(5.1)

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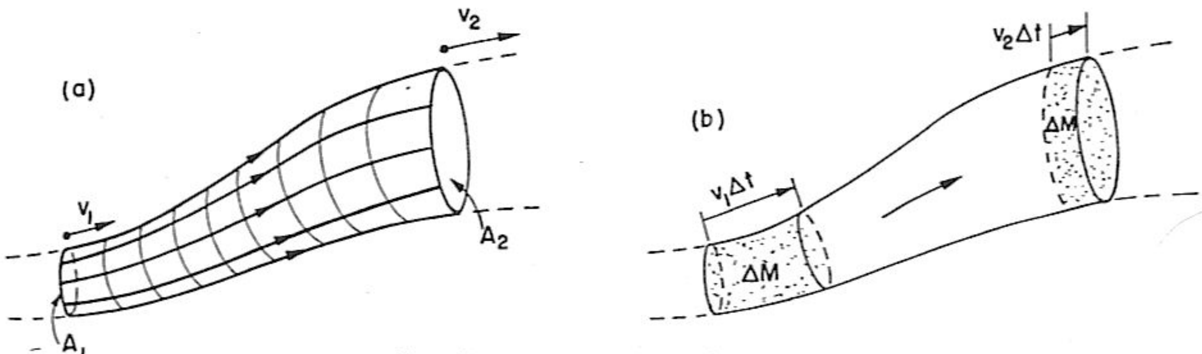

Fig. 40–6. Fluid motion in a flow tube.

The theorem of Bernoulli is in fact nothing more than a statement of the conservation of energy. A conservation theorem such as this gives us a lot of information about a flow without our actually having to solve the detailed equations. Bernoulli's theorem is so important and so simple that we would like to show you how it can be derived in a way that is different from the formal calculations we have just used. Imagine a bundle of adjacent streamlines which form a stream tube as sketched in Fig. 40–6. Since the walls of the tube consist of streamlines, no fluid flows out through the wall. Let's call the area at one end of the stream

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tube A_1, the fluid velocity there v_1, the density of the fluid ρ_1, and the potential energy ϕ_1. At the other end of the tube, we have the corresponding quantities A_2, v_2, ρ_2, and ϕ_2. Now after a short interval of time Δt, the fluid at A_1 has moved a distance $v_1\,\Delta t$, and the fluid at A_2 has moved a distance $v_2\,\Delta t$ [Fig. 40–6(b)]. The conservation of *mass* requires that the mass which enters through A_1 must be equal to the mass which leaves through A_2. These masses at these two ends must be the same:

$$\Delta M = \rho_1 A_1 v_1\,\Delta t = \rho_2 A_2 v_2\,\Delta t.$$

So we have the equality

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2. \tag{40.15}$$

This equation tells us that the velocity varies inversely with the area of the stream tube if ρ is constant.

Now we calculate the work done by the fluid pressure. The work done on the fluid entering at A_1 is $p_1 A_1 v_1\,\Delta t$, and the work given up at A_2 is $p_2 A_2 v_2\,\Delta t$. The net work on the fluid between A_1 and A_2 is, therefore,

$$p_1 A_1 v_1\,\Delta t - p_2 A_2 v_2\,\Delta t,$$

which must equal the increase in the energy of a mass ΔM of fluid in going from A_1 to A_2. In other words,

$$p_1 A_1 v_1\,\Delta t - p_2 A_2 v_2\,\Delta t = \Delta M(E_2 - E_1), \tag{40.16}$$

where E_1 is the energy per unit mass of fluid at A_1, and E_2 is the energy per unit mass at A_2. The energy per unit mass of the fluid can be written as

$$E = \tfrac{1}{2}v^2 + \phi + U,$$

where $\frac{1}{2}v^2$ is the kinetic energy per unit mass, ϕ is the potential energy per unit mass, and U is an additional term which represents the internal energy per unit mass of fluid. The internal energy might correspond, for example, to the thermal energy in a compressible fluid, or to chemical energy. All these quantities can vary from point to point. Using this form for the energies in (40.16), we have

$$\frac{p_1 A_1 v_1\,\Delta t}{\Delta M} - \frac{p_2 A_2 v_2\,\Delta t}{\Delta M} = \frac{1}{2}v_2^2 + \phi_2 + U_2 - \frac{1}{2}v_1^2 - \phi_1 - U_1.$$

But we have seen that $\Delta M = \rho A v\,\Delta t$, so we get

$$\frac{p_1}{\rho_1} + \frac{1}{2}v_1^2 + \phi_1 + U_1 = \frac{p_2}{\rho_2} + \frac{1}{2}v_2^2 + \phi_2 + U_2, \tag{40.17}$$

which is the Bernoulli result with an additional term for the internal energy. If the fluid is incompressible, the internal energy term is the same on both sides, and we get again that Eq. (40.14) holds along any streamline.

for us we have $\phi_1 = 0$

$\phi_2 = 0$

since we are in outer space away from any gravity fields etc.