

Cosmology, the Mészáros effect and Black Holes

In cosmology, the development of initial perturbations which eventually give rise to galaxies, galaxy clusters, etc. is described in the fluid approximation by perturbing the Friedmann equations. The density contrast $\delta \equiv \Delta\rho/\langle\rho\rangle$ (where ρ is matter-energy density and $\Delta\rho$ is the small excess in any given region) obeys the perturbation equation

$$\ddot{\delta} + 2\left(\frac{\dot{a}}{a}\right)\dot{\delta} + (v_s^2 k^2 - 4\pi G\rho)\delta = 0. \quad (1)$$

e.g. [1, 2, 3], where a is the Universe’s expansion scale factor, $v_s = (\partial P/\partial\rho)^{1/2}$ is the sound speed, k is wavenumber, G is Newton’s constant. For perturbations inside the horizon this leads to well-known results for the Jeans mass and growth times for non-relativistic collisionless perturbations (cold “matter” $P = 0$) and for relativistic collisional perturbations (e.g. a “radiation gas” $P = \rho c^2/3$). In the early Universe the $P = 0$ case corresponds to what is known today as **cold dark matter**, and the radiation is photons and neutrinos. After radiation-matter decoupling $z < z_{dec} \sim 10^3$ the perturbation growth is given by the $P = 0$ matter-dominated solution, while before the matter-radiation equilibrium epoch $z < z_{eq} \sim 10^4$ the growth is given by the radiation-dominated solution.

However, in a combined picture of collisionless matter in a radiation background, inside the horizon there are modes where the collisionless non-relativistic dark matter component of density ρ_m is perturbed relative to the relativistic radiation component of density ρ_r (which for $z > z_{dec}$ oscillates and on average can be considered as unperturbed and just following the Universe’s expansion). This was first considered in [4], leading to the perturbation equation

$$\ddot{\delta} + 2\left(\frac{\dot{a}}{a}\right)\dot{\delta} - (4\pi G\rho_m)\delta = 0, \quad (2)$$

where now $(\dot{a}/a)^2 = 8\pi G(\rho_m + \rho_r)/3$, and $k = 0$ is appropriate for early times. Changing variables to $y = \rho_m/\rho_r \equiv (a/a_{eq})$ and using Friedman’s equation leads to [4]

$$\delta'' + \frac{2+3y}{2y(1+y)}\delta' - \frac{3}{2y(1+y)}\delta = 0. \quad (3)$$

This equation has closed analytical solutions, the growing mode of which can be seen to be

$$\delta \propto 1 + \frac{3}{2}y. \quad (4)$$

Thus for $a < a_{eq}$ the cold matter perturbations remain “frozen”, $\delta = \text{constant}$, while for $a > a_{eq}$ they grow linearly with $y = a/a_{eq}$. This is referred to as the Mészáros effect, or Mészáros equation [5]; e.g. Google these keywords, or see, e.g. [1, 2, 3, 6, 7, 8, 9, 10, 11]. The importance of the effect is that it is the main process altering the initial perturbation spectrum, affecting the transfer function, and allowing the initial perturbations in the cold dark matter to achieve non-linearity, leading to the observed galaxies and clusters at the present epochs. This is independent of whether the dark matter consists of elementary particles or other (macroscopic) objects.

A follow-up paper [12] considered the possibility of the dark matter consisting of solar mass primordial black holes arising before the nucleosynthesis era, and the effects these would have on galaxy formation. The discovery of gravitational waves from stellar mass black holes after 2015 led to a renewed interest in the possibility of such stellar mass black holes as dark matter, e.g. [13].

References

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