Problems for M 11/9:

5.7.1 A particle moving in a force field has a position vector $x$ that satisfies $x' = Ax$. The $2 \times 2$ matrix $A$ has eigenvalues 4 and 2, with corresponding eigenvectors $v_1 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$. Find the position of the particle at time $t$, assuming that $x(0) = \begin{bmatrix} -6 \\ 1 \end{bmatrix}$.

5.7.7 Find a change of variable that decouples the equation $x' = Ax$. Write the equation $x(t) = Py(t)$ and show the calculation that leads to the uncoupled system $y' = Dy$, specifying $P$ and $D$.

$$A = \begin{bmatrix} 7 & -1 \\ 3 & 3 \end{bmatrix}$$

5.7.9 Construct the general solution of $x' = Ax$ involving complex eigenfunctions and then obtain the general real solution.

$$A = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix}$$

Problems for F 11/13:

6.1.1 Let $u = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $v = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$. Compute $u \cdot u$, $u \cdot v$, and $\frac{v \cdot u}{u \cdot u}$.

6.1.14 Find the distance between the two vectors

$$\begin{bmatrix} 0 \\ -5 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} -4 \\ -1 \\ 8 \end{bmatrix}.$$ 

6.1.16 Are the following two vectors orthogonal?

$$\begin{bmatrix} 12 \\ 3 \\ -5 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix}.$$ 

6.1.26 Let $u = \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$, and let $W$ be the set of all vectors $x$ with $u \cdot x = 0$. Explain why $W$ is a subspace of $\mathbb{R}^3$, and describe it geometrically.