Announcements

- Piazza up and running!  
  (I hope)

- class capacity now 85  
  (but full)

- #1.2.4 fixed in typed HW

- Will add some problems from 1.3; won't be on quiz.

  - next HW will have a few more problems
    than this one did.

  HW due W, quiz on W.

  covers previous MWF; will post all by W.
Finding general solution from ref.

A variable not in a pivot column is called a "free variable."

**General solution:**
- Free variables can take any value
  (plug in any number you want)
- Other variables determined by the free ones,
  using eqns from ref.

**Example:**
ref: \[
\begin{pmatrix}
0 & 0 & 3 & 1 \\
\infty & 1 & 2 & 0
\end{pmatrix}
\] \Rightarrow x, y are pivot variables / basic variable
\[z\] is free.

\[\begin{align*}
x + 3z &= 1 \\
x &= 1 - 3z \\
y - 2z &= 2
\end{align*}\]

\[\begin{align*}
x &= 1 - 3z \\
y &= 2 + z \\
z &= \text{free}
\end{align*}\]

To get a solution, plug in \(z = 10\).

then \(x = 1 - 3(10) = -29\)
\(y = 2 + (10) = 12\).

\(x = -29, y = 12, z = 10\) is a solution.
3 equations, five variable

\[
\begin{pmatrix}
1 & 2 & 1 & 4 & 1 & \mid & 4 \\
1 & 2 & 2 & 7 & -2 & \mid & -3 \\
2 & 4 & -1 & -1 & 0 & \mid & 7 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 2 & 1 & 4 & 1 & \mid & 4 \\
0 & 0 & 1 & 3 & -3 & \mid & -7 \\
2 & 4 & -1 & -1 & 0 & \mid & 7 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 2 & 1 & 4 & 1 & \mid & 4 \\
0 & 0 & 1 & 3 & -3 & \mid & -7 \\
0 & 0 & 0 & 0 & 0 & \mid & 7 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 2 & 1 & 4 & 1 & \mid & 4 \\
0 & 0 & 1 & 3 & -3 & \mid & -7 \\
0 & 0 & 0 & 0 & 0 & \mid & 7 \\
\end{pmatrix}
\]

The linear system corresponding to rref is:

\[x_1 + 2x_2 + x_4 = 3\]
\[x_3 + 3x_4 = -1\]
\[x_5 = 2\]

The free variables are? The basic variables are?

**Basic:** \(x_1, x_3, x_5\)  \(Free:\) \(x_2, x_4\)

The general solution is?

\[
\begin{align*}
x_1 &= 3 - 2x_2 - x_4 \\
x_2 &= \text{free} \\
x_3 &= -1 - 3x_4 \\
x_4 &= \text{free} \\
x_5 &= 2 \\
\end{align*}
\]

E.g. plug in \(x_2 = 1, x_4 = 2\)

\[
\begin{align*}
x_1 &= -1 \\
x_3 &= -7 \\
x_5 &= 2 \\
\end{align*}
\]

plug this into equations:

it works!
how many solutions?

0, if ref has a row that looks like
\[ \begin{array}{cccc}
0 & 0 & 0 & 0 \\
\end{array} \]
\[ \text{anything not 0} \]
no solutions to system.

\[ \infty \] solutions, if there's a free variable.

1 solution: if no free vars, and no row \( \begin{array}{cccc}
0 & 0 & 0 & 0 \\
\end{array} \).

the system is "consistent" if there's 1 or \( \infty \) solutions.
not consistent if no sols.

How to check if consistent?

→ figure out ref. is there a row \( \begin{array}{cccc}
0 & 0 & 0 & 0 \\
\end{array} \)?

yes: not consistent
no: consistent
Vectors

A vector is a matrix with only one column.

\[ \mathbf{u} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \]

A vector with \( n \) entries \( \Rightarrow \) "a vector in \( \mathbb{R}^n \)"

The "\( \mathbb{R} \)" is in a funny font: \( \mathbb{R} \)

If two vectors are the same size, we can add them.

E.g. \( \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \)

**WARNING**

- Vector + vector (same size) is OK ✓
- Vector + vector (different size) nonsense!
- Number × vector OK ✓
  
  E.g. \( 5 \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix} \)
- Number + vector doesn't make sense.
  (usually)
- Vector × vector usually doesn't make sense
  (except cross product for 3-dim vectors)
Think of a vector as a point in 2D or 3D space.

Similarly in 3D.

\[ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \]

\[ \begin{pmatrix} 2 \\ 3 \end{pmatrix} \]

\[ \begin{pmatrix} 1 \\ 2 \end{pmatrix} \]

\[ \begin{pmatrix} 3 \\ 2 \end{pmatrix} \]

\[ \vec{u} + \vec{v}, \text{ geometrically} \]

\[ \vec{u} = (2) \]

\[ \vec{v} = (1) \]

draw a parallelogram

\[ \vec{u} + \vec{v} = (3) \]

\[ 2\vec{u} = (6) \]

\[ C\vec{u}, \text{ geometrically} \]

\[ \vec{u} = (2) \]

number vector

\[ 2\vec{u} = (4) \]