Astronomy 485 — Problem Set 2 — Weeks 3 and 4

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All problems are worth 10 points.

1. Please write an $\approx$ 2 page report on the Pierre Auger Observatory (PAO). There is lots of information on this project on the World Wide Web (use a World Wide Web search engine); for example, see http://www.auger.org. Your report should address the following issues as well as other ones that you think are important.

- What is the basic idea behind the PAO? What are people trying to do by building this observatory? Why is it an advance over current cosmic ray observatories?
- Where is it being built?
- What are some of the possible physical origins of the cosmic rays it is detecting? What is the Greisen, Zatsepin & Kuz'min (GZK) cutoff?
- What is the basic idea behind the cosmic ray detection process?
- Who was Pierre Auger and why is the observatory named after him?
- Does Penn State have any connection to the PAO?

Please document your work by giving the references that you used.

2. (a) In class we derived the emitted synchrotron power from one electron spiraling along a magnetic field with a speed perpendicular to the magnetic field of $v_{\perp}$. In astronomical observations, we cannot observe one electron at a time but rather observe the average emission from many electrons. If we assume an isotropic velocity distribution of electrons each with speed v, please find the average emitted power per electron (your final equation should no longer include $v_{\perp}$ but only v). Please clearly show all your work when evaluating the relevant integral. Hint: Average the synchrotron power equation over angle.

2. (b) We will now examine an effect that is very relevant to synchrotron radiation called the beaming effect. Consider an object moving with ultrarelativistic ($\beta \approx 1$, $\gamma \gg 1$) speed in the $+x$ direction. Starting with the basic Lorentz transformations, show that most photons emitted by this object will appear to be concentrated into a cone along the x-axis with half-angle $\approx 1/\gamma$.

3. The Compton effect. A photon of energy $E_{\rm i}$ scatters off a stationary electron through an angle θ. Subsequent to the scattering, the photon has energy $E_{\rm f}$ and the electron has energy $E = \gamma m_{\rm e} c^2$. Using energy and momentum conservation, please calculate $E_{\rm f}$ in terms of $E_{\rm i}$ and θ. Also show that, to first order, $\frac{E_{\rm i}-E_{\rm f}}{E_{\rm i}} \approx \frac{E_{\rm i}}{m_{\rm e}c^2}(1-\cos\theta)$ when $E_{\rm i} \ll m_{\rm e}c^2$. Note that the scattering is nearly elastic in this case (i.e., Thomson scattering).

4. The synchrotron cooling time for an ultrarelativistic electron. Consider an ultrarelativistic ($\beta \approx 1$, $\gamma \gg 1$) electron emitting synchrotron radiation. This electron will lose energy as a result of its radiation and will thereby 'cool.' By writing down a simple differential equation (for the power radiated) and solving it, show that the electron's energy decreases with time according to

$$\gamma = \gamma_0 (1 + A\gamma_0 t)^{-1}$$

where

$$A = \frac{2e^4 B_\perp^2}{3m_e^3 c^5}.$$

Here γ_0 is the initial value of γ, and $B_\perp$ is $B \sin\alpha$ where α is the angle between the magnetic field and the electron velocity.
Hint: Note that the radiated synchrotron power can be written as $\frac{2}{3} r_0^2 c B_\perp^2 \beta^2 \gamma^2$ by changing your point of view.

Show that the time (in seconds) for the electron to lose half its energy is

$$t_{\text{cool}} = \frac{1}{A\gamma_0} = \frac{5.1 \times 10^8}{\gamma_0 B_\perp^2}.$$

This is an important timescale that you should remember.

5. (a) The inverse process to cyclotron radiation emission is known as cyclotron absorption. The binary X-ray source Hercules X-1 is composed of a neutron star in orbit around a normal star. It shows a strong feature in its spectrum at 34 keV that is thought to be due to electron cyclotron absorption arising in hot gas near the poles of the neutron star. Please estimate the magnetic field strength of the neutron star (in gauss). Also, please numerically compare this magnetic field strength with that near the Earth's surface and that in a sunspot (approximate answers here are acceptable).

5. (b) Highly relativistic cosmic rays appear to arrive at the Earth in an isotropic distribution. These particles must be deflected by Galactic magnetic fields. What is the circular orbit radius of a highly relativistic cosmic ray proton as a function of its energy? At what energy (in eV) does this become larger than the radius of the Galaxy (use 18 kpc for the radius of the Galaxy)? You may assume an interstellar magnetic of 10^{-6} gauss, and you need not worry about angular effects for this problem (i.e. you can use $v_\perp = v$). Compare your derived energy with the kinetic energy of a well hit ping pong ball. There actually are cosmic rays this energetic, so duck next time you see one!

5. (c) Could we hope to make an image and determine the position of an extragalactic cosmic ray source that emits 10^{14} eV protons? Why or why not?

6. (a) In class we derived the Eddington luminosity limit assuming the accretion of pure ionized hydrogen. Show that more generally the Eddington limit can be written as

$$L_{\rm Edd} = \frac{4\pi GMc}{\kappa}$$

where κ is the *mass absorption coefficient*. The mass absorption coefficient has units of cm^2 g^{-1} and is the absorption cross section per unit mass.

6. (b) What is the Eddington limit for a plasma composed entirely of completely ionized helium? Compute the numerical coefficient x in $L_{\rm Edd} = (x \text{ erg s}^{-1})(M/M_\odot)$.

6. (c) What is the Eddington limit for a plasma composed entirely of electron-positron pairs? Compute the numerical coefficient x in $L_{\rm Edd} = (x \text{ erg s}^{-1})(M/M_\odot)$. Note that the positron will also now Thomson scatter photons. The small Eddington limit here is one of the reasons that people think some jets may have large numbers of electron-positron pairs.

6. (d) What is the (pure ionized hydrogen) Eddington limit for an ≈ 16 M$_\odot$ black hole (like that in Cygnus X-1), an $\approx 2.6 \times 10^6$ M$_\odot$ black hole (like that in the Galactic center), and an $\approx 1 \times 10^9$ M$_\odot$ black hole (like that in a luminous quasar).

6. (e) Do you think the Eddington limit is relevant to the creation of the solar wind? Why or why not?

7. Terminal velocity of a radiation driven outflow. Now please reconsider the situation described in part (a) of the previous problem. If the luminosity is greater than the Eddington luminosity then a cloud of material placed in the system will be ejected. Taking this cloud to start from rest at a distance R from the central object, please calculate the cloud's terminal velocity ($v_{\rm t}$) in terms of G, M, R, L, c, κ, and numerical constants.

Hint: The answer is $v_{\rm t} = \sqrt{\frac{2GM}{R}\left(\frac{L\kappa}{4\pi GMc} - 1\right)}$.

8. The relativistic Doppler effect. Above we calculated the beaming effect which will be important throughout this course. Another important effect to understand is the relativistic Doppler effect. Consider an excited atom flying through a laboratory that emits a photon of frequency ν' (ignore any recoil of the atom). This atom is moving with velocity $\boldsymbol{\beta}$. What frequency ν will a scientist in the laboratory frame measure for the photon? Express your answer in terms of $\boldsymbol{\beta}$, γ, ν' and the unit vector $\boldsymbol{n}$ pointing from the atom toward the scientist at the time it emitted the photon. You will need to think of the photon as a wave and take account of the motion of the atom during the emission of the wave (in particular, the motion of the atom between the emission of one wave crest and the subsequent wave crest).

9. Based on the assigned reading for Week 3, please clearly answer the following questions. Show equations or draw pictures if needed, but long calculations are *not* needed.

9. (a) What did J.J. Thomson use the Thomson cross section for in 1906?

9. (b) Why is it not physically surprising that the total Thomson scattering cross section for unpolarized radiation is the same as that for polarized radiation? Are the differential cross sections the same?

9. (c) How does the fractional polarization of Thomson scattered radiation depend upon the scattering angle α?

9. (d) What is the Thomson mean free path (λ_{T}) in an ionized active galaxy accretion disk where $n_{\mathrm{e}} = 1 \times 10^{11}\ \mathrm{cm}^{-3}$?

9. (e) If you make a polarization map of an obscured source that we can only see indirectly via Thomson scattering, how will the polarization vectors be arranged around the source?

9. (f) Consider firing high-energy (40–400 keV) X-rays at an unionized atom with atomic number Z. What will be the total cross section for the atom? Why does the nucleus not scatter X-rays effectively?

9. (g) What is Comptonization?

9. (h) Why is pair production impossible for an isolated single photon in free space?

9. (i) What is the spectral index of a power-law spectrum? What is the effective spectral index in the Rayleigh-Jeans portion of the blackbody law?

9. (j) Consider an ensemble of electrons emitting synchrotron radiation. For a power-law distribution of electron energies, is the resulting synchrotron spectrum also a power law?

10. In class we discussed a supersonic shock moving through cold upstream gas. We worked out equations for mass conservation, momentum conservation, and energy conservation. By combining these equations, show that the physically interesting solution to them is $\frac{\rho_{\mathrm{d}}}{\rho_{\mathrm{u}}} = 4$ and $\frac{v}{u} = 4$. Do *not* simply plug this answer into the equations and reduce them to identities!