KAONIC AND OTHER EXOTIC ATOMS

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1 INTRODUCTION

In this article we attempt to tell what is known about kaonic, sigmonic, and antiprotonic atoms. These are atoms in which an electron has been
replaced by a $K^-$-meson, $\Sigma^-$-hyperon, or antiproton. Loosely speaking, the term "mesonic atom" has been applied to any atom that contains a negative particle other than a regular electron. But according to present nomenclature only pionic and kaonic atoms are truly mesonic atoms. Other exotic systems could include $\Xi^-$ (cascade), $\Omega^-$ (1), and anti-deuteron but these have not yet been observed. Sometimes pionic, kaonic, sigmonic, and antiprotonic atoms are called hadronic atoms to signify that they contain a strongly interacting particle. We confine most of our remarks to kaonic, sigmonic, and antiprotonic systems. Review articles on muonic atoms by Wu & Wilets (2) and pionic atoms by Backenstoss (3) have appeared in the Annual Review of Nuclear Science.

A complete description of mesonic atoms involves a gamut of disciplines in physics ranging from atomic and solid state through nuclear to strong interactions of elementary particles. Atomic or solid state physics is involved at the moment a meson stops in matter; the decision must be made as to which nucleus will become the host and into which Bohr orbit and angular momentum state will the meson be captured. From capture to disappearance in a reaction with a nucleon, hadronic atoms must obey the rules of atomic physics in which Auger emission and electromagnetic radiation are the dominant processes. Nuclear physics enters through the distribution of nuclear matter that the mesons encounter near the nuclear surface. Finally particle physics comes into play when annihilation in a reaction with a nucleon occurs.

Among the exotic atoms, kaonic atoms are the most studied. However, even the exact history of a kaonic atom has not yet been determined. Recently indications were found that mesons were captured into atomic angular momentum states that have a strong dependence on the atomic number $Z$, as discussed in a later section.

Kaonic atoms have been studied since 1958 when stacks of photographic emulsion were exposed to beams of $K^-$-mesons at the Berkeley Bevatron (4). Evidence of the formation of kaonic atoms came from the emission of Auger electrons of energies $> 10$ keV. Auger electron spectra were consistent with the cascade of kaons down to low-lying Bohr radii in Ag and Br atoms. From some of the early work with emulsions it was concluded that heavy nuclei might carry neutron-rich surfaces as suggested by Johnson & Teller and by Wilets (5). The question of the neutron halos has been discussed pro and con. Some recent experiments appear to support the often criticized claim that neutron-rich atmospheres are present on nuclear surfaces. This subject is discussed later. Early attempts were made to observe X-ray emission from kaonic atoms, but the experiments were not successful mainly because of the poor resolution of the detectors and the low intensities of the beams. Investigations of kaonic
atoms got underway in earnest with the invention and application of semiconductor detectors. In 1966 prominent X-ray lines of the light elements Li, Be, B, and C were recorded by Wiegand & Mack (6).

For the study of kaonic and sigmonic atoms, the experimenter must have access to an accelerator that produces protons of at least 5 GeV; for antiprotons even higher energy is required. Experiments are thus limited to a few laboratories in the world: Lawrence Berkeley, Argonne National, Brookhaven National, European Organization for Nuclear Research (Geneva), Rutherford High Energy Laboratory, Dubna (USSR) and Serpukhov (USSR).

Negative kaon production occurs mainly by the reaction

\[ p + N \rightarrow K^- + K^+ + 2N. \]

The primary target is usually heavy metal. Newly generated kaons and pions enter a secondary beam; the channel in which the beam is found is really a particle spectrometer that transports the kaons from the production target to a secondary target, in which kaonic atoms will be formed. Kaons are relatively scarce particles. Many thousands of pions are produced for each negative kaon, and most of the pions must be removed by the beam spectrometer. Typical beams can supply a few hundred stopped kaons per second. However, several hundred pions may pass through the targets without harmful consequences.

Kaon beams should be constructed as physically short as practicable; otherwise, kaons are unnecessarily lost by decay during their flight along the beam path. Typical beams are around 15 m long and operate at particle momenta from 450 to 800 MeV/c. Figure 1 shows the arrangement used by Wiegand & Godfrey (7). At 500 MeV/c, 3% of the kaons survive a beam length of 13 m and require an absorber of about 50 g cm\(^{-2}\) of graphite to bring them to rest. Figure 2 illustrates a series of scintillation and Cerenkov counters used in the experiment of (7) to signal the arrival and stoppage of kaons. Targets were 9 cm in diameter and 2 g cm\(^{-2}\) thick. At CERN thicker targets were used because the main objective of the

![Figure 1 Plan of kaon beam showing arrangement of quadrupoles Q, bending magnets M, and electrostatic separator (Berkeley) (7).](image_url)
The experiments was to observe the highest energy X-rays of a given species, and in this case target self-absorption was not a problem.

The heart of any system to study kaonic atoms is the X-ray spectrometer. At present the most practical way to measure mesonic X-rays is by semiconductor detectors. We do not discuss in detail the theory and operation of these devices, but refer the reader to a recent article in the *Annual Review of Nuclear Science* by Goulding & Jaklevic (8) in which pertinent information and references are given. Some remarks, however, are in order. Semiconductor detectors have several advantages over gas proportional counters and scintillation counters of the NaI(Tl) variety. A disadvantage at present is their relatively small size. All X-ray detectors convert photons to electrons that in turn ionize the sensitive medium whether it be in a gaseous or solid state. Si and Ge require the expenditure of about 3 eV and noble gases require about 25 eV to make an ion pair. Scintillators take more than 100 eV per effective unit charge.

Measurement accuracy can be expressed as the full width at half-maximum (FWHM) of the distribution of energy output of the detector system when the input energy is constant.

\[
\text{FWHM} = 2.35(\text{E}_\text{eF})^{1/2},
\]

where \( E \) is the energy deposited in the detector, \( e \) the energy expended per ion pair, and \( F \) is a factor that depends upon the nature of the ionization process. \( F \) is about 0.5 for gases and 0.1 for Ge. Thus the resolution is significantly better for semiconductors. For energies above about 20 keV, ionization statistics dominate the ultimate resolution. At lower energies,

![Figure 2](https://example.com/figure2.png)

*Figure 2* Diagram of the arrangement of the beam counters, target wheel, and semiconductor detectors. The targets were approximately 65 cm from the exit of quadrupole \( Q_6 \) (Berkeley) (7).
2 BASIC CHARACTERISTICS OF EXOTIC ATOMS

Before discussing the detailed physics of exotic atoms, we present in this section some basic characteristics of the atoms that are relevant to the following sections. Since information about the exotic atoms comes...
mostly from X-ray spectroscopy of the atoms, we make also a few pertinent remarks on the spectroscopy.

The exotic atoms, namely kaonic, sigmonic, and antiprotonic atoms, differ from pionic atoms in the following essential points: the negatively charged heavy hadrons interact with nuclei more strongly than pions do, and these hadrons create more complicated final states as a consequence of nuclear absorptions.

The exotic atoms have been investigated mainly when the orbiting hadron is closer to the nucleus than are the atom’s ground state electrons. The system is therefore hydrogenic, with the electron cloud contributing an almost negligible screening effect, and the fundamental equations of the hydrogen atom are applicable: In terms of “Bohr radii” of the exotic atoms, \( a_0 = \frac{\hbar^2}{e^2 \mu} \), the mean values of the radii and their inverses are given by

\[
\langle r \rangle = a_0 \left[ 3n^2 - l(l+1) \right]/2Z
\]

and

\[
\langle r^{-1} \rangle^{-1} = a_0 n^2/Z,
\]

where \( \mu \) is the reduced mass of the hadron in the atom and \( Z \) is the atomic number. The rms value of the speed of the hadron is

\[
\langle v^2 \rangle^{1/2}/c = \alpha Z/n, \quad \text{where} \quad \alpha = e^2/hc.
\]

The above quantities are evaluated nonrelativistically. Energy levels of the atoms are determined from the Dirac energy for point nuclei,

\[
E_{n,j} = \mu c^2 \left[ 1 + (\alpha Z)^2/n - j - \frac{1}{2} + \left( j + \frac{1}{2} \right)^2 - (\alpha Z)^2 \right]^{1/2} - 1
\]

\[
= \mu c^2 - \frac{1}{2}(\alpha Z/n)^2 \mu c^2 - \frac{1}{2}[n/(j + \frac{1}{2}) - \frac{3}{4}](\alpha Z/n)^4 \mu c^2 + \cdots,
\]

where \( j = l \pm \frac{1}{2} \) and the second term is the familiar Bohr atom energy. For spin zero hadrons, a substitution \( j = l \) reduces the expression to the Klein-Gordon energy.

Thus the large masses of the hadrons decrease the radii of the exotic atoms and increase the binding energies by a factor of more than 950 times those of ordinary atoms. On the other hand the “speeds” of the orbiting hadrons remain the same.

Strong nuclear absorption affects the X-ray lines in an important way; it sets a boundary on the lowest angular momentum state in which an exotic atom can exist. Hadrons with higher angular momentum have a lower probability to be found within the nucleus than do those with lower \( l \). This causes the low-lying \( n, l \) transitions to be absent from all but the lightest elements. The probability of finding the hadron at the distance between \( r \) and \( r + dr \) from the nuclear center behaves roughly as
for circular orbits \((l = n - 1)\), and with the additional factor \(n^{2l+1}/(2l+1)!\)
for \(n \gg l\) \((l\) fixed). Here we are neglecting perturbation of the hadronic
wave function due to the strong interaction. The overlap of the hadronic
density with the nucleus is a crude estimate of the region where nuclear
absorption occurs. For large \(l\) it is peaked in the neighborhood of the
nuclear surface. (See Figure 4.)

High rates of nuclear absorption have an important effect on the last
observable X-ray lines of some hadronic atoms. The lines are broadened
and shifted in energy from that predicted by the Dirac (or Klein-Gordon)
energy plus other electromagnetic energies. Line broadening is a direct
measurement of the rate of nuclear absorption. This effect is discussed
in Section 3.2.

As seen in equation 1 and the similar equation for Klein-Gordon
energy, the X-ray spectra of kaonic atoms will not exhibit fine structure
splitting (the spin of kaons is zero), whereas sigmonic and antiprotonic
lines will be doublets (analogous to those of the alkali elements) because
their spins are \( 1/2 \). These splittings are altered because of the anomalous magnetic moments of the hadrons. The effects have been observed in sigmonic and antiprotonic atoms (Section 3.3).

In addition to the more intense principal lines \( (\Delta n = -1) \), X rays have been observed from transitions of \( \Delta n = -2 \) with about 10\% of the intensity of \( \Delta n = -1 \). Progressively weaker lines have been seen for transitions up to \( \Delta n = -4 \). (See Figure 3.)

For spin 0 hadrons, energies \( E_{n,l} \) are almost equal for constant \( n \), and the degeneracy for transitions between various \( l \) states has not been resolved experimentally. However, for high \( Z \) atoms \( \Delta E = (n, l = n-2 \rightarrow n-1, l = n-3) - (n, l = n-1 \rightarrow n-1, l = n-2) \) can be significant. For example: \( \Delta E = 0.5 \text{ keV} \) for \( U, n = 10 \).

### 3 EXPERIMENTAL FINDINGS

In this section we review experimental findings in the field of exotic atoms. Quite unexpectedly it was discovered that X-ray-intensity data exhibited an intricate competition between atomic processes (capture and cascade) and the strong nuclear interaction. Energy level shifts and line widths that are direct manifestations of the strong interactions were found as anticipated.

Exotic atoms serve as tools to learn more about the properties of negative hadrons (\( K^-, \bar{p}, \Sigma^- \)). In fact the best knowledge of some of these properties came from studies of exotic atoms (Section 3.3). Phenomena related to nuclear electric quadrupole moments are treated in Section 3.4. Results of the study of hadron-nucleon reaction products are reviewed in Section 3.5.

#### 3.1 X-Ray Intensities

In this section we discuss the absolute X-ray-intensity data. The relative intensities, which have been utilized to deduce the small widths of the next to last observed levels, are treated in Section 3.2.

3.1.1 KAONIC ATOMS Wiegand & Godfrey (7) have reported on an extensive survey of the absolute X-ray intensities in kaonic atoms of natural elements and pure isotopes ranging from \( Z = 2 \) through \( Z = 92 \). Except for \( ^4\text{He} \) (10) the data supersede that previously obtained by Wiegand and his collaborator (6, 11). Figure 5 summarizes the results of most of the \( \Delta n = -1 \) transitions. Here we see the mechanism of "kaonic atoms in action." The effect of the strong nuclear absorption is reflected in the rather sudden disappearance of X-ray lines for lower-\( n \) transitions. Disappearance in small \( Z \) for higher transitions is probably...
Figure 5  Intensity versus Z of observed principal ($\Delta n = -1$) kaonic X-ray lines (7).
present as a result of competition with Auger emission. The intensities observed are at most 0.5 X rays per stopped kaon and often much less, indicating that nuclear absorption is significantly active before the kaons reach observable transitions. Other striking features of the data are the variations or dips as a function of $Z$. Figure 4 suggests that the maxima occur near closed atomic electron shells. The origin of these deviations is unknown. A less conspicuous valley was observed in pionic X-ray intensities (12). Related kinds of variations were reported in intensity ratios of $(n = 3 \rightarrow 1)/(n = 2 \rightarrow 1)$ transitions in muonic atoms (13) and also in intensities of muonic metal atoms in oxide (14). In the data of (7) no significant differences were observed among the isotopes of the same element, although it might have been expected that $n, l$ states where nuclear absorption was strong would show more absorption for added neutrons. Apparently additional neutrons caused very little change in nuclear radii.

To illustrate the state of the art of X-ray spectroscopy of exotic atoms, Figure 3 shows a spectrum of kaonic Cl (target: CCl$_4$). Several sigmonic lines and a nuclear $\gamma$ ray are also visible.

### 3.1.2 Sigmonic Atoms

$\Sigma^-$-hyperons (negative sigmons) are produced as a result of $K^-$-nucleus absorption. In experiments where kaons are stopped, most of the sigmons that emerge from target nuclei remain within the targets and either decay in flight or are captured into sigmonic atoms. It has been reported that about 10% of the kaons stopped in photographic emulsion resulted in the ejection of $\Sigma^-$ (15) and about 14% made $\Sigma^-$ in CF$_3$Br (heavy liquid bubble chamber) (16). A theoretical calculation by Zieminska predicts about 12% to 3% roughly monotonously decreasing for an increase of $Z$ (17). The predominant reaction is $K^- + N \rightarrow \Sigma^- + \pi^+ - 0$ where $N$ stands for proton or neutron. Wiegand & Godfrey have measured the absolute intensities of some X-ray lines of sigmonic atoms (7), as shown in Table 1. X-ray intensities observed are less than 4% per stopped kaon.

### 3.1.3 Antiprotonic Atoms

No absolute intensity data for X-ray lines of antiprotonic atoms are available to date.

### 3.2 Level Shifts and Widths

Shifts and widths of the energy levels in exotic atoms are a direct consequence of the hadron-nucleus strong interaction. The shift in energy $\varepsilon$ is defined as the total calculated electromagnetic energy minus the experimentally measured energy. The high nuclear absorption rate $1/\tau$ is also manifested by line broadening, $\Gamma = h/\tau$ where $\Gamma$ is the line width.
Table 1  Measured absolute intensities of $\Sigma^-$ hyperonic X-ray lines per stopped kaon (7)

<table>
<thead>
<tr>
<th>Transition</th>
<th>I X Rays per K Stop</th>
<th>Error in Relative I</th>
<th>Error in Absolute I</th>
<th>Transition</th>
<th>I X Rays per K Stop</th>
<th>Error in Relative I</th>
<th>Error in Absolute I</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^6$Li</td>
<td>4 $\rightarrow$ 3</td>
<td>0.011</td>
<td>0.25</td>
<td>0.004 CCl$_4$</td>
<td>8 $\rightarrow$ 7</td>
<td>0.008</td>
<td>0.50</td>
</tr>
<tr>
<td>$^7$Li</td>
<td>4 $\rightarrow$ 3</td>
<td>0.009</td>
<td>0.24</td>
<td>0.003</td>
<td>7 $\rightarrow$ 6$^a$</td>
<td>0.023</td>
<td>0.11</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>4 $\rightarrow$ 3</td>
<td>0.012</td>
<td>0.39</td>
<td>0.006</td>
<td>6 $\rightarrow$ 5$^b$</td>
<td>0.021</td>
<td>0.07</td>
</tr>
<tr>
<td>$^{14}$N</td>
<td>5 $\rightarrow$ 4</td>
<td>0.021</td>
<td>0.35</td>
<td>0.008</td>
<td>4$^0$Ar</td>
<td>7 $\rightarrow$ 6$^a$</td>
<td>0.034</td>
</tr>
<tr>
<td>H$_2$O</td>
<td>5 $\rightarrow$ 4</td>
<td>0.005</td>
<td>0.40</td>
<td>0.002</td>
<td>6 $\rightarrow$ 5</td>
<td>0.029</td>
<td>0.26</td>
</tr>
<tr>
<td>$^{23}$Na</td>
<td>7 $\rightarrow$ 6$^a$</td>
<td>0.019</td>
<td>0.69</td>
<td>0.013</td>
<td>K</td>
<td>8 $\rightarrow$ 7</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>6 $\rightarrow$ 5</td>
<td>0.019</td>
<td>0.39</td>
<td>0.008</td>
<td>7 $\rightarrow$ 6$^a$</td>
<td>0.015</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>5 $\rightarrow$ 4</td>
<td>0.016</td>
<td>0.23</td>
<td>0.004</td>
<td>6 $\rightarrow$ 5</td>
<td>0.007</td>
<td>0.47</td>
</tr>
<tr>
<td>$^{27}$Al</td>
<td>6 $\rightarrow$ 5</td>
<td>0.021</td>
<td>0.36</td>
<td>0.008</td>
<td>Fe</td>
<td>7 $\rightarrow$ 6</td>
<td>0.023</td>
</tr>
<tr>
<td>S</td>
<td>7 $\rightarrow$ 6$^a$</td>
<td>0.021</td>
<td>0.10</td>
<td>0.004</td>
<td>5$^9$Co</td>
<td>7 $\rightarrow$ 6$^a$</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td>6 $\rightarrow$ 5</td>
<td>0.011</td>
<td>0.11</td>
<td>0.002</td>
<td>Ge</td>
<td>9 $\rightarrow$ 8</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Se</td>
<td>8 $\rightarrow$ 7</td>
<td>0.037</td>
</tr>
</tbody>
</table>

*a Includes K, $n = 10 \rightarrow 6$ which is estimated to contribute about 0.02 $I_{\text{max}}$, where $I_{\text{max}}$ is the most intense kaonic line of the target.

*b Possibly includes a contribution from $^{19}$F* nuclear $\gamma$ rays.
In practice $\varepsilon$ and $\Gamma$ are directly observed only in transitions to the lowest $n$ levels that undergo strong but not complete nuclear absorption. An example of kaonic X-ray spectra in which $\varepsilon$ and $\Gamma$ are directly observed is shown in Figure 6.

The technique of extracting the absolute energies and widths of the X-ray lines from experimental measurements is given in the review article on pionic atoms by Backenstoss (3). The same technique was used in analyses of X-ray lines of exotic atoms.

In addition to the Klein-Gordon or Dirac energy for the point Coulomb interaction, the total electromagnetic energy includes effects due to the following: vacuum polarization, finite charge distribution of the nucleus, shielding by atomic electrons, nuclear and hadron polarization, and the center-of-mass motion of the nucleus. Among these, vacuum polarization contributes most appreciably. For the purpose of energy shift determinations, vacuum polarization can be evaluated by first order perturbation theory in the lowest order of $\alpha(\alpha Z)$ with better accuracy than that allowed by the present errors in the transition energy measurement or the errors reflected from kaon mass (discussed later). If desired, higher
order corrections of vacuum polarization can be evaluated by a known technique (18).

In contrast to pionic atoms, the effect of finite charge distribution is small because of the large angular momentum and high \( n \)-values involved in exotic atoms and is computed safely by use of first order perturbation theory. The rest of the effects are normally negligible.

The largest uncertainty in the evaluation of energy shifts in kaonic atoms comes from the uncertainty in the kaon mass. The error enters through the Klein-Gordon equation where energies are directly proportional to the reduced kaon mass. As described in Section 3.3, only recently has the kaon mass been determined to \( 8 \times 10^{-5} \) accuracy (19) so as to yield lesser uncertainties in the total electromagnetic energies than most of the errors reported in X-ray energy measurements. At the same time we expect that experimental errors will decrease in forthcoming X-ray energy measurements. Considering this rapid improvement in the field, we list the shifts and widths in kaonic atoms as they have been

### Table 2  Energy level shifts and widths of \( K^- \) atoms

<table>
<thead>
<tr>
<th>Transitions</th>
<th>( E_{em}(\text{keV}) )</th>
<th>( \epsilon(\text{keV}) )</th>
<th>( \Gamma(\text{keV}) )</th>
<th>Ref.</th>
<th>( \Gamma_{up}(\text{eV}) )</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^{10}B ) 3–2</td>
<td>43.576</td>
<td>0.208 ± 0.035</td>
<td>0.81 ± 0.10</td>
<td>( ^a )</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( ^{11}B ) 3–2</td>
<td>43.776</td>
<td>0.167 ± 0.035</td>
<td>0.70 ± 0.08</td>
<td>( ^a )</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( ^{12}C ) 3–2</td>
<td>63.317</td>
<td>0.59 ± 0.08</td>
<td>1.73 ± 0.15</td>
<td>( ^a )</td>
<td>0.98 ± 0.19</td>
<td>( ^b )</td>
</tr>
<tr>
<td>Al 4–3</td>
<td>106.45</td>
<td>0.13 ± 0.05</td>
<td>0.49 ± 0.16</td>
<td>( ^c )</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Si 4–3</td>
<td>123.51</td>
<td>0.24 ± 0.05</td>
<td>0.81 ± 0.12</td>
<td>( ^c )</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( ^{31}P ) 4–3</td>
<td>142.354</td>
<td>0.33 ± 0.08</td>
<td>1.44 ± 0.12</td>
<td>( ^a )</td>
<td>1.94 ± 0.33</td>
<td>( ^b )</td>
</tr>
<tr>
<td>S 4–3</td>
<td>162.141</td>
<td>0.57 ± 0.18</td>
<td>2.50 ± 0.32</td>
<td>( ^c )</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>S 4–3</td>
<td>162.141</td>
<td>0.33 ± 0.15</td>
<td>2.23 ± 0.20</td>
<td>( ^c )</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>S 4–3</td>
<td>162.107</td>
<td>0.55 ± 0.06</td>
<td>2.33 ± 0.20</td>
<td>( ^a )</td>
<td>3.25 ± 0.41</td>
<td>( ^b )</td>
</tr>
<tr>
<td>Cl 4–3</td>
<td>183.351</td>
<td>1.08 ± 0.22</td>
<td>2.79 ± 0.25</td>
<td>( ^c )</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Cl 4–3</td>
<td>183.306</td>
<td>0.77 ± 0.40</td>
<td>3.8 ± 1.0</td>
<td>( ^a )</td>
<td>5.69 ± 1.50</td>
<td>( ^b )</td>
</tr>
<tr>
<td>Cl 4–3</td>
<td>182.23</td>
<td>0.94 ± 0.40</td>
<td>3.92 ± 0.99</td>
<td>( ^d )</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Ni 5–4</td>
<td>231.67</td>
<td>0.18 ± 0.07</td>
<td>0.59 ± 0.21</td>
<td>( ^e )</td>
<td>6.0 ± 2.3</td>
<td>( ^e )</td>
</tr>
<tr>
<td>Cu 5–4</td>
<td>248.74</td>
<td>0.24 ± 0.22</td>
<td>1.65 ± 0.72</td>
<td>( ^e )</td>
<td>7.1 ± 3.8</td>
<td>( ^e )</td>
</tr>
<tr>
<td>Pb 8–7</td>
<td>426.27</td>
<td>—</td>
<td>0.37 ± 0.15</td>
<td>( ^f )</td>
<td>4.1 ± 2.0</td>
<td>( ^f )</td>
</tr>
<tr>
<td>U 8–7</td>
<td>538.86</td>
<td>0.26 ± 0.40</td>
<td>1.50 ± 0.75</td>
<td>( ^f )</td>
<td>45.5 ± 24.0</td>
<td>( ^f )</td>
</tr>
</tbody>
</table>

\( ^a m_k = 493.73 \text{ MeV} \) (20).
\( ^b (21, 32). \)
\( ^c m_k = 493.84 \text{ MeV} \) (7).
\( ^d m_k = 493.87 \text{ MeV} \) (22).
\( ^e m_k = 493.715 \text{ MeV} \) (23).
\( ^f m_k = 493.715 \text{ MeV} \) (24).
reported. Each experimental group has used different kaon masses that were judged to be best at the time of publication. Table 2 includes this information.

X-ray energy measurements of antiprotonic S atoms have been made to an accuracy of $3 \times 10^{-4}$ (25). The $\bar{p}$ mass determination was reported with a little less accuracy (26). However, when the theorem of the conservation of parity, charge, and time reversal invariance (PCT) is assumed to set the antiprotonic mass equal to the protonic mass [accuracy $3 \times 10^{-6}$ (27)] the problem of the uncertainty of the $\bar{p}$ mass does not occur. At present the shifts and widths data of antiprotonic atoms are scarce and generally crude, as shown in Table 3.

No direct shift or width measurement has been made in sigmonic atoms.

Before continuing the discussion on line widths, we insert a few words on what is meant by “last observable transition” or “last observed X-ray line.” From the experimentalist's viewpoint, it means the lowest $n$ assigned to identified X-ray lines, but it might not be the lowest $n$ to which all hadrons arrived without being absorbed. The lowest observed intensity of an X-ray line may depend upon how much effort was put into detecting the lines. Also, broadening tends to cause the lines to blend into the background. For example, in Figure 5, $n = 7 \rightarrow 6$ transitions are not shown for $Z > 60$, but undoubtedly $7 \rightarrow 6$ X rays would be observed for higher $Z$ if more accelerator running time had been used.

Table 3 Energy level shifts and widths of $\bar{p}$ atoms

<table>
<thead>
<tr>
<th>Transition</th>
<th>$E_{em}$ (keV)</th>
<th>$\epsilon$ (eV)</th>
<th>$\Gamma$ (eV)</th>
<th>Ref.</th>
<th>$\Gamma_{up}$ (eV)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^4$He</td>
<td>$3 \rightarrow 2$</td>
<td>191 ± 170</td>
<td>(24)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^6$Li</td>
<td>$4 \rightarrow 3$</td>
<td>$\geq 2$</td>
<td>(24)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{14}$N</td>
<td>$4 \rightarrow 3$</td>
<td>55.824</td>
<td>39 ± 51</td>
<td>173 ± 34</td>
<td>(28)</td>
<td></td>
</tr>
<tr>
<td>$^{16}$O</td>
<td>$4 \rightarrow 3$</td>
<td>73.562</td>
<td>60 ± 72</td>
<td>648 ± 150</td>
<td>(28)</td>
<td></td>
</tr>
<tr>
<td>$P$</td>
<td>$5 \rightarrow 4$</td>
<td>140.50</td>
<td>80 ± 40</td>
<td>310 ± 180</td>
<td>(25)</td>
<td>1.14 ± 0.25</td>
</tr>
<tr>
<td>$S$</td>
<td>$5 \rightarrow 4$</td>
<td>8.00 ± 0.19</td>
<td>3.04 ± 0.51</td>
<td>(21)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Cl$</td>
<td>$5 \rightarrow 4$</td>
<td>26.8 ± 7.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K$</td>
<td>$5 \rightarrow 4$</td>
<td>3.07 ± 1.79</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Sn$</td>
<td>$8 \rightarrow 7$</td>
<td>9.93 ± 4.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I$</td>
<td>$8 \rightarrow 7$</td>
<td>24.7 ± 56.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The proton mass is used for $\bar{p}$ mass (see text).
and/or detector efficiency had been greater. On the other hand, in kaonic CI where the \( n = 4 \rightarrow 3 \) line is about 3 keV wide, we expect vanishingly few kaons to have arrived at \( n = 2 \). Therefore, we should use caution in applying the term “last observed.” Likewise, high \( n \) transitions may have failed to appear because of target self-absorption, and not necessarily because of X-ray emission being overwhelmed by Auger processes. Investigations using thin targets (less absorption, less background) await the availability of “kaon factories.”

For upper levels of last observed transitions the nuclear absorption rate is often comparable to the X-ray transition rate, yet its magnitude is too small to be observed by line broadening. The widths of such levels can be indirectly determined primarily from the relative intensities of the last and next-to-last observed X-ray lines. The determination of the upper level widths \( \Gamma_{\text{abs}}(n + 1, l + 1) \) is based on a simple observation that the nuclear absorption reduces the X-ray intensities of the last observed lines that consist almost entirely of \((n + 1, l + 1) \rightarrow (n, l)\) transitions between two circular states. Only orbits of maximum angular momentum are involved because of the strongly favored radiative transitions of \( \Delta l = -1 \) and because of the large (two orders of magnitude or more) nuclear absorption width for “parallel” \((n + 1, l) \rightarrow (n, l - 1)\) transitions (see Figure 7). The amount of the reduction is expressed as

\[
I_{\text{out}}/\sum l_{\text{in}} = \Gamma_{\text{rad}}/(\Gamma_{\text{rad}} + \Gamma_{\text{abs}}(n + 1, l + 1)),
\]

where \( \Gamma_{\text{rad}} \) is the X-ray width of the \((n + 1, l + 1) \rightarrow (n, l)\) transition, and is computed by a standard dipole approximation (29) with a correction factor for the center-of-mass motion of the nucleus (30, 31).\(^1\) The Auger width of this transition is normally negligible. \( I_{\text{out}} \) is the X-ray intensity of the \((n + 1, l + 1) \rightarrow (n, l)\) transition, while \( \sum l_{\text{in}} \) is the sum of all X-ray and Auger transition intensities coming to the \((n + 1, l + 1)\) state.

\( I_{\text{out}} \) and \( \sum l_{\text{in}} \) are obtained from the X-ray spectrum after subtracting the background, correcting for detection efficiency and target self-absorption. Detector efficiency and target attenuation are well-measured factors. For weak lines the largest error comes from the statistics in the numbers of X rays and background. Also determination of the background is endangered by unexpected lines from such sources as nuclear \( \gamma \) rays. Furthermore, an accurate value of \( \sum l_{\text{in}} \) has to rely on a cascade calculation because X-ray transitions with \( |\Delta n| \) larger than about 4 or 5

\(^1\) In the field of mesonic and hadronic atoms, this large factor (e.g. about 2 for \( \bar{p} \) atoms) had been curiously neglected for a decade since the original paper by Fried & Martin (30). Most publications on exotic atoms prior to the summer of 1973 that deal with de-excitation rates must be corrected to include this factor.
and the Auger transitions are not observed. The calculated corrections are reported to amount to about 10% in kaonic atoms of light nuclei (32). We note that the correction has to be treated with caution because it inherits the uncertainties associated with cascade calculations (see Section 4.1).

The values of $\Gamma_{\text{abs}}(n+1, l+1)$ thus determined are listed as $\Gamma_{\text{up}}$ in Tables 2–4.

### 3.3 Particle Properties Derived from Exotic Atoms

Precise measurements of the radiative transition energies provide information on some particle properties that are less accurately known by other
means. In order as much as possible to avoid uncertainties arising from
the evaluation of nuclear effects, strong interaction, and nuclear polariza-
tion, the measurements are done on high-$n$ transitions that are far from
the nucleus. At the same time the transitions must not be too high,
because then the effect of electron screening will become important.
Corrections for shielding and finite nuclear size are usually so small as
to be almost negligible. They are evaluated by standard procedures. How-
ever, the choice of higher transitions causes a complication. The high $n$
levels are almost, but not quite, degenerate in energy. Some of the steps
in de-excitation involve noncircular orbits and thereby introduce X rays
of slightly different energy than those of circular orbits. Here we must
rely on cascade calculations with their inherent uncertainties in order to
determine the amount of mixing-in of “parallel” transitions as well as
Auger transitions.

3.3.1 MASS The Klein-Gordon (or Dirac) energies for the point
Coulomb interaction are directly proportional to the reduced mass of the
atoms. The transitions chosen are almost entirely dominated by these
energies. Correction for vacuum polarization amounts to several tenths
of a percent but can be computed to the desired accuracy. Therefore,
the accuracy of the hadron mass determination from exotic atoms is
practically the same as the accuracy of the energy measurement of the
chosen X-ray lines.

3.3.1.1 $K^-$ Currently the $K^-$ mass as determined by kaonic atom
measurements is considerably more accurate than that determined by
other means. It is

$$m_K c^2 = 493.691 \pm 0.040 \text{ MeV},$$

claimed by Backenstoss et al (19) from studies of kaonic Au and Ba
atoms. The determination from emulsion tracks remains at $\pm 0.17$ MeV
(27). In these kaonic atoms the transition energies are determined to an

<table>
<thead>
<tr>
<th>Transitions</th>
<th>$\Gamma_{up}$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C 4–3</td>
<td>0.031 ± 0.012</td>
</tr>
<tr>
<td>Ca 6–5</td>
<td>0.40 ± 0.22</td>
</tr>
<tr>
<td>Ti 6–5</td>
<td>0.66 ± 0.43</td>
</tr>
<tr>
<td>Ba 9–8</td>
<td>1.68 ± 3.60</td>
</tr>
</tbody>
</table>
average error $\Delta E/E$ of about $10^{-4}$. The old mass was corrected essentially by finding $\Delta m$ in

$$E_{\text{exp}} = (1 + \Delta m/m_0)E_{\text{calc}},$$

where $m_0 = 493.750$ MeV.

The final error ($8 \times 10^{-5}$) was obtained by taking a weighted average of the measurements. Note that the world average is (27)

$$m_K c^2 = 493.707 \pm 0.037 \text{ MeV}.$$  

3.3.1.2 $\bar{p}$ When the first observation of antiprotonic atoms was made in 1970 (26) the equality of the proton and antiproton masses was confirmed within $\pm 0.5$ MeV. This verification of the charge independence-parity-time invariance (CPT) theorem to an error of $\pm 5 \times 10^{-4}$ could be improved within the present status of the art.

3.3.1.3 $\Sigma^-$ Owing to the low intensities of sigmonic X-rays, a precise energy measurement of the lines leading to an improvement in the $\Sigma^-$ mass is difficult. The measured energies are in agreement with the $\Sigma^-$ mass obtained by other means. The world average is (27)

$$m_{\Sigma^-} c^2 = 1197.35 \pm 0.06 \text{ MeV}.$$  

3.3.2 MAGNETIC MOMENT The fine structure splitting in exotic atoms of spin-$\frac{1}{2}$ negative baryons is given by (29)

$$\Delta E = \frac{\hbar}{2}(1 + 2g_1)\mu c^2(xZ/n)^4 n^4 [i(l + 1)]$$

between $j = l + \frac{1}{2}$ and $l - \frac{1}{2}$ states of the same $l$. Here $g_1$ is the anomalous part of the magnetic moment in the unit of the baryon magneton and $\mu$ is the reduced mass of the atom. The magnetic moment of a negative baryon, $s$, of mass $m_s$ is written as $\mu_s = -(1 + g_1)$ in units of the baryon magneton, $e\hbar/2m_s c$. When $g_1 > -\frac{1}{2}$, the binding energy of $j = l + \frac{1}{2}$ is always less than that of $l - \frac{1}{2}$.

Equation 4 is obtained by first order perturbation theory in the Pauli approximation. The restriction $(v/c)^2 \approx (xZ/n)^2 \ll 1$ exists, where $v$ is the hadron speed in the atom. States of exotic atoms between which radiative transitions are observable satisfy this condition.

Exotic atoms convenient for measurement of fine structure splitting are large $Z$ nuclei where $Z/n$ would not be too small to be observed even for relatively higher-$n$ transitions. A precise measurement of the fine structure splitting in these atoms is difficult, especially for sigmonic atoms. In $\bar{p}$ atoms the doublet lines are resolvable, whereas in sigmonic atoms the instrumental resolution is larger than the amount of the splitting. Contamination by noncircular transitions is also a problem. Furthermore,
the sigmonic X-ray lines are found only in kaonic X-ray spectra where
the kaonic lines are more than an order of magnitude more intense. In
order to unfold the fine structure splitting, one has to perform a subtle
computer fitting that is influenced directly by results of a cascade calcu-
lation. Moreover, the results of such analyses have an ambiguity caused
by the fact that the measurements of \( \Delta E \) themselves do not determine
the sign of \( \pm (1 + 2g_1) \). In principle, the ambiguity could be removed by an
intensity difference in two major transitions between the fine-structure
doublets (33–36) when statistical population of the levels is assumed (29).
However, the difference is so small that the analyses of data (33–36) have
yielded two statistically acceptable values of the magnetic moments that
are of the opposite sign with slight preference for the negative value.
The results are presently reported to be (33, 34 respectively)

\[
\mu_\bar{p} = -2.819 \pm 0.056 \text{ N magnetons}
\]

and

\[
\mu_\Sigma^- = -1.89 \pm 0.47 \text{ \( \Sigma \) magnetons.}
\]

The errors assigned are defined to increase \( \chi^2 \) by one. For \( \bar{p} \), \( \chi^2 = 43 \)
for 57 degrees of freedom; for \( \Sigma^- \), \( \chi^2 = 172 \) for 144 degrees of freedom.
The other minima that yield positive magnetic moments occur with
\( \chi^2 = 50 \) (\( \bar{p} \)) and 173 (\( \Sigma^- \)). Each \( \chi^2 \) curve is almost symmetric with two
deep minima. Although the small errors are assigned, one has to be aware
of the complexity of the analyses and the ambiguity in the results;
especially for \( \Sigma^- \). The previous value of \( \mu_\bar{p} \) was \(-2.83 \pm 0.10 \text{ N }
\]

magnetons reported in 1972 (35), and that of \( \mu_\Sigma^- \) was between 1.0 and

\(-2.0 \Sigma \) magnetons reported in 1973 (36).

When the PCT theorem is assumed, the magnetic moment of \( \bar{p} \) is
the negative of the proton \((2.7928456 \pm 0.0000011) \text{ N magnetons (27) in}
agreement with the \( \bar{p} \) atoms result, equation 5. SU(3) theory predicts that
the magnetic moment of \( \Sigma^- \) has a value equal to the negative difference
between the proton and neutron magnetic moments \( \mu_\Sigma^- = -(\mu_p - \mu_n) = 
-0.88 \Sigma \) magnetons. According to the equal-mass quark model, \( \mu_\Sigma^- = 
-\frac{1}{2}\mu_p = -0.93 \Sigma \) magnetons (37). Equation 5 disagrees with these values.

3.3.3 POLARIZABILITY The strong electric field created between the
nucleus and the hadron in an exotic atom may induce a polarization in
the hadron structure (38–40) in addition to the nuclear polarization. The
energy shift resulting from these induced polarizations is estimated to be,
in the adiabatic treatment (39),

\[
(\Delta E)_{\text{pol}} = -\frac{1}{2}e^2(p_N + Z^2p)\langle r^{-4} \rangle.
\]

where \( p_N \) and \( p \) are the nuclear and hadron polarizabilities respectively.
Equation 6 shows a large enhancement factor $Z^2$ on $p$ that may make a direct observation of $p$ possible.

When the precise measurement of the kaon mass was made (19), such an attempt was undertaken. There was a difference of a few tens of eV between the measured and calculated transition energies, which was attributed to $(\Delta E)_{\text{pol}}$. By use of $p_N$ estimated from photonuclear cross sections (39), an upper limit of $p$ defined in Equation 6 was found to be 0.020 $F^3$ at 90% confidence (19).

The experiment was quite difficult, and it may not be justifiable to place significance on the value comparable to the experimental error. However, to observe the hadron structure directly is an exciting possibility.

3.4 Atomic Phenomena Directly Related to Nuclear Properties

3.4.1 Nuclear Quadrupole Moment An effect of hyperfine structure splitting due to the electric quadrupole moment of a nucleus has been seen in kaonic atoms. When X-ray lines of $n = 8 \rightarrow 7$ transitions in $^{167}$Er and $^{170}$Er were compared, a clear broadening in the $^{167}$Er line in contrast to the one of $^{170}$Er was observed (7). Note that $^{167}$Er has spin 7/2 and quadrupole moment of $2.83 \times 10^{-24}$ cm$^2$, whereas $^{170}$Er has spin zero. The components are not resolved because of instrumental resolution.

3.4.2 Dynamical Quadrupole ($E2$) Mixing When a massive negative particle cascades down in an atom to reach the vicinity of the nucleus, it can couple with the nucleons in the nucleus and cause combinations of nuclear excitations and nonradiative transitions in which the angular momentum of the total system, nucleus + hadron, is conserved. That is, the wave function of the total system becomes an admixture of the various atomic states and nuclear excited states, and the energy of the total system is perturbed from that of no admixture. The strength of the admixture is proportional to the ratio of the coupling strength and the energy difference between the mixed states. Thus occurs the nuclear polarization, and its effect is normally small. Deformed nuclei with large quadrupole moments, however, can induce a strong coupling involving a large admixture of angular momentum states of $\Delta j = 2$, especially when the energy difference between nuclear excited states and atomic transitions is small. This phenomenon is called dynamical quadrupole (or $E2$) mixing. In muonic atoms $E2$ mixing shows up prominently in some states involving fine structure doublets of small $n$ and $l$, e.g. $2P_{1/2}$ and $2P_{3/2}$ states (2).

In exotic atoms the phenomenon is less prominent because the strong
interaction prevents hadrons from reaching small angular momentum states. Recently an observation of dynamical quadrupole mixing has been reported in kaonic uranium atoms (41). The mixing was identified by the fact that the transition energies were perturbed by the amounts predicted from a detailed theoretical calculation (42).

The admixed states in exotic atoms may involve atomic states of small angular momentum that, as a result of nuclear absorption, cannot be seen. Consequently, X-ray intensities of transitions from mixed states may be reduced drastically when the admixture is large. Recently Leon (43) proposed to utilize this method to gain additional understanding of the \( K^- \)-nucleus strong interaction by predicting X-ray intensity reductions in several specific transitions.

Late in 1974 the effects of \( E2 \) nuclear resonance were seen in pionic (44) and kaonic (45) atoms. The Berkeley group (45) obtained kaonic X-ray spectra of \( ^{98}\text{Mo} \) and \( ^{95}\text{Mo} \). The intensity ratio \( I(n = 6 \rightarrow 5)/I(n = 7 \rightarrow 6) \) in \( ^{98}\text{Mo} \) was predicted to be attenuated from 0.93 (without mixing) to 0.18 (with mixing). The experimental ratio was found to be 0.35 ±0.2. A control target \( ^{95}\text{Mo} \) also showed attenuation, but the reason for this is not understood. The importance of these observations is that a new method has been found to obtain information on previously inaccessible states in hadronic atoms, and thereby to verify calculations of hadron-nucleus strong interactions.

3.5 Products from Nuclear Absorption of \( K^- \), \( \Sigma^- \), and \( \bar{p} \)

3.5.1 PARTICLES OBSERVED IN NUCLEAR EMULSIONS AND BUBBLE CHAMBERS Since the late 1950s there have been extensive investigations on particles produced by \( K^- \)-nuclear absorption in photographic emulsions designed to show particle tracks (4, 46−53) and in bubble chambers filled with mixtures of Freon (\( \text{CF}_3\text{Br} \)) and propane (\( \text{C}_3\text{H}_8 \)) (54, 55), pure Freon (15, 56), neon (57, 58), helium (59, 60), and deuterium (61). Recently a hydrogen bubble chamber was used to detect charged pions produced by the nuclear absorption of negative kaons (62) and antiprotons (63) in several solid targets. An investigation was also made specifically on products of \( \Sigma^- \)-nuclear absorption in emulsion in which \( K^- \) were stopped (64). Results of these investigations would be of importance in learning the distribution of nuclear matter if the reaction products could be assigned to specific hadron-nucleon interactions that occurred in the nucleus. Unfortunately there are complications. In many cases the reaction products undergo secondary reactions before they escape from the nuclei under consideration. We mention the major secondary reactions and estimates of their occurrences not that they may be used as a basis for new analyses, but simply to indicate
their general features: (a) $\Sigma$ to $\Lambda$ conversion ($\Sigma^{-} + N \rightarrow \Lambda + N$) occurs for about 0.5 of the $\Sigma^{-}$ produced, (b) $\Lambda$ trapping (hypernucleus formation) occurs 0.2 to 0.35 per $\Lambda$ produced, (c) pion absorption occurs for 0.08 to 0.15 per pion produced, (d) $\Sigma$ and $\Lambda$ decays occur with known partial decay modes.

Besides the estimate of the secondary reaction occurrences, there is a question of bias and detectability of the particles in the experimental techniques. For these reasons some reports of the primary process occurrences published in the past have been questioned because of later findings. See, for example, (53, 65, 66). Under these circumstances we show in Table 5 the primary process occurrences obtained from Freon (16) and Freon + propane (55) bubble chamber studies as these two relatively recent and independent results agree quite well.

Burhop (65, 66) made careful studies of $K^{-}$-emulsion and $K^{-}$-bubble chamber data to review the values of the quantities

$$R_{pn} \equiv \frac{N(\Sigma^{+}+\pi^{-})+N(\Sigma^{-}+\pi^{+})}{N(\Sigma^{-}+\pi^{0})}$$

and

$$R_{+} \equiv \frac{N(\Sigma^{+}+\pi^{-})}{N(\Sigma^{-}+\pi^{+})}.$$ 

### Table 5 Primary processes and their occurrences in $K^{-}$ nuclear absorption

<table>
<thead>
<tr>
<th>Primary Processes</th>
<th>Occurrences$^a$ CF$_3$Br + C$_3$H$_8^b$</th>
<th>Occurrences$^a$ CF$_3$Br$^c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^{-} + p \rightarrow \Sigma^{-} + \pi^{+}$</td>
<td>$10.3 \pm 1.7$</td>
<td>$11.1 \pm 0.5$</td>
</tr>
<tr>
<td>$\rightarrow \Sigma^{+} + \pi^{-}$</td>
<td>$14.4 \pm 2.3$</td>
<td>$11.3 \pm 1.4$</td>
</tr>
<tr>
<td>$\rightarrow \Sigma^{0} + \pi^{0}$</td>
<td>$11.8 \pm 1.4$</td>
<td>$9.6 \pm 1.5$</td>
</tr>
<tr>
<td>$\rightarrow \Lambda + \pi^{0}$</td>
<td>$13.3 \pm 1.1$</td>
<td>$11.8 \pm 1.2$</td>
</tr>
<tr>
<td>(11.8 $\pm 1.0$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K^{-} + n \rightarrow \Sigma^{-} + \pi^{0}$</td>
<td>$1.2 \pm 0.4$</td>
<td>$3.2 \pm 0.5$</td>
</tr>
<tr>
<td>$\rightarrow \Sigma^{0} + \pi^{-}$</td>
<td>$1.2 \pm 0.4$</td>
<td>$3.2 \pm 0.5$</td>
</tr>
<tr>
<td>$\rightarrow \Lambda + \pi^{-}$</td>
<td>$26.6 \pm 2.1$</td>
<td>$23.6 \pm 2.4$</td>
</tr>
<tr>
<td>(23.6 $\pm 1.9$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multinucleon absorption</td>
<td>$21.3 \pm 2.5$</td>
<td>$25.7 \pm 3.1$</td>
</tr>
</tbody>
</table>

$^a$ The total $K^{-}$ absorption events = 100.

$^b$ The $\pi$ absorption rates 0.1 (with $\Sigma$) and 0.2 (with $\Lambda$) per $\pi$ are assumed except the numbers in parentheses for which 0.1 (with $\Sigma$ or $\Lambda$) is assumed (55).

$^c$ Reference 16.

$^d$ Not given.
Table 6  Significant quantities of primary processes in $K^-$ nuclear absorption (see text on the definitions of the quantities)

<table>
<thead>
<tr>
<th></th>
<th>Bubble Chamber</th>
<th>Emulsion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H</td>
<td>D</td>
</tr>
<tr>
<td>$R_{pn}^a$</td>
<td>2.3 ± 4</td>
<td>10.5 ± 4</td>
</tr>
<tr>
<td>$R_{+ -}^a$</td>
<td>0.43</td>
<td>1.2 ± 0.3</td>
</tr>
<tr>
<td>Multinucleonb</td>
<td>~1.2c</td>
<td>16.5 ± 2.6d</td>
</tr>
<tr>
<td>(non-pion)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a$ Reference 66.

$^b$ Out of the total $K^-$-absorption events (= 100).

$^c$ Reference 68, not corrected for primary multinucleon processes. The corrections are expected to be small.

$^d$ Reference 60.

$^e$ Reference 58.

$^f$ Reference 55. See footnote $^b$ in Table 5.

$^g$ Reference 51.
where the $\mathcal{N}$ represents an occurrence of the primary processes labeled by their final states. Because of charge conservation, emission of $\Sigma^+\pi^-$ or $\Sigma^-\pi^+$ implies that the reaction was $K^-+p$. In Table 6 we list the values taken from (66). Note that the sum of these three reactions (emission of charged $\Sigma$) represents only a fraction of the total $K^-$ absorption—about 25% in CF$_3$Br, as seen in Table 5.

These quantities are important since they depend critically on the $K^-$-nucleon interactions in the nucleus. For example, the ratio of $R_{pn}$ for light to heavy nuclei is about $5$. The large ratio suggests that neutron absorption is either stronger or more frequent than proton absorption. If the latter, the nuclear surface would be rich in neutrons, as originally suggested by Johnson & Teller (5). After a controversy extending over many years this problem has been found to be subject to a frustrating complication and is not yet settled. We discuss it further in Section 4.3.

The recent measurement of charged pions emitted from kaon absorption in targets placed inside a hydrogen bubble chamber augments the previous $K^-$-nuclear emulsion and heavy liquid bubble chamber data. The rates of total $K^-$ absorption occurrences by neutrons and protons are determined by combining all of this information. Here also the deductions are subject to the question of reliability in sorting out the primary absorptions. The ratios of proton to neutron absorption were found to be 0.63 for C, 1.08 for Ti, 4.96 for Ta, and 4.01 for Pb (62).

Charged pions resulting from antiproton absorption were also measured in a similar way in a hydrogen bubble chamber (63); from these data the ratios of total $\bar{p}$ absorption in various nuclei by neutrons and protons were deduced. It has been shown in a simple model, however, that the energetic pions produced by $\bar{p}$ absorption undergo appreciable charge exchange reactions within the nuclei (67). In view of this finding, the charged pion data for $\bar{p}$ absorption have to be reanalyzed.

We turn now to kaon absorption by more than one nucleon. Rates of two-nucleon reactions of the type $K^-+N+N \rightarrow N+(a\ hyperon)$ have been obtained for some nuclei. These absorptions do not accompany pions and produce distinctly higher energy nucleons than those from one-nucleon absorptions. The rates are listed in Table 6 as reported in the literature. We discuss the problem of two-nucleon absorptions in Section 4.3.

3.5.2 Nuclear $\gamma$ Rays Interest in nuclear $\gamma$ rays induced by stopped negative kaons is concerned with the formation of excited nuclei. High resolution semiconductor detectors give the energies of the lines with accuracy sufficient to identify unambiguously the excited nuclear states responsible for the radiation. In the energy range less than 200 keV,
28 nuclear \( \gamma \) rays have been identified throughout the periodic table (7). Between 750 and 1500 keV, about 25 nuclear \( \gamma \) rays emitted from kaons stopped in Ni and Cu have been identified (69). Unfortunately one cannot tell how many of these \( \gamma \) rays result from the primary reactions—that is, how many come from the nuclei of the atoms in which K\(^-\) are captured—and how many result from secondary reactions, in which the particles produced by the primary reactions are absorbed in other target nuclei. To resolve this problem it is necessary to study \( \gamma \)-ray production versus target thickness. This has been done only for \(^{55}\text{Mn}\) using pions (70) and kaons (7). It was found that the intensities per stopped meson depended linearly on target thickness. In this particular case, secondary particles were responsible for the \( \gamma \) rays. Some speculations have been made concerning the processes of the excited state formation, specifically, \( \gamma \)-ray emission after formation of hypernuclei (7) and the removal of one, two, or three \( \alpha \) particles (69). These types of excitation are of interest because large momentum transfer components of the nuclear wave functions are involved. In particular, recent \( \pi^\pm\)-nucleus reaction experiments reported results that tend to confirm multiparticle removal (71–73); but the interpretation of another experiment seems to disagree (74). Further effort to produce concrete evidence is needed.

In order to identify the K\(^-\)-neutron and K\(^-\)-proton absorptions specifically, some nuclear \( \gamma \)-ray lines have been theoretically predicted from kaons stopped in \(^{16}\text{O}\) and \(^{208}\text{Pb}\) (75). These \( \gamma \)-ray lines have not yet been observed.

4 THEORETICAL UNDERSTANDING

The idea that kaonic atoms would be a useful means for investigating the nuclear surface was originally proposed by Jones in 1958 (76) and subsequently advanced strongly by Wilkinson (77, 78). In 1959 Wilkinson (77) pointed out that nuclear emulsion data of kaon capture products could be interpreted as the formation of \( \alpha \)-particle clusters in the nuclear surface.

The experimental and theoretical investigations that followed have revealed unexpected complications. The first is that the K\(^-\)-proton scattering is so highly energy dependent as to cause a severe difficulty for constructing the K\(^-\) nucleus optical potential in relation to the nuclear density distribution. The multichannel nature of the K\(^-\)-nucleon interactions further deepens this difficulty. At present no reliable optical potential exists that agrees with all experimental data and at the same time stands on overall firm theoretical ground. The second complication comes from the fact that less than half of the kaons captured in target
atoms were observed in the X-ray intensity experiments (7). An understanding of the atomic cascade scheme thus becomes essential if we are to determine from which atomic states the kaon is absorbed by the nucleus. Unfortunately the cascade scheme depends directly on how the kaon is captured in the atom. We thus face a highly complicated problem in atomic physics. At present this problem is very poorly understood. We present a brief account of previous theoretical investigations, including what has been learned from them. As less work has been done on the other exotic atoms, we primarily discuss kaonic atoms.

### 4.1 Atomic Capture and Cascade

The atomic capture processes for kaons are believed to be described in the same way as for pions and muons except that muons do not undergo strong interaction. The theoretical understanding of these complicated atomic processes is not an easy task. This is particularly true in the case of the capture process. Currently, however, some advances are being made in the study of this field. For the kaonic cascade calculation the same methods have been used as those for the muonic and pionic cascade processes. Some short reviews of the latter are available (3, 79, 80) and we refer the reader to these articles for further details.

#### 4.1.1 Atomic Capture

When a kaon is captured in an atom, perhaps primarily via an Auger process, the kaon velocity is smaller than the electron velocities in the atom. Most of the quantum mechanical calculations that have been done are based on Born approximation (81–84) and for this reason do not seem to give reliable results. Seemingly the most systematic and reliable current account of the capture process is given by a semiclassical model of Fermi & Teller (85–89). This model is constructed in the adiabatic limit. The Auger process is described as a continuous loss of the meson's energy and angular momentum by dissipation of the zero temperature electron gas to states above the Fermi level. Radiative processes can be accounted for by the classical radiation formula of an accelerated charge.

Based on this model, lengthy computer calculations have been performed by Leon & Seki (88) for a completely isolated single atom, and subsequently by Haff et al (89) for an agglomeration of neutral atoms. The former authors found that, as a consequence of the atomic capture, the population distribution \( P(l) \) of kaons just inside the electronic ground state orbit is strongly suppressed for the circular and nearly circular kaonic states. A cascade calculation making use of this \( P(l) \) gave crude agreement with the kaonic X-ray intensity data, but did not reproduce the \( Z \)-dependence structure. In a more realistic model of the target, the latter authors found that the energy distribution of the muon captured by an
atom in agglomeration is concentrated in a small range so as to yield an almost statistical distribution of $P(l)$. This $P(l)$ is expected, however, to yield about twice as large kaonic X-ray intensities as are found by experiment. On the other hand, during the atomic capture process in congregated atoms, the nuclear absorption of kaons is expected to take place when the perigee of the kaon orbits is close to the nuclei in the target. It is not known how many kaons are lost by this mechanism. The loss due to kaon decay was found experimentally to be insignificant.

In these calculations the meson is assumed to experience the Thomas-Fermi potential or a similar potential. Even if the known Hartree-Fock type potentials are adopted instead, these approaches may not sufficiently include the dynamical atomic structure. Meson capture involves the outer region of the atom whose dynamical response to the meson is an unexplored domain of atomic physics. The exotic atoms may turn out to be a useful means of studying the atomic surface. It has in fact been recognized that the Z-dependence structure in the kaonic X-ray intensities follows the same pattern as the statistic atomic size for metals. Beyond speculation, no detailed study has yet been made along this line.

The rate of the primary capture process (Auger) depends critically on how many electrons are available during the capture. Although in metals electron vacancies would be refilled instantaneously, electron depletion is a completely open question involving further areas of molecular and solid state physics.

The Fermi-Teller model is based on a simplification intended to replace the probability distribution of possibly discrete meson energy losses by a continuous energy loss. The attractive illustration of a definite trajectory of the meson is certainly a distortion. We do not know how much distortion is introduction in $P(l)$. Clearly, further work is needed in this area.

4.1.2 ATOMIC CASCADE A cascade calculation describes successive de-excitations of a kaon in a hydrogenic atom. Cascades are usually started inside the shell of the electronic atom's ground state where $n_K < 30$ ($n_K \approx (m_K/m_e)^{1/2}$). Because no confident calculation of the capture process is available, one has to assume a form of the initial population distribution $P(l)$. It is often assumed to be the convenient form

$$P(l) \propto (2l+1) \exp(al).$$

where $a$ is an adjustable parameter, $a = 0$ corresponding to the statistical distribution. The value of $a$ is typically less than 0.2, tends to be positive, and varies from atom to atom (32, 91–93).

The cascade calculation then follows the fate of the kaon that descends
via Auger and radiative (X-ray) transitions. The transition rates are safely computed by first order perturbation theory. Ferrell's formula (94),

$$\frac{\Gamma_{\text{Auger}}(\omega)}{\Gamma_{\text{rad}}(\omega)} = \left(\frac{3c^4}{8\pi}\right)\frac{\sigma_{\text{photo}}(\omega)}{(Z_1^2 - 1)}$$

can also be used for the transition energy, $\omega$, where experimental data of the photoelectric cross section for the neutral $Z - 1$ atom, $\sigma_{\text{photo}}(\omega)$, are available. The Auger transitions, whose rates are proportional to $\omega^{-1/2}$, dominate for large $n$ while radiative transitions, proportional to $\omega^3$, dominate for small $n$, and both transitions tend to favor $\Delta l = -1$. As with the capture process, the Auger transition rates cannot be accurately obtained unless the electron depletion problem is well understood. This seems to be the major uncertainty in the present status of the cascade calculation (92, 95).

The nuclear absorption rates are computed in practice by use of phenomenologically introduced optical potentials. As discussed in Section 4.2, these potentials are not based on a firm theoretical foundation, but seem to agree well with the experimental data. Since the basic cascade scheme is not sensitive to minor variations in the nuclear absorption rates, a phenomenological form of the potentials would not give substantially distorted results. By use of such a potential it is found (88) that nuclear absorption dominates in small angular momentum states throughout all $n$'s, and that only a fraction of kaons are absorbed from the last-observed circular state and often overwhelmingly more from the next-to-last state.

In Figure 7 we show a result of a cascade calculation that illustrates how kaons cascade down and where they are absorbed. We see a prominent competition between atomic and nuclear effects.

### 4.2 Strong Interactions with Nuclei

As discussed in the previous section, strong interactions play an important role in the atomic capture and cascade, but no reliable means is yet known for isolating the effects of the strong interaction. On the other hand, the energy-level shifts and widths of the last observed circular states are believed to be independent of these atomic processes. They are extensively used for the study of the strong interactions. The widths of the next-to-last observed circular states are also used. As discussed in Section 3.2, these widths are determined indirectly from the relative intensity data. The determination depends on atomic processes, and care must be taken in this respect.

The formation of hadronic hydrogen-like atoms via the attractive Coulomb interaction prepares the hadrons to interact with the nuclei in a way fundamentally different from the strong interactions in scattering (1). The hadrons interact with nuclei at definite kinetic energies of less
than a few MeV (2). The hadrons interact with nuclei in definite angular momentum states of non-zero values, except with the proton for which the \( l = 0 \) interaction is expected.

The observed widths and shifts reflect the strength of the strong interactions in the same way as the phase shifts do in scattering. Hadrons in non-zero angular momentum states of exotic atoms overlap the nuclei mainly in their surface region (equation 2 and Figure 4). This fact has encouraged the idea that these shifts and widths would also reflect the nuclear surface distribution. In order to proceed with this idea one has to establish a reliable means of relating the strong interaction potential to the nuclear density.

### 4.2.1 \( K^- \)-nucleus optical potential

#### 4.2.1.1 Phenomenological potentials

The simplest means of constructing the \( K^- \)-nucleus optical potential is to assume the \( K^- \)-(ith) nucleon interaction \( v_i(K^-N) \) to be point-like, described in the form of a pseudo-potential,

\[
v_i(K^-N) \approx -4\pi(\hbar^2/2\mu_K) f_i(K^-N) \delta^3(r_i - r_K),
\]

so that in Born approximation the \( K^- \)-nucleon amplitude is exact at the energy and momentum of interest. In equation 8 \( \mu_K \) is the reduced mass of the kaon and nucleon system, \( \mu_K = m_K m_N/(m_K + m_N) \) and \( r_i \) and \( r_K \) are the coordinates of the ith nucleon and the kaon respectively. Once \( v_i(K^-N) \) is set, the \( K^- \)-nucleus potential \( V \) is obtained as

\[
V = \sum_i \langle \psi | v_i | \psi \rangle
\]

with neglect of nuclear excitations, \( \psi \) is the ground-state nuclear wave function and \( \langle \rangle \) denotes integrations over \( r_i \)'s.

For the sake of simplification one makes a further approximation,

\[
f_i(K^-N) = -a,
\]

where \( a \) is the \( K^- \)-(ith) nucleon scattering length in free space. The negative sign is a nuclear physics convention, while most of the literature in elementary particles adopts the opposite sign. Nuclear physics convention always gives \( Ima \leq 0 \) and, when the potential is weak, this convention yields the same sign of the real parts of the potential and its scattering length. The approximation, \( f(K^-N) = -a \), involves neglect of all nuclear effects on \( f(K^-N) \) such as on- and off-shell energy, momentum dependence, and nuclear correlation. The \( K^- \)-nucleus optical potential is then obtained in the form

\[
V = 4\pi(\hbar^2/2\mu)(m_K/\mu_K)(Za_n \rho_p(r) + Na_n \rho_n(r)),
\]

2 Mesonic atoms are "natural angular momentum selectors," as depicted by T. E. O. Ericson.
where $a_p(n_a)$ and $\rho_p(n_p)$ are the $K^-$-proton (neutron) scattering lengths and the proton (neutron) density distribution normalized to unity, respectively; $\mu$ is the reduced mass of the kaon-nucleus system. The kinematic factor $m_K/\mu_K$ arises due to a shift of the center of mass from $K^-$-nucleon to the $K^-$-nucleus system.

After the various approximations above, the nuclear potential of equation 9a is now proportional to $\rho_n(r)$ and $\rho_p(r)$, and its form is convenient for investigating them. Nevertheless, before pursuing the investigation, one has to insure that the potential form of equation 9a is reliable, at least in the surface region where the potential is expected to be most sensitive to the kaonic atom data. In practice, the potential has generally been applied in a form

$$V = 4\pi(h^2/2\mu)(m_K/\mu_K)A\tilde{\rho}(r),$$

where $\tilde{a} = (Za_p + Na_n)/A$ and $A = Z + N$. The further assumption leading to equation 9b from equation 9a is $\rho_n(r) = \rho_p(r) \equiv \rho(r)$; the Wood-Saxon and harmonic-well forms have been commonly adopted as $\rho(r)$. These would not, of course, be a rigorous description of the nuclear density distributions, but have been used for the sake of a technical simplification.

When the known $K^-$-nucleon scattering lengths (Table 7) were applied in equation 9b, the shifts and widths computed were found to disagree with the experimental data, yielding less than about half of the values of the observed widths (96–99). A remedy was then taken to keep the form of equation 9b and to treat $\tilde{a}$ as an “effective scattering length in nuclei,” the exact value of which was to be determined phenomenologically. It has been found that a single value of $\tilde{a}$ produces kaonic atom data of various transitions very well (100, 101). Figure 8 exemplifies the overall good description of the experimental data by a phenomenological potential. The most recent value of $\tilde{a}$, fit to 1972 CERN data (20), is (98)

$$a = -0.44 \pm 0.04 - i(0.83 \pm 0.07) F,$$

which corresponds to a square well $K^-$-nucleus potential of $(-42-i79)$ MeV depth when the radius is taken to be $1.25 A^{1/3} F$. More complicated versions of equation 9 were examined, but it was found that adding $\nabla \rho \cdot \nabla$ terms (the $K^-$-nucleon p-wave interaction) (20), a $\rho^2(r)$ term (two-nucleon absorption) (20), or a $(N - Z)/A$ dependence term in $\tilde{a}$ (101) does not alter the good agreement and is not required at least for the present experimental accuracy. The nuclear density dependence on $\tilde{a}$ was also examined: for given shift and width data, the explicit value of $\tilde{a}$ is rather sensitive to the parameters in $\rho(r)$ (102, 103).

The conclusion that the kaonic atom data provide no evidence to revoke $\rho_n = \rho_p$ is tempting, but must be avoided until the theoretical
Table 7  $K^-$-nucleon scattering lengths ($a$ and $\tilde{a}$), and the mass ($M$) and width ($\Gamma$) of $\Lambda(1405)$ obtained by multichannel analyses.

<table>
<thead>
<tr>
<th></th>
<th>Martin-Sakkita</th>
<th>Kim b</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a(l = 0)$ (F)</td>
<td>$1.66 \pm 0.02 - i(0.69 \pm 0.02)$</td>
<td>$1.65 \pm 0.04 - i(0.73 \pm 0.02)$</td>
</tr>
<tr>
<td>$a(l = 1)$ (F)</td>
<td>$0.09 \pm 0.03 - i(0.54 \pm 0.02)$</td>
<td>$0.13 \pm 0.03 - i(0.51 \pm 0.03)$</td>
</tr>
<tr>
<td>$\tilde{a}$ (F)</td>
<td>$0.48 \pm 0.03 - i(0.58 \pm 0.02)$</td>
<td>$0.51 \pm 0.03 - i(0.57 \pm 0.03)$</td>
</tr>
<tr>
<td>$M + i\Gamma/2$ (MeV)</td>
<td>$1416 \pm 4 + i(29 \pm 6)/2$</td>
<td>$1403 \pm 3 + i(50 \pm 5)/2$</td>
</tr>
</tbody>
</table>

a Reference 113, 134.
b Reference 112, 134.

The origin of $\tilde{a}$ is well understood in terms of the $K^-$-nucleus interaction in the nucleus. We discuss this problem below in this section, and more in detail in the following section. The good agreement found so far must also be examined critically through more accurate data since some indication of alarming disagreements has recently been reported (21, 23). In connection with this matter, there exists an interesting region of the kaonic transitions, $n = 6 \rightarrow 5$ and $7 \rightarrow 6$, which often involve deformed nuclei (Figure 8). Dynamic effects of the deformed nuclei would appear in the X-ray spectra as additional shifts and widths of the energy levels (104, 105), admixtures of level splitting (105), and $E2$ dynamics (Section 3.4). A systematic measurement of these transitions has not been made, but its careful analysis may turn out to be an important means of examining the validity of the phenomenological potentials and other potentials discussed below and in the following section.

Comparing equation 10 and Table 7, one observes that the phenomenological $\tilde{a}$ has a negative real part (an attractive $K^-$-nucleus interaction by our convention), whereas the known $K^-$-nucleon scattering lengths have positive real parts (yielding a repulsive $K^-$-nucleus interaction via equation 9b). The confusing sign change originates from the fact that the $K^-$-nucleon interaction is so strong and absorptive that the potential and its scattering length no longer have the same sign of the real parts (in our convention). This fact is neglected in the point-like or zero-range approximation (equation 8). In fact, the $K^-$-nucleon potential of a reasonable range is found to be attractive and strongly absorptive (98, 106–108). The strong absorption then causes the scattering length to be positive by overcoming the attractive real part of the interaction. In the case of the $K^-$-proton interaction, this effect is greatly amplified because “the kaon and proton spend more time together” (106) via $K^- + p \rightarrow \Lambda(1405) \rightarrow \Sigma + \pi(100\%)$.

The underlying physics discussed here has been exemplified by direct
Figure 8  Kaonic energy level shifts ($\epsilon$), broadening ($\Gamma$), and $I_{\text{out}}/\Sigma I_{\text{in}}$ via equation 3 computed from a phenomenological potential. The "effective scattering length in nuclei" $\tilde{a}$ is $-0.60 - i0.71$ F, by fitting only to 1970 CERN $^3$S data (133) and a 1971 Cl data (22). The experimental data shown are from the Berkeley group ($\bigcirc$), the CERN group ($\times$), and the Argonne group ($\blacktriangle$). Most of the experimental $I_{\text{out}}/\Sigma I_{\text{in}}$ data are not corrected by cascade calculations (Section 3.2) and the experimental errors are not shown when they are greater than 20%. No data are shown for $6 \rightarrow 5$ $I_{\text{out}}/\Sigma I_{\text{in}}$ because the transitions $6 \rightarrow 5$ and $8 \rightarrow 6$ have nearly the same energies, which made intensity measurements difficult.
applications of finite range $K^-$-nucleon potentials. $K^-$-nucleus optical potentials were constructed either by “smearing the $K^-$-nucleon potentials over the nucleus” (98, 106, 107) or “folding in the nuclear density distribution” (106, 108). Both methods have been shown to yield potentials similar to the phenomenological ones. As seen in the above discussion, however, it is clear that a substantial $K^-$-nucleon dynamics is at work in the nucleus. In order to obtain a $K^-$-nucleus optical potential reliable enough to extract a detailed nuclear density distribution, one has to establish its microscopic foundation starting from the detailed $K^-$-nucleon interactions in the nucleus. We discuss theoretical efforts on this in the following section.

Before closing this section, we make a pertinent remark. In computing the shifts and widths of two observed states from an optical potential, one must avoid first order perturbation theory. In 1971 it was found independently by Krell (96) and Seki (97) that perturbation theory for a repulsive, complex potential, such as with the known $K^-$-nucleon scattering lengths, yields substantially smaller widths than the values obtained by numerical integration of the Klein-Gordon equation. In some situations perturbation theory gives only one third of the values obtained by a numerical integration. In the case of an attractive, complex potential, such as with a phenomenologically fitted $\bar{a}$, the attractive real part of the potential enhances the widths so as to bring about a fair agreement for the widths by the two computational methods. This is an accidental agreement depending critically on the magnitude of both real and imaginary parts, exemplified by the fact that the shifts obtained by two methods have opposite signs. In principle, the smallness of the shifts and widths is a necessary, but not a sufficient, condition for first order perturbation theory to be applicable, as previously pointed out in the case of the pionic atoms (109). One must use care in interpreting results of analyses in the literature, particularly prior to 1971. But this is a numerical problem, rather than one of physics.

4.2.1.2 $K^-$-nucleon interactions in nuclei and theoretical $K^-$-nucleus optical potentials

The $K^-$-nucleon interactions are as follows:

$$K^- + N \rightarrow K^- + N, \quad \bar{K}^0 + N$$

$$\rightarrow \Sigma + \pi$$

$$\rightarrow \Lambda^0 + \pi.$$ 

The prominent feature in the $K^-$-proton interaction is formation of $\Lambda$ (1405) about 27 MeV below the $K^-$-proton threshold (zero kinetic energy). $\Lambda$ (1405) has isospin-0 and decays only to $\Sigma + \pi$. The Particle
Data Group (27) gives the mass and width of Λ (1405) as 1405 ± 5 and 40 ± 10 MeV, respectively. The K⁻-nucleon scattering amplitudes below the threshold are obtained up to now by extrapolations based on multichannel, effective-range-type methods (110, 111).

Figure 9 and Table 7 summarize the results most frequently quoted (112, 113). In Figure 9 we observe the highly energy-dependent K⁻-proton scattering amplitude that differs recognizably in the two analyses despite the very close values of the scattering lengths shown in the table. Because of Λ (1405) the K⁻ proton amplitude is not so well determined that it can be relied on explicitly. The K⁻-neutron amplitude is expected to be a much less "pathological case." Recently Chao et al (114)
showed that $\Sigma \pi$ production data give severe constraints on the mass and width of $\Lambda$ (1405). Further studies are required to determine with confidence the $K^-\text{nucleon}$ amplitudes below threshold.

In order to incorporate the strong energy dependence in the $K^-p$ scattering amplitude, Bardeen & Torigoe (99, 115) proposed using an effective amplitude in equation 8. The effective $f_i(K-N)$ was obtained by weighting $f_i(K-N)$ with the probability of finding the $K^-\text{nucleon}$ reaction taking place in the nucleus at various $K^-\text{nucleon}$ center-of-mass energies. Most of the $K^-\text{nucleon}$ interactions are between about $-10$ and $-40$ MeV in the $K^-\text{nucleon}$ center-of-mass kinetic energy system, peaking around $-20$ MeV. $20$ MeV is close to the nucleon binding energy in nuclear matter plus the kaon binding energy in the atom. The averaged $f_i(K-N)$ for the proton turns out to have the sign of the real part opposite to that at threshold. That is, the $K^-\text{nucleus}$ optical potential thus constructed is attractive and proportional to the nuclear density distribution. This potential gave a reasonable agreement with 1972 CERN data (20). Ideas similar to the above modification of $f_i(K-N)$ have been also discussed by Bethe & Siemens (116), Burhop (66), Bloom, Johnson & Teller (117), and recently Chattarji & Ghosh (118).

However, the $K^-\text{nucleon}$ interactions in the nucleus take place in the domain of the off-energy shell caused by nuclear binding. Furthermore, the presence of the other nucleons near the interacting kaon-nucleon pair may induce dynamical effects via the Pauli exclusion principle and nuclear correlations. Wycech (119) performed a detailed calculation of the $K^-\text{nucleon}$ scattering amplitudes in the nucleus by examining their dependence on the position, momentum, and energy of the kaon in the nuclear Fermi gas. Since the scattering amplitudes obey multichannel Lippmann-Schwinger equations that involve the $K^-\text{nucleus}$ potential itself, the scattering amplitudes and in turn the $K^-\text{nucleus}$ potential are calculated in a self-consistent way. The scattering amplitudes are approximated so as to yield a local $K^-\text{nucleus}$ potential, but not proportional, to the nuclear distribution. The shifts and widths calculated with this potential give a fair agreement with the experimental data (120). Two-nucleon correlation and the Pauli exclusion principle have been shown to have a prominent effect on $\Lambda(1405)$ in the nucleus. In the region where the nuclear density is greater than about $20 \sim 30\%$ of the central value, the width is found to be broadened to about $70$ MeV and the position of the bound state to be shifted to about $-10$ or $-15$ MeV.

The effects of the nonlocality and off-energy shell on the $K^-\text{nucleon}$ interactions have been explicitly examined and found to be appreciable
by Alberg, Henley & Wilets (121), and Alberg (122). The off-energy-shell \( K^- \)-nucleon scattering amplitudes are obtained from separable \( K^- \)-nucleon potentials that describe the multichannel \( K^- \)-nucleon scatterings in free space. The \( K^- \)-nucleus potential is then constructed from these scattering amplitudes via a multiple scattering theory without nuclear correlations. The resulting potential is nonlocal, so different localizations are examined. None of the localizations is found to follow the nuclear distribution, but agreement with shift and width data turns out to be fair for these localized potentials.

The \( K^- \)-nucleon interactions in the nucleus thus seem to be highly sensitive to all nuclear effects considered. One feature appears to be clear, however: the behavior of the \( K^- \)-proton interaction dominates the \( K^- \)-nuclear interaction. In order to learn the detailed nuclear distribution, then, it seems that we require more reliable descriptions of the \( K^- \)-nucleon interactions and how they are influenced by nuclear effects. It is not yet entirely clear whether the good overall agreement between the data and the phenomenological potentials described in the previous section is accidental, due to the choices of the nuclear density distribution, or a consequence of some important physics.

In closing this section, we take note of a peculiar contradiction of the above works. Revai (123) performed a three-body calculation for a system consisting of the kaon, a nucleon, and the rest of the nucleus. He found that the effect of \( \Lambda(1405) \) due to the \( \Sigma \pi \) channel yields little variation in the \( K^- \)-proton amplitude as a function of the energy below threshold. Despite recent work on the \( K^- \)-deuteron three-body calculation (124), this puzzle seems to remain unsolved.

### 4.2.2 \( \bar{p} \)- and \( \Sigma^- \)-NUCLEUS OPTICAL POTENTIAL

Because reliable information is not available on the \( \bar{p} \) and \( \Sigma^- \)-nucleon scattering near threshold, little is known about the \( \bar{p} \) and \( \Sigma^- \)-nucleus optical potential. The effective scattering length \( \tilde{a} \) in the phenomenological potential (equation 9) has been determined by fitting the available shift and width data in \( \bar{p} \) atoms. It is (28)

\[
\tilde{a} = -2.9^{+1.0}_{-1.4} - i(1.5^{+0.6}_{-0.2}) \text{ F.}
\]

This is to be compared with the \( \bar{p} \)-nucleon scattering length (125)

\[
\tilde{a} = +0.88 - i0.81 \text{ or } +0.91 - i0.69 \text{ F,}
\]

which is computed from a semitheoretical \( \bar{p} \)-nucleon optical potential that agrees reasonably well with the \( \bar{p} \)-nucleon scattering data below 300 MeV. Again we seem to have the sign change in \( \text{Re} \tilde{a} \) that appeared in the \( K^- \)-nucleus optical potentials (Section 4.2.1). The \( \bar{p} \)-
nucleus potential constructed by the method of "folding in the nuclear distribution" yields the correct sign change (126). The $\bar{p}$-nucleon scattering amplitudes in nuclei should, however, be carefully examined in order to understand the $\bar{p}$-nucleus interaction. For this purpose more reliable information would be needed on the $\bar{p}$-nucleon scattering amplitudes in free space.

No information is available on the $\Sigma^-$-nucleus optical potential. Based on poorly known $\Sigma^-$-nucleon scattering lengths (127), an educated guess on $Im\tilde{a}$ is within a few $F$.

4.3 Two- and One-Nucleon Absorption Products

According to Table 6 in Section 3.5, kaonic two-nucleon absorption occurs in deuterium for only about 1% of the total absorption, whereas in $^4$He and other nuclei it occurs with a probability of about 20%. Wilkinson (78) proposed this experimental observation as possible evidence of $\alpha$-particle cluster formation in the nuclear surface. However, detailed analyses by Wycech (128) and Aslam & Rook (129) showed that the $\alpha$-cluster formation is not necessary to explain the large two-nucleon absorption rates in nuclei other than deuterium. The mechanism considered by these authors is that the virtual $\Lambda(1405)$ produced by proton absorption interacts with a second nucleon in a finite range via a virtual pion that is a decay product of $\Lambda(1405)$. Thus $\Sigma$ and $\Lambda$ and one nucleon are produced. It is argued that this mechanism occurs even in that part of the nuclear surface region in which two nucleons are more tightly bound than deuterium. This region corresponds to nuclear densities greater than roughly 10% of the nuclear density at the center.

Nevertheless, there is a question of how much $K^-$ absorption takes place in the nuclear surface region. As discussed in Section 4.1, answering this question requires a better understanding of the atomic capture and cascade processes. The same problem exists in understanding the various rates that result from one-nucleon absorption, such as $R_{pn}$ and $R_{+-}$, and the occurrence ratio of neutron and proton absorption. In Section 3.5 we mentioned the question of reliability in deducing the values of these quantities from the observed data. Besides this question, an uncertainty in understanding the atomic schemes worsens the possibility of verifying the theoretical $K^-$-nucleus optical potentials by comparing their predictions with these quantities. Several attempts (65, 66, 115–117, 119, 130) have been made to combine these quantities in relation to one-nucleon absorption and the $K^-$-nucleon interactions in the nucleus. Efforts (131, 132) have also been made to explicitly include the atomic cascade scheme. Yet the complexity of the problem seems to prevent us from obtaining confident, detailed information.
5 CONCLUDING REMARKS

We have seen that exotic atoms are complex phenomena, rich in physics that remains to be completely understood. Here three stages of physics—atomic, nuclear, and elementary particle physics—are interrelated, and better understanding of all three disciplines is needed. This is particularly true of the early hope of learning about the nuclear surface structure. Regrettably, we are far from reaching that goal. Whether or not this complicated mixture of disciplines will turn out to be a fertile source of physics, must be answered in the future.

Clearly, theoretical and experimental effort should continue. "Kaon factories," whose construction should be studied seriously, could foster major experimental advances. Nuclear physics involving strangeness could be one of the interesting fields to be pursued.

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