Superconducting-Gravimeter Tests of Local Lorentz Invariance

Natasha A. Flowers, Casey Goodge, and Jay D. Tasson

Physics and Astronomy Department, Carleton College, Northfield, Minnesota 55057, USA
(Received 9 February 2017; revised manuscript received 28 July 2017; published 16 November 2017)

Superconducting-gravimeter measurements are used to test the local Lorentz invariance of the gravitational interaction and of matter-gravity couplings. The best laboratory sensitivities to date are achieved via a maximum-reach analysis for 13 Lorentz-violating operators, with some improvements exceeding an order of magnitude.

DOI: 10.1103/PhysRevLett.119.201101

Local Lorentz invariance is among the foundational building blocks of general relativity (GR). Though GR provides an impressive description of the wide variety of gravitational phenomena, standard lore holds that GR may be the low-energy limit of an underlying theory that merges gravitation and quantum physics, such as string theory. Local Lorentz violation may arise in such an underlying framework [1]. Hence, tests of local Lorentz invariance probe the core construction of GR and may provide clues about the structure of new physics at the quantum-gravity scale. These ideas triggered the development of a comprehensive effective field theory based framework [2,3] for testing Lorentz symmetry violation, and the best laboratory sensitivity to six coefficients not previously explored in gravimeter experiments is achieved. In some cases, sensitivities are improved by more than a factor of 10.

The SME is constructed as an expansion about the actions of GR and the standard model in Lorentz-violating models of new physics at the Planck scale [1], and the gravitational standard-model extension (SME) provides a field-theory based framework for organizing a systematic search [2,3,15]. While sensitivities to SME coefficients for Lorentz violation have been achieved in a variety of gravitational systems [16–23], including pioneering work with an atom-interferometer gravimeter [16,17], this work provides the first exploration of superconducting gravimeters in the SME framework and the first search for matter-sector Lorentz violation using gravimeters of any kind. Sensitivity improvements over prior gravimeter work [16,17] are achieved for seven coefficients for Lorentz violation, and the best laboratory sensitivity to six coefficients not previously explored in gravimeter experiments is achieved. In some cases, sensitivities are improved by more than a factor of 10.

The SME is constructed as an expansion about the actions of GR and the standard model in Lorentz-violating operators of increasing mass dimension. In the present work we focus on the minimal gravitational SME, in which attention is restricted to operators of mass dimension 3 and 4. We consider both the pure-gravity sector [24] and the spin-independent gravitationally coupled fermion sector [25] in the limit of linearized gravity. Though work extending the framework to include higher dimension operators [22,23,26] and nonlinear gravity [27] is now well underway, treatment of these operators lies beyond our present scope. Here, we summarize aspects of the SME framework relevant for this work. For additional detail, the reader is referred to Refs. [3,24,25].

The SME action in this limit can be written $S = S_G + S_p + S'$. Here, $S_G$ is the minimal pure-gravity sector,

$$
S_G = \frac{1}{16\pi G} \int d^4x e(R - uR + s^\mu R_{\mu} + \psi^{[\mu} C_{\nu]s}),
$$

where $G$ is Newton’s constant, and $R$, $R_{\mu}$, and $C_{\nu s}$ are the Ricci scalar, Ricci tensor, and Weyl tensor, respectively. The symbol $e$ is the determinant of the vierbein $e_{a}^{\mu}$, and $u$, $s^\mu$, and $\psi^{[\mu}$ are coefficient fields having dynamics contained in $S'$. Lorentz-violating signals in the post-Newtonian analysis to follow are associated with $s^\mu$, without contribution from $\psi^{[\mu}$ [28].
Similarly, spin-independent effects in the minimal gravitationally coupled fermion sector take the form

\[
S_ϕ = \int d^4x \left( \frac{1}{2} i e \bar{\psi}^α \Gamma^α β D_β ψ - e \bar{\psi} M ψ \right).
\] (2)

Here, ψ is the fermion field and \( D_μ \) is the covariant derivative, which, along with the vierbein, provides the coupling to gravity, \( Γ^μ \equiv γ^μ - c_μ e^a μ γ^a_μ - e_μ e^a μ γ^a_μ \), and \( M \equiv m + \bar{α}_μ e^a μ γ^a_μ \). The matter-sector coefficient fields \( α_μ \) and \( e_μ \) also have dynamics contained within \( S_ϕ \). The dynamics are assumed to trigger spontaneous Lorentz violation in which the coefficient fields acquire vacuum expectation values, a process for generating Lorentz violation in which the coefficient fields acquire vacuum expectation values, a process for generating Lorentz violation. Following generic treatment of SME coefficients for Lorentz violation and the development of the conventional Lorentz-invariant mass of the test body and source body, respectively, and \( N^w \) and \( N^S \) are the number of particles of type \( w \) in the test body and Earth, respectively. Here, the test body is a niobium sphere with a mass of a few grams, and the source body is Earth. The summation index \( w \) takes the values proton, neutron, and electron. The frequencies \( ω_ν \) are drawn from the set

\[
ω_ν \in \{ 2ω, ω, 2ω + Ω, 2ω - Ω, ω + Ω, ω - Ω, Ω \},
\] (5)

where \( ω \) is the sidereal angular frequency and Ω is the annual angular frequency. Note that \( 2ω \) arises due to the rotation of two-index coefficients. The corresponding phase \( φ_ν \) can be obtained from the frequency via the replacement \( ω → φ, Ω → 0 \), where \( φ \) is a phase that specifies the orientation of the laboratory at time \( T = 0 \). The time \( T \) along with the spacial coordinates \( X, Y, Z \) are the coordinates of the Sun-centered celestial equatorial frame in standard use for SME studies \[4\]. The contributions to the Lorentz-violating amplitude \( G^w, H^w, E^w \), and \( F^w \) can be found in Table I, while the contributions \( G_{n,0} \) and \( H_{n,0} \) are constructed via Eq. (142) of Ref. [25]. These results developed in Ref. [25] presented here with a few corrections. Here, \( V_0 = ω R, \) where \( R \) is Earth’s radius and \( V_0 \) is the speed of Earth on its path around the Sun. The angle \( ζ \) is between the local Lorentz-invariant free-fall direction and the direction of Earth’s center, \( χ \) is the colatitude of the experiment, \( η \) is the inclination of Earth’s orbit, and \( m^w \) is the mass of species \( w \).

Our method for extracting measurements of the coefficients for Lorentz violation from the Global Geodynamics Project data proceeds as follows. We use corrected minute data, which provide a measurement of the gravitational force each minute obtained from the raw data via some repairs performed by the station manager including the removal of some transients such as major earthquakes. Where possible, we follow the methods developed for the atom-interferometer gravimeter analysis [16]. The basic idea is to perform a discrete Fourier transform on relevant sets of gravitational force versus time data to extract the amplitudes \( A_n, B_n \). Equation (4) is then used to interpret the amplitudes as measurements of the SME coefficients.

As is typical of SME searches, the amplitudes \( A_n, B_n \) extracted from data collected by a particular device at a given site provide a measurement of a linear combination of SME coefficients rather than a measurement of a single term in the underlying theory. The numbers multiplying the coefficients for Lorentz violation in these linear combinations can contain the colatitude of the experiment \( χ \) and the dependence on the particle species content of the bodies involved. Hence, different sets of data from different locations and/or different devices measure different linear combinations of SME coefficients. Two procedures are common in the literature for extracting sensitivities to
TABLE I. Amplitudes for the force \( F_{LV} \).

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G_{\mu}^{\nu} = 2m^w \zeta((\omega^w)<em>{(X)}) - \frac{2}{3} V</em>{(L,\hat{g}<em>{\mu\nu})} X / (\omega^w)</em>{(XX)} )</td>
<td>( \phi )</td>
</tr>
<tr>
<td>( + \frac{2}{3} V_{(L,\hat{g}<em>{\mu\nu})} X / (\omega^w)</em>{(XX)} )</td>
<td>( \phi )</td>
</tr>
<tr>
<td>( H_{\mu}^{\nu} = 2m^w \zeta((\omega^w)_{(XY)}) )</td>
<td>( 2\phi )</td>
</tr>
<tr>
<td>( G_{\mu}^{\nu} = m^w \zeta((\omega^w)<em>{(UX)} - (\omega^w)</em>{(YY)}) )</td>
<td>0</td>
</tr>
<tr>
<td>( H_{\mu}^{\nu} = 2m^w \zeta((\omega^w)_{(XY)}) )</td>
<td>0</td>
</tr>
<tr>
<td>( E_{\mu}^{\nu} = -V_L(2\alpha(\hat{a}_{\mu\nu})<em>X + \frac{2}{3} m^w (\omega^w)</em>{(TX)}) )</td>
<td>( \phi )</td>
</tr>
<tr>
<td>( F_{\mu}^{\nu} = -V_L(2\alpha(\hat{a}_{\mu\nu})<em>X + \frac{2}{3} m^w (\omega^w)</em>{(TX)}) )</td>
<td>( \phi )</td>
</tr>
</tbody>
</table>

A daily variation associated with tidal effects is clearly visible in the three peaks. Following Ref. [16] we remove the dominate tidal contributions from the signal using a model of solid Earth tides [33]. Figure 1 also shows the data after the subtraction of the model. Application of a discrete Fourier transform to the residuals,

\[
A_n, B_n = \frac{2}{K} \sum_k d(t_k) \cos(\omega_n t_k + \phi),
\]

yields the amplitudes shown in Table II. Here, \( K \) is the total number of measurements; \( d(t_k) \) are the residual gravity measurements at times \( t_k \). Estimated uncertainties are obtained following Refs. [16,17] by performing the analysis at several frequencies near the characteristic frequencies and computing the root mean square.

The amplitudes in Table II together with the maximum-reach procedure yield the sensitivities to the coefficients for Lorentz violation shown in the second column of Table III. A dagger (\( \dagger \)) indicates a sensitivity that exceeds previous laboratory tests, though better constraints exist from Solar System or astrophysical observations [19–21,23,34]. The maximum reach listed here for the \( \delta^w_{\mu\nu} \) coefficients, which have previously been explored via gravimeter analysis [16,17], is an improvement upon that work for all seven coefficients listed.

TABLE II. Bad Homburg amplitudes.

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Measurement ((10^{-9}g))</th>
<th>Amplitude</th>
<th>Measurement ((10^{-9}g))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{260} )</td>
<td>(-0.02 \pm 0.01)</td>
<td>( B_{260} )</td>
<td>(0.04 \pm 0.01)</td>
</tr>
<tr>
<td>( A_{600} )</td>
<td>(-0.01 \pm 0.06)</td>
<td>( B_{600} )</td>
<td>(-0.1 \pm 0.1)</td>
</tr>
<tr>
<td>( A_{20\Omega} )</td>
<td>(-0.003 \pm 0.004)</td>
<td>( B_{20\Omega} )</td>
<td>(0.003 \pm 0.004)</td>
</tr>
<tr>
<td>( A_{25\Omega} )</td>
<td>(-0.01 \pm 0.01)</td>
<td>( B_{25\Omega} )</td>
<td>(0.006 \pm 0.005)</td>
</tr>
<tr>
<td>( A_{40\Omega} )</td>
<td>(-0.00 \pm 0.02)</td>
<td>( B_{40\Omega} )</td>
<td>(-0.01 \pm 0.02)</td>
</tr>
<tr>
<td>( A_{50\Omega} )</td>
<td>(0.01 \pm 0.03)</td>
<td>( B_{50\Omega} )</td>
<td>(0.06 \pm 0.03)</td>
</tr>
<tr>
<td>( A_{1\Omega} )</td>
<td>(-1 \pm 1)</td>
<td>( B_{1\Omega} )</td>
<td>(1 \pm 1)</td>
</tr>
</tbody>
</table>

FIG. 1. Bad Homburg data taken January 1–3, 2012, before and after tidal model subtraction. Discrete points are plotted that appear as a continuous curve at this scale.
We perform the same analysis on data collected from the device in Metsahovi, Finland, from 2007–2012, and on a year’s worth of data from Strasbourg, France and from Apache Point, USA taken in 2012. While the maximum reach available from these sites is typically less than that obtained from Bad Homburg, their locations at different colatitudes permit some coefficient separation. We do this following the procedure outlined in Ref. [17] in which each measurement of $A_n, B_n$ provides a probability distribution that we assume to be Gaussian with the measurement and uncertainty providing the center and standard deviation. The probability distribution can then be understood as a function of the coefficients for Lorentz violation through Eq. (4). These probability distributions can then be multiplied together for each of the relevant measurements from each of the four sites to obtain an overall probability distribution. Integrating the distribution over all of the coefficients except the one of interest then yields an estimate and uncertainty for that coefficient. The result of this process provides our estimates for the coefficients achieved by coefficient separation shown in the right column of Table III. This procedure for achieving initial coefficient separation estimates assumes the error sources in the four experiments are completely independent, while some geophysical noise sources may be somewhat coherent. Though beyond our current scope, it may be possible to address this potential issue through coherent combination of the original data. Relative to Ref. [17], correlations between amplitudes due to finite data are small and are neglected here.

Coefficients $\tilde{s}^{XX-YY}$ and $\tilde{s}^{XY}$ are obtained from amplitudes in which they are the only coefficient for the Lorentz violation involved. Hence, these entries in the maximum reach column of Table III could equally be regarded as the results of coefficient separation, and they are omitted from the four-site analysis. Sufficient information is not available to separate the $a(\tilde{s}_{\text{eff}}^{+})_{Z}$, $a(\tilde{s}_{\text{eff}}^{-})_{Z}$, and $\tilde{\beta}^{+} Z$ coefficients from each other. Hence, individual constraints are not available for column 3 of Table III, and the combination is treated as a single coefficient in the separation analysis resulting in $a(\tilde{s}_{\text{eff}}^{+})_{Z} + 1.1 a(\tilde{s}_{\text{eff}}^{-})_{Z} + 1.1 GeV(\tilde{\beta}^{+} Z) = 0.6 \times 10^{-4}$ GeV.

We do not include data from other experiments beyond the four gravimeter sites except in excluding from consideration other coefficients that have been constrained much more tightly by nongravitational tests. As this work was completed, the $\tilde{\beta}^{+} Z$ were also constrained by nongravitational tests [35]. Note that the results of coefficient separation generate improvements over prior lab work for $\tilde{s}^{uu}$ coefficients while constraints for matter-sector coefficients are weak. This feature can be traced to the fact that all four sites involve niobium test masses and Earth and hence the same proton/neutron ratios. Note also that proton and electron coefficients are listed together as separating them would require charged matter.

The sensitivities to coefficients in Table III, found via the standard approach to gravimeter analysis in the SME [16,17], provide a basic sense of upper bounds on coefficients. However, some care should be used in interpreting the results. Though we find no compelling evidence of Lorentz violation, some notable deviations from zero are seen in a few cases. In addition to the statistical expectation of weak signals when seeking this number of effects, these weakly reflect some challenges inherent to the search that we outline here.

The search involves subtracting dominant tidal effects from the gravimeter signal and attributing any remaining periodicity at the characteristic frequencies to Lorentz violation, with uncertainty estimated by the average level of the local Fourier spectrum near the characteristic frequency. The method relies on the assumption that any potential Lorentz-violating signal is not also contained in the tidal model. Modeling of additional local effects is avoided to minimize this concern. We also note that a Fourier transform of the raw data with no tidal modeling yields the same level of reach for annual variations, which is the aspect of the measurement associated with many of the most significant sensitivity improvements. The method also assumes that residual environmental effects at the characteristic frequencies have a size similar to neighboring Fourier amplitudes. This assumption is most challenged by Lorentz-violating frequencies that coincide with dominant tidal components. Here, the relatively small residual signal is the result of subtracting a comparatively large modeled tide from a similarly sized signal. One could also imagine the Lorentz-violation signal of a special linear combination of coefficients that matches the tidal phase being masked by a tidal effect.

A variety of opportunities for further improvements with related experiments exist. One key challenge in gravimeter tests is managing periodic environmental effects without
using models constructed by fitting to gravimeter data. A way
to side step this issue for matter-sector coefficients is to
consider analogous weak-equivalence principle tests that
search for a variation in the relative gravitational force or
acceleration of two or more bodies. It may also be possible to
use the phase information associated with environmental
systematics to separate them from the effects of certain
combinations of coefficients for Lorentz violation.
Correlations between signals at multiple sites may also be
useful. Gravimeter data involving bodies of other compositions
would aid in performing coefficient separation for the
matter sector. Free-fall gravimeter tests such as atom interferometers are also of interest, particularly for the matter
sector, as they involve a different dependence on the matter-
sector coefficients. An increase in the long-term stability
of gravimeters would further improve sensitivities at the annual
frequency. Beyond gravimeters, searches for Lorentz viola-
tion with satellite geodesy data may be of interest. In all,
exciting prospects remain for further searches for Lorentz
violation with gravimeters and related systems.

This work was supported in part by the Carleton College
Clinton Ford Physics Research Fund.

(1989).
6760 (1997); 58, 116002 (1998).
slu.edu/GGP/ggphome.html; J. Hinderer and D. Crossley,
(1976).
[10] Proceedings of the Seventh Meeting on CPT and Lorentz
Symmetry, edited by V. A. Kostelecký (World Scientific,
[11] For reviews of the SME approach, see, and J. D. Tasson,
[12] For a review of the SME gravity sector, see, A. Hees, Q. G.
Bailey, A. Bourgoin, H. Pihan-Le Bars, C. Guerlin, and C.
Le Poncin-Lafitte, Universe 2, 30 (2016).
[13] For discussion of other approaches to Lorentz violation see,
for example, S. Liberati, Classical Quantum Gravity 30,
133001 (2013); S. Mirshekari, N. Yunes, and C. M. Will,
[14] For a review, see C. M. Will, Theory and Experiment in
Gravitational Physics (Cambridge University Press,
(2009); 85, 096005 (2012); 88, 096006 (2013).