1. Consider the matrix \( A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix} \).

(a) What are the eigenvalues of \( A \)?
(b) What are the eigenvectors of \( A \)?
(c) Find a diagonalization of the matrix \( A \).
(d) Compute the matrix \( A^5 \).

2. The matrix \( B \) has reduced echelon form \( U \), where
\[
B = \begin{bmatrix}
1 & 1 & 2 & 0 & 0 \\
0 & 1 & 1 & -1 & -1 \\
1 & 1 & 2 & 1 & 2 \\
2 & 1 & 3 & -1 & -3
\end{bmatrix}, \quad U = \begin{bmatrix}
1 & 0 & 1 & 0 & -1 \\
0 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 2 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}.
\]

(a) Give a basis for \( \text{Row } B \). What is the dimension?
(b) Give a basis for \( \text{Col } B \). What is the dimension?
(c) Give a basis for \( \text{Nul } B \). What is the dimension?
(d) For what values of \( a \) and \( b \) does the matrix
\[
A = \begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
0 & 0 & a & 2 & 2 \\
0 & 0 & 0 & b & 2
\end{bmatrix}
\]
have rank 2?

3. Let \( \mathcal{E} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \) be the standard basis for \( \mathbb{R}^2 \).

(a) Show that \( \mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right\} \) is another basis for \( \mathbb{R}^2 \).
(b) Write down the change of basis matrix \( \mathcal{P}_{\mathcal{E} \leftarrow \mathcal{B}} \) from \( \mathcal{B} \) to the standard basis \( \mathcal{E} \).
(c) Find the \( \mathcal{B} \)-coordinate vector \( [\mathbf{v}]_\mathcal{B} \) of the vector \( \mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \).

4. Consider the matrix
\[
C = \begin{bmatrix}
1 & 2 & 3 \\
0 & 2 & 2 \\
0 & 0 & 2
\end{bmatrix}.
\]
(a) What is the dimension of the eigenspace corresponding to the eigenvalue 1? (You
do not need to compute a basis.)

(b) What is the dimension of the eigenspace corresponding to the eigenvalue 2? (You
do not need to compute a basis.)

(c) Explain why the matrix $C$ is not diagonalizable.

5. (a) Find an LU decomposition of the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 3 & 7 & 3 & 3 \\ 2 & 2 & 3 & 4 \end{bmatrix}.$$

(b) Use your LU decomposition to find a solution to $Ax = \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix}$.

6. Consider the matrix

$$A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 1 & 1 \\ 0 & 3 & 0 \end{bmatrix}.$$

(a) Compute the determinant of $A$ using row reduction.

(b) Compute the determinant of $A$ using cofactor expansion.

(c) Use Cramer’s rule to solve

$$Ax = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

for $x_3$. (You do not need to solve for $x_1$ and $x_2$.)