- Can't register / stuck in this section?

  email me + UN: jdl@uic.edu

- HW problems this week posted on site.
A 3×3 system of eqns.

\[ x + y + z = 6 \]
\[ x - 2y + z = 0 \]
\[ z - y = 1 \]

"Augmented matrix"

\[
\begin{pmatrix}
1 & 1 & 1 & | & 6 \\
1 & -2 & 1 & | & 0 \\
0 & -1 & 1 & | & 1 \\
\end{pmatrix}
\]

Idea: eliminate \( x \) from 2\(^{nd} \) and 3\(^{rd} \) eqns.

Eliminate \( y \) from 3\(^{rd} \) eqn.

Then we'll be almost done!

\[
\begin{pmatrix}
1 & 1 & 1 & | & 6 \\
1 & -2 & 1 & | & 0 \\
0 & -1 & 1 & | & 1 \\
\end{pmatrix}
\] subtract row 1 from row 2 \( \rightarrow \)

\[
\begin{pmatrix}
0 & 0 & 0 & | & 6 \\
0 & -30 & 6 & | & 0 \\
0 & -11 & 18 & | & 1 \\
\end{pmatrix}
\] multiply row 2 by \( -\frac{1}{3} \) \( \rightarrow \)

\[
\begin{pmatrix}
1 & 1 & 1 & | & 6 \\
0 & 10 & 2 & | & 0 \\
0 & -1 & 1 & | & 1 \\
\end{pmatrix}
\]

i) \( x + y + z = 6 \)

\[ x + y + z = 6 \]
\[ y = 2 \]
\[ z = 1 \]

ii) \( x - 2y + z = 0 \)

\[ x - 2y + z = 6 \]
\[ -3y = -6 \]
\[ -y + z = 1 \]

Subtract \( \text{ii) from i) } \)

\[ -3y = -6 \]
\[ -y + z = 1 \]

\[ x - y + z = 6 \]
\[ y = 2 \]
\[ z = 1 \]

Add row 2 to row 3 \( \rightarrow \)

\[
\begin{pmatrix}
1 & 1 & 1 & | & 6 \\
0 & 10 & 2 & | & 0 \\
0 & 0 & 1 & | & 1 \\
\end{pmatrix}
\] subtract row 3 from row 2 \( \rightarrow \)

\[
\begin{pmatrix}
1 & 1 & 1 & | & 6 \\
0 & 0 & 1 & | & 2 \\
0 & 0 & 0 & | & 3 \\
\end{pmatrix}
\] subtract row 2 from row 1 \( \rightarrow \)

\[
\begin{pmatrix}
1 & 0 & 0 & | & 1 \\
0 & 0 & 1 & | & 2 \\
0 & 0 & 0 & | & 3 \\
\end{pmatrix}
\]

\[ x = 1 \]
\[ y = 2 \]
\[ z = 3 \]

this is "triangular"

means \( x \) is eliminated from eqn 2 & 3

\( y \) is eliminated from 3
solving a general system of linear eqns (§1.2)

two main steps.

1. simplify as much as possible using row operations.
   ("eliminating variables"
   "put in echelon form/reduced echelon form")

2. find all solutions to simplified equations

example:
\[
\begin{align*}
x + 3z &= 1 \\
x + ty + 2z &= 1
\end{align*}
\Rightarrow
\begin{align*}
x + 3z &= 1 \\
y - z &= 0 \\ & \quad \text{can't simplify further.} \\
\end{align*}
\]
\( \hat{\text{1 done.}} \)

\( z \) can have any value.

Once we choose a value of \( z \), values of \( x \& y \)
are forced on us.

Infinitely many solutions!

Solution is
\[
\begin{align*}
x &= 1 - 3z \\
y &= z \\
z & \text{is free}
\end{align*}
\]
plug in any \( z \), get a sol!

\( \checkmark \) can be anything
Elementary row operations

algebraic manipulations of eqns \[\leftrightarrow\] changes in rows of matrix

three legal changes:

I. adding multiple of one eqn to another eqn \[\leftrightarrow\] add multiple of a row of matrix to another row.

II. multiply equation by a constant \[\leftrightarrow\] multiply row by a number

III. switch two equations \[\leftrightarrow\] swap two rows of matrix.

What's the point of III?

want first eqn to tell us x, second to tell us y, ...

e.g. \[y=1 \implies 2x+3y=6\]
\[2x+3y=6 \quad y=1\]

goal. given any system of linear eqns, systematically use row operations to get to eqn we can solve.
A matrix is in "echelon form" if

1) any rows of all Os are at the bottom
2) the leading entry of each row (first nonzero thing) is to right of leading entry of row above
3) all entries in a column below leading entries of a row are 0

"reduced echelon form" means

4) leading entry of every row is 1
5) each leading 1 is the only nonzero thing in its column.

Idea: use row operations, get matrix in echelon form. System of equations can get any simpler!

echelon:

\[
\begin{pmatrix}
1 & 2 & 3 & 4 & | & 1 \\
0 & 0 & 2 & 1 & | & 2 \\
0 & 0 & 0 & 5 & | & 3 \\
0 & 0 & 0 & 0 & | & 0 \\
\end{pmatrix}
\]

these leading entries are called "pivot entries".

not reduced!

this one is:

\[
\begin{pmatrix}
1 & 2 & 0 & 0 & 1 & | & 3 \\
0 & 0 & 0 & 3 & | & 4 \\
0 & 0 & 0 & 0 & | & 2 \\
\end{pmatrix}
\]

3) means: if some eqn in subsequent list involves x, listed equations don't! so not/simplifying.
how to put a matrix in reduced echelon form "rref"

1. start with leftmost nonzero column
2. pick a nonzero number in column, do row exchange to put at top.
   ("move an equation with an x in it to the top")
3. subtract multiples of top row from all other rows to make 0s below top entry.
   ("eliminate x from all eqns after the first")
4. cover up first eqn in your list, repeat steps 1-3 with remaining equations.
   ("eliminate y from equations after the second")
   (will be in echelon form)
5. start with rightmost pivot, and create 0s above pivot by subtracting multiples of this row from rows above.
next time:

1) example of rref

2) finding all solutions from rref.