Is there a role for stylized models?

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Complex, interdependent networks: at the core of modern society

Modern power-girds: cyber-physical, socio-technical systems.
Commonly found features

1. “Small Worlds”
   - Diameter $\sim \log(N)$,
   - + high “clustering”

2. Clustering in social networks
   -(Triangle motifs / transitive closure)

3. Broad heterogeneity in node degree

4. Modularity
3. Broad heterogeneity in node degree

Power-law, log-normal, stretched exponential, etc., degree distributions

Social contacts
Szendrői and Csányi

Airport traffic
Bounova 2009

Protein interactions
Giot et al Science 2003
4. Modularity / Community structure

Zhiao Shi, Jing Wang & Bing Zhang
Modeling networks as random graphs

- **Configuration models** (Bollobás 1980, Molloy and Reed RSA 1995).
  Enumerating over all networks with specified degree distribution \( \{ p_k \} \).

  Start with half-edges. Assign a random matching to create an instance.

- **Generating functions**:
  \[
  G_0(x) = \sum_k p_k x^k
  \]

  Edge following properties \( \rightarrow \) component size distributions

  *(You can see this assumes a tree.)*
Modeling networks as random graphs, cont.

- **Master equations**:
  Mean-field evolution of clusters/graph structures.

  \[ x_k(t + 1) = F(\bar{x}(t)) \]

  e.g., node and edge arrival via “preferential attachment”,

  \[ x_k \equiv \text{fraction of nodes of degree } k: \]

  \[ x_k(t + 1) = \frac{k-1}{n} x_{k-1}(t) - \frac{k}{n} x_k(t) \]

  Analyzed for asymptotic properties: \( n \to \infty \) and \( t \to \infty \).

Probabilistic properties of the ensemble of graphs.
Caveat: Random graphs versus a specific realization of an engineered or biological system

- Ensemble not necessarily representative!
  Doyle, et. al., PNAS 102 (4)2005.

- Degree distribution is not specific/doesn’t constrain ensemble.
Modeling networks: The “classic” random graph, $G(n, p)$


- Start with $n$ isolated vertices.
- Consider each possible edge, and add it with probability $p$.
- Expected number of edges,
  $e = pn(n - 1)/2$.
- Edge density: $t = e/n = p(n - 1)/2$.

What does the resulting graph look like?
(Typical member of the ensemble)
$G(n=300,p)$

$p = 1/400 = 0.0025$

$p = 1/200 = 0.005$
Emergence of a *unique* "giant component"
Phase transition in connectivity

- $p_c = 1/n$.
- $p < p_c$, $C_{\text{max}} \sim \log(n)$
- $p = p_c$, $C_{\text{max}} \sim n^{2/3}$
- $p > p_c$, $C_{\text{max}} \sim A \cdot n$

Expected # of edges per node

\[ t = e/n = p(n - 1)/2 \]

so $t_c = 1/2$
Erdős-Rényi, a continuous, second order transition: Mean-field scaling behaviors

- Magnetic susceptibility: \( \chi = \frac{\partial m}{\partial h} \sim |T - T_c|^{-\gamma} \)
- Random graph “susceptibility” (second moment of the component sizes): \( \chi = \sum_{i=1}^{\infty} i^2 n_i \)
- For Erdős-Rényi, \( \chi \sim |t_c - t|^{-\gamma}, \text{ with } \gamma = 1. \)

Power law correlation lengths and response functions →

Potential EARLY WARNING SIGNALS
(e.g., Scheffer et al. Nature 461, 2009)
Product Rule

- **Enhance** – similar to ER but with earlier onset.

- **Delay** – Extremely abrupt

Achlioptas, D’Souza, Spencer, *Science*, **323** (5920), 2009
From single to interacting networks
(Recall, even a single network has emergent properties)
Dynamics on interdependent networks

The “power grid” is a collection of independently owned and operated plants and transmission lines.

- We can’t store power. Need supply=demand.
- Blackouts cascade from one grid to another.
- Increasingly distributed and interconnected (in both US and Europe)

Sandpile model on networks

- Start with a network
- Drop units of load randomly on nodes
- Each node has a threshold. Here = degree.
- Load on a node ≥ threshold ⇒ node topples, moves load to neighbors
Sandpile model on networks

- Start with a network
- Drop units of load randomly on nodes
- Each node has a threshold. Here = degree.
- Load on a node ≥ threshold → node topples, moves load to neighbors
- Neighbors may topple. Etc. Cascade (or avalanche) of topplings.

Power-law distribution of avalanche sizes, $P(s) \sim s^{-3/2}$
Power law tails (Universal behavior)

Avalanche size follows power law distribution $P(s) \sim s^{-3/2}$

Power law tails seem to characterize the sizes of electrical blackouts, financial fluctuations, neuronal avalanches, earthquakes, landslides, overspill in water reservoirs, forest fires and solar flares.

Two-type network: \( a \) and \( b \)
Impact of increased \( a-b \) links.

\[
p_a(k_a, k_b), p_b(k_a, k_b)
\]
(Configuration model)

Branching process treatment

\[
q_{ab}(r_{ba}, r_{bb}) := \text{the branch (children) distribution for an } ab\text{-shedding.}
\]
Main findings: optimal $p^*$ for an individual network

- (Blue curve) self-inflicted cascades (second network is reservoir).
- (Red curve) inflicted from the second network
- (Gold curve) Neglecting the origin of the cascade
Main findings: Increased systemic risk

- **More interconnections fuel larger system-wide cascades.**
  
  - Each new interconnection adds capacity and load to the system (Here capacity is a node’s degree, interconnections increase degree).

- **Tradeoff:** an individual operator adding edges to achieve $p^*$ may inadvertently cause larger global cascades.

- **Frustrated equilibrium** between networks possible.
Lesson: Optimal interdependence

“Some networking is good. Too much is overwhelming.”

Break connections in times of stress

Controlling the BTW model away from the SOC state


Control parameter $\mu$: probability grain lands on a node at threshold

- Avoid cascades, $\mu = 0.05 \rightarrow$ larger cascades when they do occur.
- Ignite cascades, $\mu = 0.99 \rightarrow$ smaller cascades, but more frequent.

![Graph showing probability distribution of cascade sizes with different control parameters $\mu$.]
Accounting for cost of cascading events

Assuming larger events more costly.

cost

\[ c \ [\text{size}]^{\alpha} \]

\[ -1 \]

\[ 0 \]

size

\[ m_{\text{OK}} \]

\[ m_{\text{bad}} \]

\[ \text{size} \]

\[ s_{\text{tip}} \]
Accounting for costs: Optimal control levels

Too much control can be detrimental.

Accounting for costs with larger events more costly.

Optimal $\mu^*$

- Frequently triggering cascades mitigates large events but sacrifices short-term profit.
- Avoiding cascades maximizes short-term profit but suffers from rare, massive events.
Real power grids – Non-local failures

(1996 Western blackout NERC report, 3 → 4 → 5; 7 → 8, etc.)


  (Featured as *Science* Editor’s Choice, 2010.)
Random graphs with nested loops

Expanding to random graphs with nested cycles of all lengths,
Pierre-Andre Noël:
Caveat: Real power systems – a web of feedbacks

Conclusions / Discussion

How can statistical physics & random graphs help us understand power grids?

– percolation, cascades, synchronization, epidemic spreading, ....

Challenges:

• Bringing in databases with the rich collection of node and edge attributes.
• Applying predictions from ensembles to real instances.
• Networks with long loops, in particular nested loops.
• Interdependence.
• Dynamics of network structure coupled to dynamical processes on networks.