of the discharge tube atomic hydrogen source.

![Fig. 1. The separation energy spectrum obtained at 400 eV and \( \phi = 0^\circ \) with the hydrogen discharge on (solid curve) and off (dashed curve).](image1)

The coincidence count rate measured as a function of the separation energy \( \varepsilon = E_0 - E \) for \( E = 400 \text{ eV} \) and \( \phi = 0^\circ \) is shown in fig. 1. A peak can clearly be seen centred at 13.6 eV due to ionization of atomic hydrogen with the discharge on. The peak at approximately 16 eV is due to ionization of molecular hydrogen. A detailed discussion of the separation energy spectrum is given by Hood et al. [5].

Angular correlations obtained for the atomic hy-
Hydrogen transition at $E = 400, 800, \text{ and } 1200 \text{ eV}$ are shown in fig. 2. They are compared with the curve given by the square of the atomic hydrogen ground-state momentum space wave function. Since absolute cross sections are not obtained, the data at each energy are arbitrarily normalized by fitting the three data points of lowest $q$ to the curve given by $(1 + q^2)^{-4}$. The inclusion of the Mott-scattering factor makes a negligible difference to the calculated cross section shapes. Small corrections were made to the raw data to deconvolute the effects due to the finite angular resolution of the spectrometers. These corrections, which are discussed in detail by Frost and Weigold [6], were always smaller than the one standard deviation statistical errors shown on fig. 2. The results at all energies are in excellent agreement with the momentum profile given by the absolute square of the probability amplitude obtained from the solution of Schrödinger's equation for the ground state of hydrogen.

The present data therefore provide a direct experimental demonstration of the interpretation of wave functions as probability amplitudes in the simplest and most important application of Schrödinger's equation to atomic physics, namely the hydrogen atom.

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References