1. Consider the matrix
\[
A = \begin{bmatrix}
1 & 1 & 2 \\
2 & 3 & 3 \\
1 & -1 & 1 \\
\end{bmatrix}
\]
(a) [10] Compute an LU decomposition for \( A \).
(b) [10] What is the determinant of \( A \)?

2. Consider the matrix
\[
B = \begin{bmatrix}
-1 & 1 \\
3 & 1 \\
\end{bmatrix}
\]
(a) [12] Find the eigenvalues and eigenvectors of \( B \).
(b) [8] Give a diagonalization of \( B \) (i.e. find \( P \) and \( D \) so \( B = PDP^{-1} \), where \( D \) is diagonal).

3. Row reduction on a matrix \( C \) yielded the echelon form
\[
U = \begin{bmatrix}
1 & -2 & 0 & 0 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
(a) [5] I’m not going to tell you the original matrix \( C \). You still have enough information to find 2 out of the 3 spaces Row \( C \), Col \( C \), and Nul \( C \). Which ones?
(b) [10] For each of the spaces in your answer to (a), give a basis. What are the dimensions of the three subspaces? (Include the dimension of the one for which you did not find a basis.)
(c) [10] Let \( B \) be the basis for Row \( C \) that you found in part (b). Another basis \( C \) is given by the two vectors
\[
c_1 = \begin{bmatrix}
1 \\
-2 \\
3 \\
\end{bmatrix}, \quad c_2 = \begin{bmatrix}
0 \\
0 \\
1 \\
\end{bmatrix}.
\]
Find the change of basis matrix \( P_{B\leftarrow C} \).

4. (a) [10] Use cofactor expansion to compute the determinant of
\[
E = \begin{bmatrix}
1 & 1 & 2 \\
-1 & -2 & -2 \\
2 & 0 & 4 \\
\end{bmatrix}
\]
(Half credit if you use some other method instead.)
(b) [5] What does your answer to (a) tell you about the columns of $E$?

5. (a) [10] A Markov process is defined by the stochastic matrix

$$M = \begin{bmatrix} 0.6 & 0.1 \\ 0.4 & 0.9 \end{bmatrix}.$$ 

What is $\lim_{n \to \infty} M^n x_0$ if $x_0 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$? (In other words, what vector does $M^n x_0$ approach when $n$ is very large?)

(b) [10] Consider the three polynomials

$$p_1(t) = 1 + t, \quad p_2(t) = 1 + 2t, \quad p_3(t) = 2 + 3t.$$ 

Are these three polynomials linearly independent? Justify your answer. (Hint: think about the coordinate vectors for these polynomials in some basis.)