# Reselling Information*

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**Abstract**

Information is replicable in that it can be simultaneously consumed and sold to others. We study how resale affects a decentralized market for information. We show that even if the initial seller is an informational monopolist, she captures non-trivial rents from at most a single buyer: her payoffs converge to 0 as soon as a single buyer has bought information. By contrast, if the seller can also sell valueless tokens, there exists a "prepay equilibrium" where payment is extracted from all buyers before the information good is released. By exploiting resale possibilities, this prepay equilibrium gives the seller as high a payoff as she would achieve if resale were prohibited.

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# 1 Introduction

**Overview:** The market for information is central to today's economy. But as has been appreciated since Arrow (1962) (and likely before), information is a difficult commodity to trade for at least two reasons. First, it is difficult to prove that one has valuable information without revealing it, and second, information is easily replicated and re-sold once it has been purchased. Our interest is in studying that second problem, namely how replication affects the pricing of information. When a good is replicable, can a seller charge a high price for it?

We ask this question in a model of decentralized bargaining where a monopolist for information may have significant bargaining power, can personalize prices, negotiates bilaterally with buyers who face frictions in negotiations, and can coordinate behavior on her favorite equilibrium of this game. We find that nevertheless, she appropriates very little of the social surplus from selling information across all equilibria; a robust upper-bound for her payoff is the value of information of a *single* buyer. This negative result suggests that without protection from resale, sellers of information have little incentive to acquire information even if they have significant market power.

Why does the seller fail to appropriate much of the social surplus? The challenge is that of commitment: neither a seller nor buyers can commit to not sell information to third-parties in the future. Thus these players anticipate future competition. This commitment problem endemic to a market for information is both dynamic and multilateral. Buyers are unwilling to buy information at a high price today if they can buy it cheaply in the future. The reason that they can do so is that those who have information cannot commit to not compete with each other. If players were able to commit—either if the initial seller had exclusive rights to sell information (e.g., as in a copyright with a permanent duration), or if each party could commit to selling information only once—then the seller may appropriate a substantial fraction of social surplus. The information good monopolist is stymied not only by the buyers' ability to resell information but also her own ability to do so.

We find a non-contractual solution to this commitment problem that exploits both resale possibilities and bargaining delays. We allow the seller to create worthless tokens. We study an equilibrium of that game—which we call a prepay equilibrium—where she holds up the release of the information good until she has sold tokens to all but one buyer. Using these tokens and delay, the seller achieves the same payoff that she would if resale were prohibited. In this scheme, buyers are willing to pay substantial amounts for worthless tokens because they anticipate being able to buy information in the future at very cheap prices. Effectively, the seller exploits the commitment problem to encourage buyers to pay before they receive information. Thus, within the commitment problem lies the seed to its solution.

**Framework and Results:** We study decentralized Nash bargaining in a market where a single seller is connected to multiple buyers. At time 0, that single seller has information, which is valuable to each buyer. The information may correspond to knowing some payoff-relevant state variable or the knowledge of how to use a technology or some transferable skill or be some replicable content that the seller has produced (and cannot protect with a copyright). The seller and each buyer face a trading opportunity at a random time. When presented with a trading opportunity, the pair trade if their joint continuation value from trading exceeds that from not trading. If trade occurs, the buyer consumes the information and pays a price. But the challenge is that not only can the seller continue selling information to others but now so can the buyer. Our model of a market involves a complete graph so in selling information to that buyer, the seller now faces competition with respect to each other buyer. Our results concern prices in the frequent-offer limit, as the time between trading opportunities converges to 0.

Our first main result (Proposition 3 in Section 4.2) is that prices converge to 0, robustly across all (Markovian) equilibria, as soon as two parties possess the information, i.e., once a single buyer has purchased information. The intuition for this result is subtle because there are two opposing forces. On the one hand, no seller wishes to lose the opportunity to sell to a buyer, and so competition between sellers reduces the price of information. But on the other hand, our decentralized framework features trading frictions in which each buyer meets at most one seller at any instance, and bears some delay in waiting for the next trading opportunity. Analogous to the Diamond Paradox (Diamond, 1971), one may anticipate that these trading frictions would benefit sellers. In our setting, the competitive effect dwarfs the "Diamond effect" so that prices converge to 0.

Given this future competition, how can a monopolist gain any surplus at all (without the use of tokens)? Her ability to capture rents depends solely on how she conducts her first trade when she is the only one who possesses information. We show that a seller-optimal equilibrium necessarily uses delay as a strategy to capture some rents: the seller sets a high price for her first sale of information and designates a particular buyer to be her exclusive "first buyer." After that first sale, prices converge to 0. Because a designated buyer knows that he, in equilibrium, cannot obtain information from any other source, it is as if he and the seller are negotiating bilaterally for that information, and thus, the price is bounded away from 0. Given this possibility, we show that it is self-enforcing for the seller and every other buyer to disagree until that first sale is made. While this strategy offers the seller some surplus, the seller can do this only once. Thus, across equilibria, resale severely limits the seller's ability to capture rents.

Our solution to this dynamic and multilateral commitment problem combines the scheme identified above with tokens that have no intrinsic value. When there are $n_B$ buyers, the seller trades both the information good itself and $n_B - 1$ tokens. She first sells these $n_B - 1$ tokens and then trades the information good exclusively to the buyer who could not buy a token. We show that the seller can then appropriate approximately the same surplus from each buyer as if information could not be resold.

Here is the strategic logic. Each buyer knows that he must either buy a token or be the exclusive first information good buyer. In the latter case, he gives up substantial surplus to the seller. Buying a token avoids this latter case, and lets him obtain information in the future at a price of approximately 0. Thus, in equilibrium, the token becomes almost as valuable as information itself. Because these tokens are scarce, the seller extracts the value of the token from each buyer. The seller herself faces no incentive to deviate and sell information early because it impedes her ability to extract surplus from future buyers.

This token scheme requires no commitment on the part of the seller and is self-enforcing. One can view the token scheme as a non-Markovian equilibrium where tokens add just the right amount of history-dependence to resolve the commitment problem. The resolution *isn't* through a punishment scheme (in the spirit of repeated games) but instead exploits (a) bargaining dynamics when there are fewer tokens than buyers, and (b) information prices converging to 0 once a single buyer has bought information. While we implement this solution via tokens, it could be implemented just as well using public communication channels where the seller and buyer announce who has prepaid for information.

**Broader Connections:** Our work is motivated by inefficiencies in social learning when there is peer-to-peer sharing of knowledge and information; in other words, *non-markets for information*. Knowledge and information diffusion are central to many household economic decisions and technology adoption choices. But it is well-understood that information and knowledge need not be acquired or diffuse efficiently (e.g. Galeotti and Goyal, 2010; Niehaus, 2011; Banerjee, Breza, Chandrasekhar, and Golub, 2018). A clear tension is that in these non-markets for information, individuals do not internalize the gains that their knowledge and information accrue to others. Moreover, who one knows can affect what one learns, and so learning can be stymied by an incomplete network.

Motivated by this tension, one naturally wonders if the issue is that of a missing market for information. If information and expertise could be bought and sold, or traded easily for favors, would this facilitate investments in acquiring and sharing information?[1] Our negative result highlights a difficulty: once a knowledgeable farmer shares her know-how

[1] While one can view information in our model as the results of a Blackwell experiment, our preferred interpretation is that it represents the knowledge (or know-how) of how to take a particular action, or alternatively some conceptual content that can be easily replicated.

with another, she is unable to extract further rents or favors from others on the basis of that knowledge, especially since no one can commit to not reselling information. But our positive result suggests that if there is a way to obtain favors in advance, then individuals may have appropriate incentives to acquire information even without the use of contracts. What might these tokens look like in practice? One way to view them is as the ability to divide information into complementary bits, or splitting information into noisy relatively uninformative bits. By dividing information into complementary bits, the seller can extract prices or favors for each bit separately.

While our interest began with thinking about non-markets for information, there are vast literatures that have explored related issues in the context of information markets and intellectual property. We view our stylized model as clarifying some issues that emerge there. One view, dating back to at least Schumpeter (1942) and also in Grossman and Helpman (1994), is that imperfect competition and bargaining are necessary ingredients for people to have incentives to acquire information. Our negative result shows that even if a seller is a monopolist and has substantial bargaining power, she may not be able to capture much of the social surplus that comes from acquiring information without restrictions to resale. But our positive result shows that an information-good monopolist can capture much of these rents through the use of both tokens and delay.

Information markets have been of broad interest recently; see Bergemann and Bonatti (2019) for a survey. One vein of this literature on the problem of verifiability: *how does a seller prove that she has valuable information without giving it away?* Anton and Yao (1994) study a contracting solution where the information monopolist commits to sell information to competing buyers if a buyer steals her idea without payment. Horner and Skrzypacz (2014) offer an elegant dynamic solution that involves gradually selling information and collecting payments. Our solutions also use delay but for the different reason that it helps with commitment problems in a setting with resale possibilities.

This resale-commitment problem is the focus of the innovative (and, in our view, underappreciated) study of Polanski (2007). He studies information resale on arbitrary networks in an environment without discounting and restricts attention to an immediate agreement equilibrium. In this equilibrium, prices equal 0 along any cycle in the graph. Also studying this environment, Manea (2020) provides a complete payoff characterization (for the undiscounted limit) of the immediate agreement equilibrium in terms of the global network structure for general networks. In his setting, buyers may be heterogeneous in their intrinsic value for information, and some buyers may not value information intrinsically at all. His analysis uses "bottlenecks" and "redundant links" to offer an elegant perspective of competition and intermediation in the networked market.

We focus on a different set of issues, and importantly, we abstract from the complexity of incomplete graphs that features in these papers. Instead of restricting attention to the immediate agreement equilibrium in the undiscounted game, we derive bounds on prices across *all* (Markov) equilibria in the frequent-offers limit of games with discounting.[2] Looking at other equilibria is important because the seller-optimal equilibrium features delay and not immediate agreement. Our negative result shows that sellers' inability to capture significant rents (without tokens) emerges robustly across equilibria. Our positive result, where we combine delays with a token-scheme to collect payments before the sale of information, has not been studied in these papers.

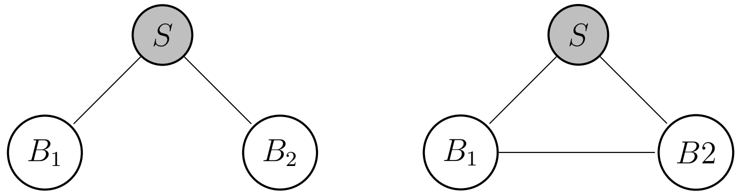
Our stylized analysis omits a number of important features. One of these is direct "consumption-externalities" for buyers, which has been studied extensively in the context of financial markets (Admati and Pfleiderer, 1986, 1990). Muto (1986, 1990) studies how these negative externalities from others holding information can deter buyers from reselling information. Polanski (2019) extends results from his prior work to illustrate how they apply with consumption externalities.

Our analysis uses approaches pioneered in studies of decentralized bargaining and matching. Much of this literature concerns non-replicable goods. Condorelli, Galeotti, and Renou (2016) and Manea (2018) study *intermediation* where buyers can choose to resell non-replicable goods rather than consume them. In our setting, because resale prices converge to 0, buyers are not buying information for the sake of reselling it. Much of the strategic logic of our paper exploits the fact that intertemporal competitive effects are strong when there are small search frictions. Elliott and Talamàs (2019) show that intertemporal competition resolves holdup problems in matching markets.

# 2 Examples

We illustrate the key ideas of our paper using a simple example. A single seller $S$ ("she") has information that is valuable to two buyers, $B_1$ and $B_2$ (each of whom is a generic "he"). This information has an "intrinsic value" of 1 to each buyer. All players have a discount rate of $r$, and each link meets with probability $\approx \lambda\, dt$ in a period of length $dt$. When a pair meets, tranfers are determined through symmetric Nash Bargaining where players' outside options are their continuation values without trade occurring, but following the same equilibrium. The ratio $\lambda/r$ measures the frequency of trading opportunities per unit of effective time, and we think of $\lambda/r \to \infty$ as the frequent-offers limit of the game.

[2] One reason to study a frequent-offers limit of the game rather than the undiscounted game directly is that it's unclear that the equilibrium correspondence is continuous at the undiscounted limit.




(a) No Resale (b) Resale

Figure 1: A Single Seller Trades Information with 2 Buyers.

## 2.1 No Resale: A Benchmark

First, consider the setting depicted in Figure 1a: each buyer can buy information from the seller but cannot resell information to the other buyer. The setting is therefore of two separate bilateral bargaining interactions and in each interaction, the Nash Bargaining price $p$ is that which divides the gains from trade equally between the buyer and seller. These gains from trade are dynamic: the total gain from trade is the difference between the joint surplus from trading today and waiting for the next opportunity to do so. Therefore, the price solves

$$\underbrace{p-\int_0^\infty e^{-rt}e^{-\lambda t}\lambda\, p\, dt}_{\text{Seller's Gain from Trading Today}} = \underbrace{(1-p)-\int_0^\infty e^{-rt}e^{-\lambda t}\lambda\,(1-p)\, dt}_{\text{Buyer's Gain from Trading Today}}. \tag{1}$$

The solution, $p=\frac{1}{2}$, splits the surplus within each trading relationship, and thus, the seller obtains half of the social surplus.

## 2.2 An Immediate Agreement Equilibrium with Resale

We now consider the setting shown in Figure 1b where all three players are connected and can trade information. We first consider an immediate agreement equilibrium where parties expect every trading opportunity to result in trade happening.

For our discussion below, it is useful to define $\gamma \equiv \int_0^\infty e^{-rt}e^{-2\lambda t}\lambda\, dt$. For a player, fixing one of his links $\ell$, the term $\gamma$ is the discounted weight that of his two links, $\ell$ is the next link that is selected. In the frictionless limit, as $\lambda \to \infty$, $\gamma$ approaches $\frac{1}{2}$.

We first consider the history after only one of the buyers is informed, and then consider the original game in which no buyer is informed.

*Only one buyer is informed*: Suppose that buyer $B_1$ is informed but buyer $B_2$ is not; the latter can then purchase information from one of two parties. Denote the equilibrium price in this history by $p(2)$ (because there are two players who can sell information). Splitting the gain in surplus from trade between the buyer and seller, $p(2)$ solves

$$\underbrace{p(1-\gamma)}_{\text{Seller's Gain from Trading Today}} = \underbrace{(1-p)(1-2\gamma).}_{\text{Buyer's Gain from Trading Today}} \tag{2}$$

The current seller's gain from trading today is that she secures the sale; by contrast, if she waits, she can only sell to the uninformed buyer if that buyer does not meet the other seller. Therefore, for any strictly positive price $p$, the LHS is at least $p/2$. By contrast, the buyer's only gain from waiting is avoiding delay, which vanishes to 0 as we approach the frictionless limit. For Eq. (2) to hold as $\frac{\lambda}{r} \to \infty$, the price $p(2)$ converges to 0.[3]

*No buyer is informed*: That $p(2)$ is low influences negotiations at the earlier stage before any buyer has purchased information. The payoff of buying information at price $p$ for the first buyer then is an immediate benefit of $(1-p)$, and the potential for reselling that information if he meets the other buyer first, which has a discounted value of $\gamma p(2)$. His "outside option" is his payoff from waiting and not trading today: in the future, either the seller meets him first in which case he is in the same position as today, or the seller meets the other buyer first, in which case he can buy information at a price of $p(2)$ from either of the other two players. The first contingency obtains a stochastic discount of $\gamma$ (reflecting that the seller does not meet the other buyer first), and the second contingency obtains a stochastic discount of $2\gamma^2$ (reflecting that the other buyer buys information first, after which there are twice as many opportunities to buy information). Thus, his gain from buying information is

$$1-p+\gamma p(2)-\gamma\left(1-p+\gamma p(2)\right)-2\gamma^2(1-p(2)) \to -\frac{p}{2},$$

which implies that for trade to occur, the price that the first buyer pays for information must converge to 0. The intuition is straightforward: because each buyer recognizes that he can wait to be the second buyer and obtain the information for virtually free, he has no gain from securing the information now at a strictly positive price.

The strategic logic is that neither the seller nor the first purchaser is able to commit to not resell information. Once both are selling information, the price of information that the second purchaser faces is close to 0. Because each buyer can wait to be that second buyer (in this equilibrium), no buyer is willing to be the first to buy information unless that price also converges to 0. Thus, the seller obtains 0 in the frequent-offers limit.

The issue here is that all parties expect every trading opportunity to end with trade. Our next scheme shows how the seller can use a scheme with delay and disagreement so that

[3] The equilibrium price, $p(2)$, equals $p(2) = \frac{1-2\gamma}{2-3\gamma}$, which converges to 0 as $\frac{\lambda}{r} \to \infty$. We note that this outcome replicates Bertrand competition even though the buyer never meets both sellers simultaneously.

she obtains 1/2 from the first buyer.

## 2.3 The Seller's Optimal Equilibrium (without tokens)

Here, we construct an equilibrium where on the equilibrium path, only one specific buyer can have the opportunity to be the second purchaser of information. Suppose that the seller never trades information with buyer $B_2$ *until* she has sold information to buyer $B_1$; after she sells information to $B_1$, then she and $B_1$ shall compete to sell information to $B_2$ at the price $p(2)$ characterized above. Thus, buyer $B_1$ never anticipates being able to be the second purchaser of information. The price that the seller and $B_1$ agree to solves

$$(p^* + \gamma p(2)) \left(1 - \int_0^\infty e^{-rt} e^{-\lambda t} \lambda \, dt\right) = (1 - p^* + \gamma p(2)) \left(1 - \int_0^\infty e^{-rt} e^{-\lambda t} \lambda \, dt\right). \tag{3}$$

The LHS reflects the seller's gain from trading today versus waiting for tomorrow: she obtains price $p^*$ today and after selling information, she can potentially be the first to sell information to the other buyer (which is thus discounted by $\gamma$). The RHS is buyer $B_1$'s gain from trading today versus waiting. If he agrees, then he obtains a payoff of $1 - p^*$ today and potentially resells information successfully at $p(2)$ (discounted by $\gamma$). If he rejects, he anticipates that the next trading opportunity where there may be trade is where once again, he faces the prospect of being the first to buy information from the seller. Solving Eq. (3) yields that $p^* = \frac{1}{2}$.

For this to be an equilibrium, it must be that the seller and $B_2$ do not trade until the seller has sold information to $B_1$. No trading occurs only if their joint surplus from trading is below their joint surplus from not trading:

$$\underbrace{\frac{\lambda}{r+\lambda}(p^* + \gamma p(2))}_{\text{Seller's Surplus with No Trade}} + \underbrace{\frac{\lambda}{r+\lambda}(2\gamma(1 - p(2)))}_{\text{Buyer's Surplus with No Trade}} \geq \underbrace{1 + 2\gamma p(2)}_{\text{Joint Surplus with Trade}}.$$

As $\frac{\lambda}{r} \to \infty$, the LHS converges to $\frac{3}{2}$ whereas the RHS converges to 1. Therefore, the seller and buyer $B_2$ would not trade before the seller sells information to $B_1$.

## 2.4 A Prepay Equilibrium: Combining Delay with Tokens

We illustrate how the seller can do better by using tokens that have no intrinsic value. The seller creates a single token. Once the token has been sold (to either buyer), then we exploit the equilibrium constructed in Section 2.3 where the other buyer has to be the first to buy information. Thus, if buyer $B_i$ buys the token, the seller sells information first only to the other buyer $B_j$. Once $B_j$ purchases information, then the seller and $B_j$ compete to sell information to $B_i$, the buyer who had purchased the token.[4] Purchasing the token effectively gives a buyer the ability to be the second purchaser of information.

In this equilibrium, once the first buyer has bought the token, the other buyer pays a price of $p^*$ for information (as in Section 2.3). Now, consider the initial token trading stage. Disagreement means that the seller may next meet with either buyer to trade the token. The price $p^{**}$ for the token equalizes the gains from trade to the seller and buyer, and thus solves

$$\frac{r}{r+\lambda}\left(p^{**}+\frac{\lambda}{r+\lambda}p^*+\gamma p(2)\right) = \begin{array}{c}(1-\gamma)\left(-p^{**}+\frac{\lambda}{r+\lambda}2\gamma[1-p(2)]\right)\\ -\gamma\left(\frac{\lambda}{r+\lambda}[1-p^*+\gamma p(2)]\right)\end{array}. \quad (4)$$

The LHS is the seller's gain from selling the token now versus waiting for the next trading opportunity,[5] and the RHS is the buyer's gain.[6] Because $p^* = 1/2$ and $p(2) \to 0$, we see that the price of the token, $p^{**}$ converges to $\frac{1}{2}$ in the frequent-offer limit. Thus, the seller receives approximately 1/2 for selling the token to one buyer and 1/2 for selling information to the other buyer. Notice that this payoff is identical to that when resale was prohibited (Section 2.1). By selling the token, the seller is effectively receiving a prepayment for information.

Why is the token so valuable? Buying the token gives a buyer the opportunity to buy information as the second purchaser, at which point, the price for information is $\approx 0$. By contrast, if the other buyer buys the token, one has to purchase information at a price of 1/2. Thus, the equilibrium value of the token is $\approx 1/2$. Because, by design, tokens are scarce goods in that only one buyer can buy a token, the seller can extract close to 1/2 for that token in the frequent-offers limit. Rather than being stymied by future resale possibilities, this prepay equilibrium exploits them to secure rents for the seller.

[4] In an off-equilibrium path history where the seller sells information before the token, the seller and buyer who has bought information compete in selling it to the other buyer.

[5] How this is derived: the seller first sells the token at price $p^{**}$, then after some delay sells information to the other buyer at price $p^*$, and finally may get the opportunity to sell information to the remaining uninformed buyer at price $p(2)$. The seller's disagreement payoff is the same as her agreement payoff, subject to some delay.

[6] How this is derived: if the buyer agrees, he pays $p^{**}$, then waits until information is sold to the other buyer, and finally pays $p(2)$ for information from either of the other players. If he disagrees, there are two possibilities. The first possibility is that the next trading event is the same and leads to the same payoff as the agreement payoff; this explains the first term, in which the $(1-\gamma)$ discount reflects the gain from agreement today versus waiting for this event in the future. The other possibility is that the seller sells the token to the other buyer. In that case, this buyer anticipates paying $p^*$ for information in the future and possibly selling it to the other buyer (the one who bought the token) at price $p(2)$.

# 3 Model

## 3.1 Environment

A set of buyers $B \equiv \{b_1, \ldots, b_{n_B}\}$ interacts with a set of sellers $S \equiv \{s_1, \ldots, s_{n_S}\}$, where $n_B$ and $n_S$ are the total number of buyers and sellers respectively. We sometimes focus attention on the special case of a single seller, i.e., $n_S = 1$. The set of all agents is denoted by $A \equiv B \cup S$ and $n \equiv n_B + n_S \geq 3$ is the total number of agents. Each seller has identical information that is of value $v > 0$ to each buyer.

Information is replicable so a seller can sell information separately to each buyer, and each buyer who gains information can resell it to others. Accordingly, the set of buyers and sellers changes over time, and so we distinguish between "informed" and "uninformed" agents. Each pair of players has a "link", which reflects the potential to trade information for money if one party to that link is informed and the other is uninformed; we denote the link between players $i$ and $j$ as $ij$.

Let $T \equiv [0, \infty)$ denote the interval of *real-time*. Trading opportunities occur on the discrete-time grid $T_\Delta \equiv \{0, \Delta, 2\Delta, \ldots\}$, where $\Delta > 0$ is the period length between trading opportunities. At each time period $t$ in $T_\Delta$, a pair of agents $i$ and $j$ in $A$ is selected uniformly at random to potentially trade information, independently of the past. Therefore, the probability that a particular pair is selected is $\rho \equiv \frac{2}{n(n-1)}$. If the pair that is selected is either two uninformed players or two informed players, then play proceeds to the next period. Otherwise, the matched pair has an opportunity to trade information for a price. In period $t$, the payoff-relevant state of the system is the set of informed agents at that time. Define $\mathcal{M} \equiv \{M \subseteq A : S \subseteq M\}$ as the payoff-relevant state space, where each element $M \in \mathcal{M}$ corresponds to a feasible set of informed agents. Let $m = |M|$ be the number of informed agents when the set $M$ is clear from the context. All players have discount rate $r \geq 0$ with a per-period discount factor of $\delta \equiv e^{-r\Delta}$. Our results concern the *frictionless limit* of the market, namely as $\Delta \to 0$.

## 3.2 Solution-Concept

Our solution-concept combines *rational expectations* in how players derive their continuation values from future actions and *Nash Bargaining*, which specifies their actions given those continuation values. Continuation values, trading prices, and trading decisions condition on the set of informed players, which is the payoff-relevant state variable.

We define a value function $V : A \times \mathcal{M} \to \Re$ such that $V(i, M)$ is player $i$'s expected payoff when the set of informed players is $M$. It is measured from the start of a period, before a pair is selected for a trading opportunity. The set of feasible trades in a state $M$ depends on who can potentially sell information, and who still would like to buy information. This space is denoted as $\mathcal{T} \equiv \{(i,j,M) \in A^2 \times \mathcal{M} : i \in M, j \notin M, M \in \mathcal{M}\}$, where a trading opportunity $(i,j,M)$ denotes the possibility for $i$ to sell information to $j$ in state $M$. *Trading functions* comprise prices $p : \mathcal{T} \to \Re$ and decisions $\alpha : \mathcal{T} \to \{0,1\}$ such that $\alpha(i,j,M) = 1$ if player $i$ sells information to player $j$ in state $M$; if trade occurs, it does so at price $p(i,j,M)$. Definition 1 uses trading functions to derive equilibrium continuation values while Definition 2 uses these continuation values to derive trading functions consistent with Nash Bargaining. Definition 3 brings these two together to define an equilibrium. We use $R(m) \equiv \frac{m(m-1)+(n-m)(n-m-1)}{2}$ to denote the number of redundant links—between two informed players or two uninformed players—when the number of informed players is $m$.

**Definition 1.** *Given trading functions $(\alpha, p)$, $V$ satisfies **Rational Expectations** if for every $M \in \mathcal{M}$, and every informed player $i \in M$,*

$$V(i,M) \equiv \rho\Big[\underbrace{\sum_{j \notin M} \alpha(i,j,M)p(i,j,M)}_{Sales} + \underbrace{\sum_{i' \in M, j \notin M} \begin{pmatrix} \alpha(i',j,M)\delta V(i, M \cup \{j\}) \\ +(1-\alpha(i',j,M))\delta V(i,M) \end{pmatrix}}_{Change\ in\ Continuation\ Value} + \underbrace{\delta R(m) V(i,M)}_{Redundant\ Links}\Big],$$

*and for every uninformed player $j \notin M$,*

$$V(j,M) \equiv \rho\Big[\underbrace{\sum_{i \in M} \alpha(i,j,M)(v - p(i,j,M))}_{Purchases} + \underbrace{\sum_{i \in M, j' \notin M} \begin{pmatrix} \alpha(i,j',M)\delta V(j, M \cup \{j'\}) \\ +(1-\alpha(i,j',M))\delta V(j,M) \end{pmatrix}}_{Change\ in\ Continuation\ Value} + \underbrace{\delta R(m) V(j,M)}_{Redundant\ Links}\Big].$$

The above expressions describe a player's expected payoff from buying or selling information (and the associated change in continuation values), and how her continuation value evolves based on any other interaction that occurs in that time period.

**Definition 2.** *Given the value function $V$, trading functions $(\alpha, p)$ satisfy **Nash Bargaining** if for all $(i,j,M) \in \mathcal{T}$, $\alpha(i,j,M) = 1$ if and only if*

$$\underbrace{\delta V(i, M \cup \{j\}) + v + \delta V(j, M \cup \{j\})}_{Joint\ Surplus\ with\ Trade} \geq \underbrace{\delta V(i,M) + \delta V(j,M)}_{Joint\ Surplus\ with\ No\ Trade}, \tag{5}$$

*and $p(i,j,M)$ solves the following equation for $\tilde{p}$:*

$$(1-w)\left[\underbrace{\tilde{p} + \delta V(i, M \cup \{j\}) - \delta V(i,M)}_{Change\ in\ Seller's\ Surplus}\right] = w\left[\underbrace{v - \tilde{p} + \delta V(j, M \cup \{j\}) - \delta V(j,M)}_{Change\ in\ Buyer's\ Surplus}\right]. \tag{6}$$

The first condition stipulates that information is traded between players $i$ and $j$ if and only if their bilateral joint surplus from trade exceeds their joint surplus from not trading. The second condition stipulates that should trade occur, it does so at a price that splits the gains from trade according to the seller's bargaining weight $w$. We assume that $w \in [\frac{1}{2}, 1)$; this assumption highlights that our negative results emerge even though sellers of information have substantial bargaining power.

**Definition 3.** *An **equilibrium** is a triple $(\alpha, p, V)$ such that given $(\alpha, p)$, the continuation value function $V$ satisfies Rational Expectations and given $V$, the trading functions $(\alpha, p)$ satisfy Nash Bargaining.*

Our solution-concept combines Nash Bargaining with dynamic considerations. All decisions condition only on the set of informed players, which is the payoff-relevant state variable; thus, the solution-concept has the flavor of a Markov-Perfect Equilibrium (Maskin and Tirole, 2001). This kind of solution-concept is standard, widely used both in the prior literature and that on decentralized search and bargaining.[7] As we elaborate in Section 5, one way to view our prepay equilibrium is as a non-Markovian solution-concept where the payoff-irrelevant tokens introduce a modicum of history-dependence.

# 4 The Challenge of Information Resales

Without the prospect for resale, if there were a single seller of information, she would obtain $wv$ from each buyer, accumulating a total of $n_B wv$. In this section, we prove our negative result that without the use of tokens, an upper bound to the seller's payoff across equilibria is $wv$. Section 4.1 proves some preliminary results on prices and existence of equilibria, and Section 4.2 proves our main results.

## 4.1 Preliminary Results

We first fix a trading function $\alpha$, and given that, we derive continuation values and prices corresponding to Nash Bargaining. This trading function may be inconsistent with Nash Bargaining in that we do not impose (5) of Definition 2.[8] This intermediate step of deriving Nash Bargaining prices and continuation values that emerge from any trading function is useful for our subsequent results.

[7] While we do note pursue a non-cooperative microfoundation here, one may view each bilateral bargaining game as the outcome of matched players playing "quick" random proposer bargaining games that breakdown with some probability, as in Binmore, Rubinstein, and Wolinsky (1986).

[8] Thus the trading function could be compelling pairs to trade (resp. not trade) even if their joint surplus from trading is below (resp. above) that from not trading.

Given a trading function $\alpha$, we say that link $ij$ is *active* when the set of informed players is $M$ if $\alpha(i, j, M) = 1$. In other words, if $i \in M$ and $j \notin M$, then when players $i$ and $j$ have a trading opportunity, they trade information so that their behavior induces a transition of the set of informed players from $M$ to $M \cup \{j\}$.

**Proposition 1.** *For any trading function $\alpha$, the following hold:*

1. *There exist unique prices consistent with Rational Expectations and sellers receiving a share of surplus equal to $w$.*
2. *If for every proper subset $M' \subset A \setminus M$, there is more than one active link in state $M \cup M'$, then for all links $ij$ that are active in state $M$, $p(i, j, M) \to 0$ as $\Delta \to 0$.*
3. *If for some state $M$, there is only one active link $ij$, then*

$$\lim_{\Delta \to 0} p(i, j, M) = wv + w \lim_{\Delta \to 0} V(j, M \cup \{j\}) - (1 - w) \lim_{\Delta \to 0} V(i, M \cup \{j\}).$$

Proposition 1 establishes that the trading function pins down, for every period length, the prices and continuation values corresponding to Nash Bargaining. Furthermore, it specifies an important consequence of competition: if there are multiple active links, then prices must converge to 0 as the period length vanishes. Finally, if there is only a single active link, then the players must be dividing their joint surplus from trading according to their Nash bargaining weights.

We use Proposition 1 to prove that an equilibrium exists. Our approach is constructive. We consider a trading function where every link is active: for every $(i, j, M) \in \mathcal{T}$, $\alpha(i, j, M) = 1$. In other words, this is an immediate agreement trading function. We prove that this trading function generates prices and continuation values such that this *immediate agreement* trading function is also consistent with Nash Bargaining.

**Proposition 2.** *For every $\Delta > 0$, there exists an equilibrium.*

Because every link is active in this equilibrium, Proposition 1 implies that prices converge to 0 as $\Delta \to 0$. Immediate agreement equilibria are the focus of prior work (Polanski, 2007; Manea, 2020), which have shown that these equilibria exist in general environments where all players are *perfectly patient*. Because our model involves frictions, our method of proof is different. We use the simplicity of a complete graph to construct an equilibrium regardless of the degree of frictions.

Our focus is not on the immediate agreement equilibrium per se, other than as a way to claim equilibrium existence. Instead, our goal is to understand whether a seller's inability to capture a large fraction of the social surplus is a conclusion that holds robustly across equilibria of this setting. We turn to this question next.

## 4.2 Main Results: Bounding Prices Across All Equilibria

In Section 2.3, we see that a seller may use delay as a credible tool to obtain a non-trivial share of the surplus (in contrast to the immediate agreement equilibrium) from one of the buyers. We prove that this upper-bound is general: a monopolist can capture a non-trivial fraction of the surplus from *at most* one buyer. We prove that this is an upper-bound by first showing that once there are two or more informed players, prices converge to 0 across all equilibria in the frequent-offers limit.

**Proposition 3.** *If there are at least two informed agents, then equilibrium prices converge to* 0 *in the frictionless limit. Formally, for any equilibrium* $(\alpha, p, V)$ *and* $M \in \mathcal{M}$*, if* $|M| \geq 2$*, then for all* $s \in M,\ b \in A \backslash M$*,* $\lim_{\Delta \to 0} p(s, b, M) = 0$ *if* $\alpha(s, b, M) = 1$*.*

*Proof.* Given Proposition 1, it suffices to show that for any equilibrium, if $|M| = m \geq 2$, then there must be more than one active link. The proof proceeds by induction on the number of uninformed players.

**Base Case:** Consider the base case where there is a single uninformed player, $b$, and therefore, $M = A \backslash \{b\}$. Suppose that buyer faces a trading opportunity with an informed agent. Because, $V(i, A) = 0$ for every $i \in A$, it follows that the joint agreement surplus is $v$. The highest possible joint disagreement surplus is $\delta v < v$. Therefore, in every equilibrium, $\alpha(i, b, A \backslash \{b\}) = 1$ for every $i \in A \backslash \{b\}$. Hence, there are at least two active links, and so Proposition 1 implies that prices converge to 0.

**Inductive Step:** Now, suppose that whenever $n - |M| \leq k$, there is more than one active link. Consider the case where $n - |M| = k + 1$. Suppose towards a contradiction that there is at most one active link. In equilibrium, it cannot be that there are zero active links because this would imply disagreement payoffs of 0, in which case trading between any pair of informed and uninformed players increases joint surplus. Now, suppose that there is one active link $sb$ in state $M$. It follows that

$$\lim_{\Delta \to 0} p(s, b, M) = wv + w \lim_{\Delta \to 0} V(s, M \cup \{b\}) + (1 - w) \lim_{\Delta \to 0} V(b, M \cup \{b\}) = wv,$$

where the first equality follows from Part 3 of Proposition 1 and the second equality follows from the inductive hypothesis. Because there are at least two informed agents in state $M$ (by assumption), there exists an informed agent $s' \neq s$ for whom $\alpha(s', b, M) = 0$. For this to be consistent with Nash Bargaining,

$$\delta V(s', M \cup \{b\}) + v + \delta V(b, M \cup \{b\}) < \delta V(s', M) + \delta V(b, M).$$

By the inductive hypothesis, the LHS converges to $v$ as $\Delta \to 0$. Therefore, taking the limit of the above inequality, $\lim_{\Delta\to 0}[V(s', M) + V(b, M)] \geq v$. Moreover, $\lim_{\Delta\to 0} V(s', M) = \lim_{\Delta\to 0} V(s', M \cup \{b\}) = 0$, because $s'$ does not trade in state $M$. Thus, $\lim_{\Delta\to 0} V(b, M) \geq v$. However, $\lim_{\Delta\to 0} V(b, M) = v - \lim_{\Delta\to 0} p(s, b, M) = (1-w)v < v$, leading to a contradiction. Q.E.D.

Proposition 3 implies that once there are at least two sellers, competition forces prices to converge to 0. This Bertrand-like outcome emerges even though all trading occurs in bilateral meetings, with small frictions from delay and where pricing can be personalized.

When are prices strictly positive and bounded away from 0? A corollary of Proposition 3 is that this can happen only if (a) an information seller is a monopolist, and (b) she has not yet sold information to any buyer. Thus, prices are strictly positive only on the first sale. The result below constructs an equilibrium that attains this upper-bound: the single seller of information, $s$, designates a single buyer as the first to whom she would sell information, disagreeing with every other buyer before she does so. Such behavior garners her a limit payoff of $wv$ and as we show below, is consistent with Nash Bargaining. Her payoff here is a $1/n_B$ fraction of what she achieves were resale prohibited, and thus, when facing a large group of buyers, resale possibilities severely limit her capability to appropriate a significant fraction of social surplus. This result is a generalization of what we illustrate in Section 2.3.

**Proposition 4.** *Suppose that there is a single seller of information, $s$. There exists $\overline{\Delta} > 0$ such that for all $\Delta < \overline{\Delta}$, and for every buyer $b$, there exists an equilibrium where the seller only sells information* first *to buyer $b$, disagreeing with all other buyers until she does so. After she trades information with that buyer, the equilibrium features immediate agreement. As $\Delta \to 0$, the price charged to $b$ converges to $wv$ and all other prices converge to 0.*

*Proof.* By Proposition 2, from any starting state, there exists an equilibrium with immediate agreement. Therefore, all we need to construct is behavior in the initial state $M^0 = \{s\}$. We consider a trading function $\alpha$ such that for some buyer $b$, $\alpha(s, b, M^0) = 1$ and for all buyers $b' \neq b$, $\alpha(s, b', M^0) = 0$. It follows that

$$\lim_{\Delta\to 0} p(s, b, M^0) = wv + w \lim_{\Delta\to 0} V(b, M^0 \cup \{b\}) - (1-w) \lim_{\Delta\to 0} V(s, M^0 \cup \{b\}) = wv,$$

where the first equality follows from Part 3 of Proposition 1, and the second follows from Proposition 3. To prove that this is an equilibrium, we have to show that the the trading function $\alpha$ specified above is consistent with Nash Bargaining.

Since $sb$ is the only active link in $M^0$, the joint disagreement surplus is less than the joint agreement surplus because disagreement merely delays payoffs. Therefore, it follows that $\alpha(s, b, M^0) = 1$. Consider interactions between $s$ and any buyer $b' \neq b$. Disagreement is consistent with equilibrium as long as the following inequality holds:

$$\delta V(s, \{s, b'\}) + v + \delta V(b', \{s, b'\}) < \delta V(s, \{s\}) + \delta V(b', \{s\}).$$

Proposition 3 implies that as $\Delta \to 0$, $V(s, \{s, b'\}) \to 0$ and $V(b', \{s, b'\}) \to 0$. Because of the initial trading price, $V(s, \{s\}) \to wv$. Finally, since $b'$ does not trade until after $b$ has become informed and thus pays nothing, $V(b', \{s\}) \to v$. Therefore, as $\Delta \to 0$, the LHS converges to $v$, and the RHS converges to $(1 + w)v$, so the inequality holds for sufficient low values of $\Delta$. Thus, this trading function is consistent with Nash Bargaining. Q.E.D.

# 5 The Prepay Equilibrium

Having shown that a monopolistic seller appropriates very little of the social surplus from information, we turn to non-contractual avenues by which a monopolistic seller can do better. We construct the "prepay equilibrium" where the seller collects prepayments by selling tokens that have no intrinsic value. In this equilibrium, the seller obtains approximately $n_B wv$ in the frictionless limit, which is her payoff if buyers were unable to resell information, and thus, this prepay equilibrium solves the resale problem.

We begin by formally extending our model and solution-concept to allow for tokens. We continue to use $i$ and $j$ to denote sellers and buyers, respectively. We introduce $k$ as an index for the type of item being sold where $k = 1$ denotes a token and $k = 2$ denotes the information good itself. A generic state is now $(M, K)$ where $M$ still refers to the set of agents with the information good and $K$ refers to the set of buyers who have bought a token from the seller. The function $\alpha(i, j, k, M, K)$ is an indicator of whether $i$ sells an item of type $k$ to $j$ in state $(M, K)$. Price $p(i, j, k, M, K)$ is the price paid in the same transaction (if it occurs). Each player $i$ has value function $V(i, M, K)$.

The model of trading opportunities is the same as before, with equal probabilities of each pair trading in each period. However, a trading pair now simultaneously considers both token trades and information good trades. Nash Bargaining is defined slightly differently but in the standard way. If any trade occurs, $\alpha(i, j, 1, M, K)$ and $\alpha(i, j, 2, M, K)$ will maximize the joint surplus from trade, and the joint surplus from that trade is at least as great as the joint surplus with no trade. There could be multiplicity due to multiple types of trades giving the maximum joint surplus from trade, but since we are only constructing an equilibrium here, this does not matter. The definition of Rational Expectations undergoes a straightforward modification to account for the different types of trades possible. Rational Expectations and

Nash Bargaining are combined as before to define equilibrium.

Let us describe the trading decisions of this prepay equilibrium. For any state $(M, K)$ with $M = \{s\}$ and $|K| < n_B - 1$,

$$\alpha(i, j, k, M, K) = \begin{cases} 1 & \text{if } i = s, j \in B \backslash K, k = 1, \\ 0 & \text{otherwise.} \end{cases}$$

That is, trades occur only between the initial seller and buyers who have not yet bought tokens, and trades involve only tokens. Once $n_B - 1$ tokens have been sold, the market for tokens shuts down: $\alpha(i, j, 1, M, K) = 0$ for all $i, j$ and $K$ such that $|K| = n_B - 1$. At that stage, the information good is initially sold exclusively to the only buyer who does not have a token: for any $K$ such that $|K| = n_B - 1$,

$$\alpha(i, j, k, \{s\}, K) = \begin{cases} 1 & \text{if } i = s, j \in B \backslash K, k = 2, \\ 0 & \text{otherwise.} \end{cases}$$

If the seller faces a buyer to whom she has sold a token, that trading opportunity ends in disagreement. After the sale of information to the buyer $j$ that did not buy a token—and more generally for any state where information is possessed by at least two players—we transition to the immediate agreement equilibrium where every trading opportunity between an informed and an uninformed player results in trade: if $M \neq \{s\}$,

$$\alpha(i, j, k, M, K) = \begin{cases} 1 & \text{if } i \in M, j \notin M, k = 2, \\ 0 & \text{otherwise.} \end{cases}$$

This fully describes the trading decisions. The following result proves existence of this prepay equilibrium, its consistency with Nash Bargaining, and derives the seller's payoff in the frequent-offer limit.

**Proposition 5.** *Suppose that there is a single seller of information, $s$. There exists $\overline{\Delta} > 0$ such that for all $\Delta < \overline{\Delta}$, the prepay equilibrium exists. Moreover, as $\Delta \to 0$, the prices paid for the tokens and the first sale of the information good in the prepay equilibrium all converge to $wv$, while all other prices converge to $0$.*

The formal proof of Proposition 5 can be found in the Appendix, but the strategic logic combines that of Nash Bargaining for a scarce good (tokens) with that of Proposition 4. Let us trace this backwards, once all the tokens are sold. The remaining buyer is willing to pay the bilateral bargaining price for information ($\approx wv$) for the reasons described in

Proposition 4 (since the game is, at this stage, identical to that of Section 4.2). After this buyer buys information, then competition in the resale market ensures that the price of information incurred by all other buyers—those who bought tokens—converges to 0 (as outlined in Propositions 3 and 4).

We see that buying a token allows a buyer to buy information in the (near-)future at a price of $\approx 0$, generating a price discount of approximately $wv$ relative to the buyer who doesn't buy a token. In equilibrium, a token's value is then approximately $wv$. We now argue that the scarcity of tokens guarantees that the price of tokens must converge to $wv$.

There are $n_B - 1$ tokens but there are $n_B$ buyers. So before all the tokens are sold, when the seller meets a buyer who has not yet bought a token, the buyer has to worry about competition for tokens. On his end, the gain from buying now is that he can guarantee himself a token; if he delays, there is a chance that the next time he meets the seller, all the tokens are sold and he has to pay $wv$ for information. By contrast, for the seller, the only gain from selling this token now versus waiting is the cost of delay, which is vanishing in the frequent-offer limit. For these gains to remain magnitudes of the same proportional size, it must be that in the frequent-offer limit, the buyer's gain from buying a token is vanishing, and hence, the price of a token must approximate $wv$.

Above, we have outlined the on-path equilibrium logic for how the seller extracts $wv$ from each buyer. She must also have incentives to not sell information too early (before she has sold all of the tokens) and to not sell information initially to a buyer who has already bought a token. As we show in the proof, neither issue arises.[9]

One way to view this result, and its contrast with Propositions 3 and 4 is that our prior negative results pertained to "Markov Perfect Equilibria" where neither the seller nor buyers could condition on payoff-irrelevant state variables. In introducing tokens, we have added a degree of history-dependence where both the seller's and the buyers' behaviors condition on a payoff-irrelevant variable, namely $K$. We view the degree of history-dependence that has been added to be somewhat minimal and the strategic logic does not use reward and punishment schemes that typically feature in models of repeated games. Instead, the tokens leverage the market competition already inherent in bargaining when there is a short side of the market. Thus, one way to see this result is that this mild form of history-dependence allows the seller to not merely sidestep the resale issue but to instead exploit it.

[9] The logic in each case is straightforward. Selling the information good early (before $n_B - 1$ tokens have been sold) is inconsistent with Nash bargaining, because the market for tokens would shut down, eliminating a source of extra surplus for the monopolist seller. Moreover, selling the information good initially to a buyer who has already bought a token is inconsistent with Nash bargaining, because that buyer would not be willing to pay anything in the frictionless limit, as they could simply wait until the buyer who does not have (or will not have) a token trades for the information good.

# 6 Discussion

**Summary:** In this paper, we investigated how the possibility for resale influences the pricing of information. Robustly across equilibria, prices converge to 0 as soon as two players possess information. Accordingly, if a monopolistic seller is constrained to the option of selling or not selling information, the best she can do is obtain the share of one buyer's value of information. Once she has sold information to a buyer, then the subsequent competition from resale impedes her from charging high prices. Our results suggest that even market power, decentralized bargaining, search frictions, personalized prices, and equilibrium selection do not overcome the commitment problem that comes from resale possibilities.

Inherent in this commitment problem is an antidote to it. We show that if the seller can trade both information and worthless tokens, she can use a prepay scheme to obtain a payoff approximately equal to that of perfect intellectual property protections. In this equilibrium, the seller sells $n_B - 1$ tokens (where $n_B$ is the number of buyers). The buyer who is unable to buy a token pays the standard bilateral bargaining price for information. Buyers who buy tokens are able to buy information at a discount because buying a token ensures that one can buy information after someone else already has; in other words, a token buyer exploits the competitive effect of resale. In equilibrium, the value of a token to a buyer is the price discount that it generates, which converges to the bilateral bargaining price of information. Because tokens are scarce, the equilibrium price of a token converges to its value. Thus, the seller obtains the bilateral bargaining price of information from each buyer.

**Interpretations and Caveats:** While we have described our prepay equilibrium as involving the sale of tokens, one potential way to implement it is for the seller to divide information into complementary bits (which are uninformative on their own) where each bit is sold to a different buyer. From this perspective, our model offers a perspective on why an information seller may benefit from releasing noisy information over time when she worries about that information being resold.

Our work necessarily connects to the rich debate on intellectual property protections, whether they are necessary, and how copyrights and patents might stimulate or impede innovation. While our negative result suggests a basic tension that information sellers face without protection from resale, our positive result offers a non-contractual solution. This solution does require an ability to secure prepayment from all parties prior to selling information. Thus, in contrast to permanent intellectual property protection, the seller can extract surplus from the set of buyers present at a time but may be unable to do so from new buyers that enter the market. That said, this may be efficient in that it generates sufficient incentives for the initial seller to acquire information, and the low price that future buyers face encourages their entry.

We have used a stylized model to make transparent the strategic forces inherent in information resale. We assume that players are identical in their value for information and pairwise bargaining weights. These assumptions are made for analytical tractability. The difficulty with heterogeneity is in extending our existence result (Proposition 1) because we can no longer construct an equilibrium directly. But if an equilibrium exists, the building block of both our negative and positive result—Proposition 3—still applies. Thus, without tokens, a monopolistic seller cannot charge much for information after selling it to one player. But with tokens, the monopolistic seller can exploit low information prices in the future to charge more for tokens upfront.

Our setting is one where information is non-rival in consumption but where buyers are excludable. In a number of settings, one may envision payoff externalities in the consumption of information. In financial markets, information may be more valuable the less of it that others have. In technology adoption decisions, there may be direct complementarities in others using the same technology or having the same knowhow. We abstract from these externalities. We anticipate that the strategic forces that we study remain in richer settings, although the strength of these forces will be moderated or amplified by these externalities.

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# A Appendix

*Proof of Proposition 1 on page 13.* For a trading decision function $\alpha$, let

$$B(i, M) \equiv \{j \in A \setminus M : \alpha(i, j, M) = 1\},$$
$$S(j, M) \equiv \{i \in M : \alpha(i, j, M) = 1\}.$$

When the set of informed players is $M$, $B(i, M)$ is the set of buyers who trade with seller $i$, and $S(j, M)$ is the set of sellers who trade with buyer $j$. Let $\mathcal{L}(M) \equiv \{ij|\alpha(i, j, M) = 1\}$ denote the set of active links and $\mathcal{L}^c(M)$ denote its complement. Finally, let $\hat{\rho}(\delta, M) \equiv \frac{\rho}{1-\delta\rho|\mathcal{L}^c(M)|}$, where recall that $\rho \equiv \frac{2}{n(n-1)}$ is the probability that a specific link is recognized. To economize on notation, we suppress the arguments of $\hat{\rho}$ but note that in the frequent offer limit, $\hat{\rho}(\delta, M) \to \frac{1}{|L(M)|}$.

Our proof follows by induction.

Base Case: Suppose that $|A \setminus M| = 1$, with the remaining uninformed buyer being player $j$. For any $i \in S(j, M)$, Nash Bargaining implies that

$$\frac{(1-w)}{w} \times \underbrace{p(i, j, M)\,(1 - \delta\hat{\rho})}_{\text{Seller } i's \text{ gain from trade}} = \underbrace{v - p(i, j, M) - \delta\hat{\rho}\left(|\mathcal{L}(M)|v - \sum_{i' \in S(j,M)} p(i', j, M)\right)}_{\text{Buyer } j\text{'s gain from trade}},$$

where recall that $w$ is the seller's bargaining weight. This equation can be re-written as

$$p(i, j, M) - \frac{w\delta\hat{\rho}}{1-\delta\hat{\rho}} \sum_{i' \in S(j,M) \setminus \{i\}} p(i', j, M) = \frac{w(1 - \delta\hat{\rho}|\mathcal{L}(M)|)}{1-\delta\hat{\rho}} v.$$

An analogous equation holds for any other $i'$ such that $i' \in S(j, M)$, and subtracting that equation from the one above implies that $p(i, j, M) = p(i', j, M)$, so prices are symmetric. Therefore, for every $ij \in \mathcal{L}(M)$,

$$p(i, j, M) = \frac{w(1 - \delta\hat{\rho}|\mathcal{L}(M)|)}{1 - \delta\hat{\rho}(w|\mathcal{L}(M)| + (1-w))} v,$$

which proves that a unique solution exists. Note that if $|\mathcal{L}(M)| = 1$, then $p(i, j, M) = w$. If $\mathcal{L}(M) > 1$, then $\lim_{\Delta \to 0} p(i, j, M) = 0$.[10]

Inductive Step: Suppose that for every $M'$ with $|M'| \geq x$, prices $p(i, j, M')$ are uniquely

[10] As $\Delta \to 0$, $\delta\hat{\rho}|\mathcal{L}(M)| \to 1$, which implies that the numerator converges to 0. The denominator converges to $(1-w)(1 - (|L(M)|)^{-1})$, which is strictly positive for $|L(M)| \geq 2$.

determined for every $ij \in \mathcal{L}(M')$, and if $|\mathcal{L}(M')| \geq 2$ these prices all converge to 0 as $\Delta \to 0$. Prices being uniquely determined implies that for each player $k$, $V(k, M')$ is also unique (by Rational Expectations and the inductive hypothesis). We first argue that prices in $M$ with $|M| = x - 1$ must also be unique.

For any $ij \in \mathcal{L}(M)$, Recursive Nash Bargaining implies that

$$\begin{aligned}(1-w)\left[p(i,j,M) + \delta V(i, M \cup \{j\}) - \delta V(i,M)\right] \\ = w\left[v - p(i,j,M) + \delta V(j, M \cup \{j\}) - \delta V(j,M)\right]. \quad (7)\end{aligned}$$

Using Rational Expectations to expand the value functions in (7), we obtain

$$\begin{aligned}(1-w)&\left[p(i,j,M) + \delta V(i, M\cup\{j\}) - \delta\hat{\rho}\left[\sum_{j' \in B(i,M)} p(i,j',M) + \delta \sum_{i'j' \in \mathcal{L}(M)} V(i, M \cup \{j'\})\right]\right] \\ = w&\left[v - p(i,j,M) + \delta V(j, M \cup \{j\}) - \delta\hat{\rho}\left[\sum_{i' \in S(j,M)} [v - p(i',j,M)] + \delta \sum_{i'j' \in \mathcal{L}(M)} V(j, M \cup \{j'\})\right]\right].\end{aligned}$$

We collect all terms that involve prices in state $M$ on the LHS and others on the RHS. Define

$$\begin{aligned}\kappa(i,j,M) \equiv w(1 - \delta\hat{\rho}|S(j,M)|)v + w\delta&\left[V(j, M \cup \{j\}) - \hat{\rho}\delta \sum_{i'j' \in \mathcal{L}(M)} V(j, M \cup \{j'\})\right] \\ - (1-w)\delta&\left[V(i, M\cup\{j\}) - \hat{\rho}\delta \sum_{i'j' \in \mathcal{L}(M)} V(i, M \cup \{j'\})\right].\end{aligned}$$

Then it follows that

$$\kappa(i,j,M) = [1 - \delta\hat{\rho}]p(i,j,M) - (1-w)\delta\hat{\rho} \sum_{j' \in B(i,M)\setminus\{j\}} p(i,j',M) - w\delta\hat{\rho} \sum_{i' \in S(j,M)\setminus\{i\}} p(i',j,M).$$

Note that $\kappa(i,j,M)$ is uniquely determined in equilibrium since it depends only on parameters and future continuation values, which are uniquely determined. Similar equations hold for every active link between a buyer and a seller in state $M$.

We index these links as $1, \ldots, |\mathcal{L}(M)|$ (the specific indexing is unimportant) and define a $|\mathcal{L}(M)| \times |\mathcal{L}(M)|$ matrix $\Phi(M)$ where $\Phi_{uv}(M) = w$ if link $u$ and link $v$ are distinct active links and share a common buyer, $\Phi_{uv}(M) = 1 - w$ if link $u$ and link $v$ are distinct active links and share a common seller, and $\Phi_{uv}(M) = 0$ otherwise. Combining the $|\mathcal{L}(M)|$ equations yields the following matrix equation:

$$\left[I_{|\mathcal{L}(M)|} - \frac{\delta\hat{\rho}}{1-\delta\hat{\rho}}\Phi(M)\right]\vec{p}(M) = \frac{1}{1-\delta\hat{\rho}}\vec{\kappa}(M) \tag{8}$$

Here, $\vec{p}(M)$ and $\vec{\kappa}(M)$ are $|\mathcal{L}(M)| \times 1$ vectors consisting of all of the state $M$ prices and values of $\kappa$. Since the right hand side of this last equation is unique, the current state price vector $\vec{p}(M)$ is unique if matrix $\Psi(M) \equiv \left[I_{|\mathcal{L}(M)|} - \frac{\delta\hat{\rho}}{1-\delta\hat{\rho}}\Phi(M)\right]$ is invertible.

Note that $\Psi(M)$ is a Z-matrix because the off-diagonal elements are all negative. Additionally, we can show that $\Psi(M)$ exhibits semipositivity; that is, there exists a vector $\vec{x} > 0$ such that $\Psi(M)\vec{x} > 0$.[11] Being a Z-matrix that exhibits semipositivity is equivalent to $\Psi(M)$ being a non-singular M-matrix and thus invertible (Plemmons, 1977). This completes the proof of existence and uniqueness of the price vector.

To show that prices converge to 0 whenever more than one link is active in the current and all subsequent states (the $M \cup M'$ in the statement of the lemma), consider the values of $\kappa(i,j,M)$. From the inductive hypothesis, every informed agent's continuation payoff on a subsequent state converges to 0 (they capture nothing from resale). Moreover, every buyer's continuation payoff on a subsequent state converges to $v$ (they pay nothing to buy, but they capture nothing from resale). Also, note that $\hat{\rho} \to \frac{1}{|\mathcal{L}(M)|}$. This implies that

$$\kappa(i,j,M) \to w\left(1 - \frac{|S(j,M)|}{|\mathcal{L}(M)|}\right)v - w\left[\frac{1}{|\mathcal{L}(M)|}\sum_{i'j' \in \mathcal{L}(M), j' \neq j} v\right] = 0$$

Since the solution price vector is unique, and the right hand side of the price equation $\to 0$ (when $|\mathcal{L}(M)| > 1$), the solution price vector must also $\to 0$.

When $|\mathcal{L}(M)| = 1$, the right hand side of Equation (8) takes on an indeterminate form in the limit. In this case, $I_{|\mathcal{L}(M)|} = 1$. By the definition of matrix $\Phi(M)$, the only non-zero entries correspond to pairs of distinct active links, of which there are none, so $\Phi(M) = 0$. Substituting in the expression for $\kappa(i,j,M)$ and taking the limit $\Delta \to 0$,

$$\lim_{\Delta \to 0} p(i,j,M) = \lim_{\Delta \to 0} \frac{\kappa(i,j,M)}{1-\delta\hat{\rho}} = wv + w \lim_{\Delta \to 0} V(j, M \cup \{j\}) - (1-w) \lim_{\Delta \to 0} V(i, M \cup \{j\})$$

[11] This is shown by using for $\vec{x}$ a vector of all ones and noting that for all $u$,

$$[\Psi(M)\vec{x}]_u = 1 - [(1-w)|B(i,M)| + w|S(j,M)| - 1]\frac{\delta\hat{\rho}}{1-\delta\hat{\rho}} > 1 - (|\mathcal{L}(M)| - 1)\frac{\frac{\rho}{1-\rho|\mathcal{L}^c(M)|}}{1 - \frac{\rho}{1-\rho|\mathcal{L}^c(M)|}}$$
$$= 1 - \frac{\rho(|\mathcal{L}(M)| - 1)}{1 - \rho[|\mathcal{L}^c(M)| + 1]} = 0.$$

Q.E.D.

*Proof of Proposition 2 on page 13.* We define "immediate agreement" as $\alpha(i, j, M) = 1$ for all $(i, j, M) \in \mathcal{T}$. We first prove that for every $\Delta$, immediate agreement prices are symmetric: $\forall M$ and $\forall i, i' \in M, \forall j, j' \in A \setminus M, p(i, j, M) = p(i', j', M) \equiv p(m)$. We establish this claim by induction. The base case $(m = n - 1)$ symmetry was already proven for Proposition 1.

Inductive Step: Suppose that for every $M'$ with $|M'| \geq x$, prices $p(i, j, M')$ are symmetric. Since all seller/buyer links are active, we write the price equation by indexing links as $1, \ldots, [m(n-m)]$. $\Phi(M)$ and $\Psi(M)$ are defined in the same way, so the price equation is

$$\Psi(M)\vec{p}(M) = \frac{1}{1 - \delta\rho(\delta, m)}\vec{\kappa}(M),$$

where $\rho(\delta, m) \equiv \frac{\rho}{1-\delta\rho R(m)}$.

To see that prices are symmetric, first notice that price symmetry in the inductive hypothesis implies that all future buyer continuation payoffs are the same and all future seller continuation payoffs are the same, and these depend only on the number of sellers. Therefore, $\kappa(i, j, M)$ does not depend on the identities of the particular buyer or seller or the precise configuration, so $\vec{\kappa}(M)$ can be written as $\vec{\kappa}(M) = \kappa(m)\vec{1}$, where $\kappa(m)$ is some scalar, and $\vec{1}$ is the vector of all ones. Note that if prices are symmetric, there exists a scalar $p(M)$ such that $\vec{p}(M) = p(M)\vec{1}$. Then, $\Psi(M)\vec{p}(M) = p(M)\Psi(M)\vec{1}$. Because of the symmetry of the network (in each state, every buyer is linked to the same number of sellers, and every seller is linked to the same number of buyers), every row of $\Psi(M)$ has the same sum. Moreover, this row sum depends only on the number of sellers. Therefore, $\Psi(M)\vec{1} = s(m)\vec{1}$, where $s(m)$ is the (scalar) row sum. Now, the price equation can be written as $p(M)s(m)\vec{1} = \frac{1}{1-\delta\rho(m)}\kappa(m)\vec{1}$. For any given $\kappa(m)$, there must exist a $p(M)$ satisfying this equation. Moreover, it must depend only on $m$. Because the pricing equation determines unique prices, $p(M)$ is this unique, symmetric price. In other words, for all $M$ and for all $i, i' \in M$ and all $j, j' \in A \setminus M$, $p(i, j, M) = p(i', j', M)$, and this price may be written as $p(m)$.

We use this symmetry result to write state $M$ prices as $p(m)$, buyer continuation payoffs as $V^b(m)$, and seller continuation payoffs as $V^s(m)$.

$$\begin{aligned}
V^s(m) &= \rho(\delta, m)\left[(n-m)p(m) + \delta m(n-m)V^s(m+1)\right], \\
V^b(m) &= \rho(\delta, m)\left[m[v - p(m)] + \delta m(n-m-1)V^b(m+1) + \delta m V^s(m+1)\right], \\
p(m) &= w(v + \delta V^s(m+1) - \delta V^b(m)) - (1-w)(\delta V^s(m+1) - \delta V^s(m)).
\end{aligned}$$

We use the following lemma, which we prove at the end of the Appendix.

**Lemma 1.** *Suppose that trading decisions are immediate agreement, i.e.,* $[\alpha(i,j,M) = 1, \forall i,j,M]$*. Then for every* $m$*, the Nash Bargaining prices,* $p(m)$ *are non-negative.*

Using Lemma 1, to show that an immediate agreement equilibrium exists it suffices to show that the joint surplus from agreement exceeds the joint surplus from disagreement, for all $m$ (where $m$ is the number of informed players):

$$v + 2\delta V^s(m+1) - \delta[V^s(m) + V^b(m)] \geq 0$$

We start with the difference between the seller's immediate and delayed next step continuation payoffs. Then, we use the rational expectations equation to step the value functions forward:

$$\begin{aligned}
&V^s(m) - \delta V^s(m+1) = \rho(\delta,m)[(n-m)p(m) + m(n-m)\delta V^s(m+1)] - \delta V^s(m+1)\\
&= \rho(\delta,m)(n-m)p(m) - [1 - m(n-m)\rho(\delta,m)]\delta V^s(m+1)\\
&\leq \rho(\delta,m)(n-m)p(m)
\end{aligned}$$

Since $V^s(m) \geq 0, p(m) \geq 0, \delta < 1$, and $\rho(\delta,m)(n-m) < 1$, we can reduce the left hand side and increase the right hand side of this inequality to yield

$$\delta[V^s(m) - V^s(m+1)] \leq p(m)$$

Using the formula for Nash Bargaining prices,

$$\begin{aligned}
&\Leftrightarrow \delta[V^s(m) - V^s(m+1)] \leq w(v + \delta V^s(m+1) - \delta V^b(m)) - (1-w)(\delta V^s(m+1) - \delta V^s(m))\\
&\Leftrightarrow \delta[V^s(m) - V^s(m+1)] \leq wv + w\delta[V^s(m+1) - V^b(m)] + (1-w)\delta(V^s(m) - V^s(m+1))\\
&\Leftrightarrow w\delta[V^s(m) - V^s(m+1)] \leq wv + w\delta[V^s(m+1) - V^b(m)]\\
&\Leftrightarrow \delta[V^s(m) - V^s(m+1)] \leq v + \delta[V^s(m+1) - V^b(m)]\\
&\Leftrightarrow v + 2\delta V^s(m+1) - \delta[V^b(m) + V^s(m)] \geq 0
\end{aligned}$$

Note that the bounding argument above used the non-negative prices result in Lemma 1 and the fact that all seller continuation payoffs are non-negative, which is a direct consequence of non-negative prices. Q.E.D.

*Proof of Proposition 5 on page 17.* In any state where $M = \{s\}$ and $|K| = n_B - 1$, it is clear that the equilibrium conditions are satisfied for low $\Delta$ by Proposition 4. Moreover, the equilibrium conditions are satisfied in all off the equilibrium path states, as the immediate agreement equilibrium always exists (Proposition 2). In both of these cases, note that tokens could hypothetically continue to be traded, but afterwards there is immediate agreement regardless, so there are no gains from token trade and $\alpha(i, j, 1, M, K) = 0$ is consistent with Nash Bargaining. We proceed by induction on $|K|$.

As the base case, suppose $M = \{s\}$ and $|K| = n_B - 1$, so that all tokens have been sold but the information good has not. This is analogous to the setting of Proposition 4, so everything is consistent with equilibrium and the price converges to $wv$. Futhermore, the prices and continuation payoffs are trivially symmetric. That is, they depend only on the number of tokens sold thus far and in the case of a buyer's continuation payoff, whether or not the buyer has already bought a token.

For the inductive step, suppose that for all $(M, K)$ with $M = \{s\}$ and $|K| = x + 1$ for some $x \in \{0, 1, \ldots, n_B - 2\}$, the equilibrium conditions are met (for low $\Delta$), limit prices are as in the statement of the Proposition, and prices and continuation payoffs are symmetric as previously described. We now consider states where $M = \{s\}$ and $|K| = x$.

First, consider trade of tokens to buyers $j \in K$. If the trade were to take place, the state would not change, and thus the joint surplus from trade and the joint surplus with no trade are equal. Thus, the equilibrium conditions are satisfied by $\alpha(i, j, 1, M, K) = 0$.[12] The same argument also justifies no trade in the second-hand market for tokens, i.e., from seller $i \in K$ to buyer $j \in B \setminus K$. Since initial seller $s$ does not record this buyer as having bought *from them*, the state does not advance, so there are no joint gains from trade.

Next, we consider trade of tokens from initial seller $s$ to buyers $j \in B \setminus K$. We use $\hat{\rho}$ as in the proof of Proposition 1, noting that the active links are those between $s$ and buyers $j \in B \setminus K$. The price is

$$p(s, j, 1, M, K) = w\delta \left[V(j, M, K \cup \{j\}) - V(j, M, K)\right] - (1 - w)\delta \left[V(s, M, K \cup \{j\}) - V(s, M, K)\right],$$

which can be re-written as

$$\begin{aligned} p(s, j, 1, M, K) =& w\delta \left( V(j, M, K \cup \{j\}) - \hat{\rho} \left[ -p(s, j, 1, M, K) + \sum_{j' \in B \setminus K} \delta V(j, M, K \cup \{j'\}) \right] \right) \\ & - (1 - w)\delta \left( V(s, M, K \cup \{j\}) - \hat{\rho} \sum_{j' \in B \setminus K} \left[ p(s, j', 1, M, K) + \delta V(s, M, K \cup \{j'\}) \right] \right). \end{aligned}$$

[12] If this seems to be an artifact of our particular definition of the state space, note that the only value created by trading tokens is in allowing the state to advance to where the information good is distributed. Since trading a token to someone who already has one does not affect the progression of states in the future (in this particular equilibrium), there is no net joint surplus from the trade.

Re-arranging terms yields

$$
\begin{aligned}
&(1-\delta\hat{\rho})p(s,j,1,M,K) - \delta\hat{\rho}(1-w)\sum_{j'\in B\setminus(K\cup\{j\})} p(s,j',1,M,K) \\
&= w\left[(1-\delta\hat{\rho})\delta V(j,M,K\cup\{j\}) - \delta\hat{\rho}\sum_{j'\in B\setminus(K\cup\{j\})}\delta V(j,M,K\cup\{j'\})\right] \\
&- (1-w)\left[(1-\delta\hat{\rho})\delta V(s,M,K\cup\{j\}) - \delta\hat{\rho}\sum_{j'\in B\setminus(K\cup\{j\})}\delta V(s,M,K\cup\{j'\})\right] \qquad (9)
\end{aligned}
$$

We can write this more compactly as

$$
\Psi(M,K)\vec{p}(M,K) = \frac{1}{1-\delta\hat{\rho}}\vec{\kappa}(M,K)
$$

where $\Psi(M,K)$ is the $[n_B-|K|]\times[n_B-|K|]$ matrix with 1 along the diagonal and $-\frac{\delta\hat{\rho}}{1-\delta\hat{\rho}}(1-w)$ everywhere else, $\vec{p}(M,K)$ is the $[n_B-|K|]$-vector of prices of tokens being traded in the current state, and $\vec{\kappa}(M,K)$ is the $[n_B-|K|]$-vector of entries as in the right hand side of Equation (9). $\Psi(M)$ is a non-singular M-matrix (see the proof of Proposition 1 for an analogous proof). By the inductive hypothesis, this implies that the Nash bargaining prices in the current state exist and are unique. Also by the inductive hypothesis, $\frac{1}{1-\delta\hat{\rho}}\vec{\kappa}(M,K) \to wv-(1-w)wv$. This implies that as $\Delta \to 0$, $p(s,j,1,M,K) \to wv$ for all $j \in B \setminus K$.

Also, note that each element of $\vec{\kappa}(M,K)$ is the same by the symmetry part of the inductive hypothesis, and since the matrix $\Psi(M,K)$ has a constant row sum that depends only on $|K|$, the prices only depend on $|K|$. We will write these prices as $p(|K|)$. Moreover, since all current continuation payoffs depend only on $p(|K|)$ and future continuation payoffs (which by the inductive hypothesis only depend on $|K|$), the current continuation payoffs depend only on $|K|$, whether the agent is the seller $s$, and whether the agent is a buyer who already bought a token or not. Continuation payoffs will be denoted $V^s(|K|)$ for the seller, $V^{b+}(|K|)$ for a buyer who already possesses a token, and $V^{b-}(|K|)$ for a buyer who does not yet possess a token. Note that the discussion of this paragraph has been all conditional on $M=\{s\}$, and thus $M$ is suppressed in the notation; different sets $M$ give different continuation payoffs.

We now consider the continuation payoffs at the token bargaining stage and that trading occurs only if the joint surplus from agreement is weakly more from that of disagreement:

$$
\begin{aligned}
V^s(|K|) - \delta V^s(|K|+1) =& \hat{\rho}(n_B-|K|)[p(|K|)+\delta V^s(|K|+1)] - \delta V^s(|K|+1) \\
=& \hat{\rho}(n_B-|K|)p(|K|) - [1-\hat{\rho}(n_B-|K|)]\delta V^s(|K|+1) \\
\leq& \hat{\rho}(n_B-|K|)p(|K|)
\end{aligned}
$$

which implies that $\delta[V^s(|K|) - V^s(|K|+1)] \leq p(|K|)$. This is equivalent to $\delta[V^s(|K|) - V^s(|K|+1)] \leq w(\delta V^{b+}(|K|+1) - \delta V^{b-}(|K|)) - (1-w)(\delta V^s(|K|+1) - \delta V^s(|K|))$, which is equivalent to $w\delta[V^s(|K|) - V^s(|K|+1)] + w\delta[V^{b-}(|K|) - V^{b+}(|K|+1)] \leq 0$, and therefore, $\delta[V^{b+}(|K|+1) + V^s(|K|+1)] \geq \delta[V^{b-}(|K|) + V^s(|K|)]$.[13]

Finally, consider potential trade of the information good in this same inductive step (that is, when $|K| < n_B - 1$). For any buyer $j$, we will show that the following inequality holds in the limit:

$$v + \delta V(s, M \cup \{j\}, K) + \delta V(b, M \cup \{j\}, K) \leq \delta V(s, M, K) + \delta V(j, M, K)$$

Since selling the information good to $j$ triggers immediate agreement in the future, both $V(j, M \cup \{j\}, K)$ and $V(s, M \cup \{j\}, K) \to 0$. Since the seller has at least one token remaining to sell, $\lim_{\Delta \to 0} V(s, M, K) > wv$. At worst, the buyer will still have to pay in order to get the information good, so $\lim_{\Delta \to 0} V(j, M, K) \geq (1-w)v$. Therefore, for sufficiently low $\Delta$, the inequality holds.

Combining these two inequalities (one for tokens in isolation, and one for the information good in isolation), it is clear for this pair that trading only the token is at least as good as trading nothing, which is at least as good as trading only the information good. Finally, note that conditional on trading the information good (which triggers immediate agreement and no future token trade), trading the token as well changes nothing about the future continuation payoffs, so the joint surplus from trading both is tied for lowest. Thus, trading the token but not the information good is consistent with equilibrium (for low enough $\Delta$). This completes the inductive step, as we have demonstrated the correct limit prices, price and continuation payoff symmetry, and trading functions that are consistent with equilibrium (for low $\Delta$). Q.E.D.

*Proof of Lemma 1 on p. 25.* Consider all markets of at least 3 agents ($n \geq 3$), because the case with two agents is trivial. The proof proceeds by induction over the number of informed agents

Base Case: If $m = n - 1$ then by adapting the base case pricing formula from the proof of Proposition 1, we get

$$p(m) = \frac{w[1 - \delta\rho(\delta, m)(n-1)]}{1 - \delta\rho(\delta, m)[w(n-1) + (1-w)]} v \geq 0$$

Inductive Step: Given that $p(m+1) \geq 0$, we now consider the case of $p(m)$.

[13]Note that the above bounding argument assumed that $p(|K|) \geq 0$, which must be true for sufficiently small $\Delta$, as prices are continuous as a function of $\Delta$.

The proof proceeds by writing the Nash bargaining pricing equation, stepping the value functions forward, and rewriting in terms of the next step price $p(m+1)$:

$$
\begin{aligned}
p(m) &= w(v + \delta V^s(m+1) - \delta V^b(m)) - (1-w)(\delta V^s(m+1) - \delta V^s(m)) \\
&= w(v + \delta V^s(m+1) \\
&\qquad - \delta\rho(\delta, m)\left[m[v - p(m)] + \delta m(n-m-1)V^b(m+1) + \delta m V^s(m+1)\right]) \\
&\quad - (1-w)(\delta V^s(m+1) - \delta\rho(\delta, m)\left[(n-m)p(m) + \delta m(n-m)V^s(m+1)\right]) \\
&= w[1 - \delta\rho(\delta, m)m]v + w\delta V^s(m+1) + [wm + (1-w)(n-m)]\delta\rho(\delta, m)p(m) \\
&\quad - \delta^2 w\rho(\delta, m)m(n-m-1)V^b(m+1) - \delta^2 w\rho(\delta, m)mV^s(m+1) - (1-w)\delta V^s(m+1) \\
&\qquad + (1-w)\delta^2\rho(\delta, m)m(n-m)V^s(m+1) \\
&= w[1 - \delta\rho(\delta, m)m]v + [wm + (1-w)(n-m)]\delta\rho(\delta, m)p(m) \\
&\quad + \delta(2w-1)V^s(m+1) - \delta^2\rho(\delta, m)m[w(n-m-1)V^b(m+1) \\
&\quad + (w - (1-w)(n-m))V^s(m+1)] \\
&= w[1 - \delta\rho(\delta, m)m]v + [wm + (1-w)(n-m)]\delta\rho(\delta, m)p(m) \\
&\quad + \delta(2w-1)\rho(\delta, m)[(n-m)p(m+1) + \delta m(n-m)V^s(m+2)] \\
&\quad - \delta^2\rho(\delta, m)m[w(n-m-1)V^b(m+1) + (w - (1-w)(n-m))V^s(m+1)] \\
&= w[1 - \delta\rho(\delta, m)m(n-m)]v + [wm + (1-w)(n-m)]\delta\rho(\delta, m)p(m) \\
&\quad + \delta(2w-1)\rho(\delta, m)(n-m)p(m+1) \\
&\quad - \delta^2(2w-1)\rho(\delta, m)m[V^s(m+1) - V^s(m+2)] \\
&\quad + \delta\rho(\delta, m)m(n-m-1)[w(v + \delta V^s(m+2) - \delta V^b(m+1)) \\
&\quad - (1-w)(\delta V^s(m+2) - \delta V^s(m+1))]
\end{aligned}
$$

Thus, $p(m) \geq 0$ if the following holds:

$$
\begin{aligned}
&\delta(2w-1)\rho(\delta, m)(n-m)p(m+1) - \delta^2(2w-1)\rho(\delta, m)m[V^s(m+1) - V^s(m+2)] \\
&\quad + \delta\rho(\delta, m)m(n-m-1)p(m+1) \geq 0
\end{aligned}
$$

Recall that $w \geq \frac{1}{2}$. Using the fact that $V^s(m+1) - V^s(m+2) \leq p(m+1)$ and replacing $w$ with 1 and replacing $\delta^2$ with $\delta$:

$$
p(m) \geq \delta(2w-1)\rho(\delta, m)(n-m)p(m+1) + \delta\rho(\delta, m)m(n-m-2)p(m+1)
$$

The base case had $n - m - 1 = 0$, so it must be that $n - m - 2 \geq 0$. Thus, $p(m) \geq 0$.

Q.E.D.